Appendix A – Calculation of electrical performance indicators

This appendix is an example of the calculus regarding electrical performance indicators from a single electrode level to a reactor, or in the last phases of experiments, sub-reactor level. Therefore, this calculus will include voltage, current, power, power density, coulombic efficiency, pump energy consumption, normalised energy recovery and energy balance.

Voltage, current and power are calculated at a single electrode level and by Ohm's law, for the single electrode R_1, part of the multi-electrode MFC,

$$I_1 = \frac{V_1}{R_{ext}}$$

Where V_1 is the voltage (V-volts) measured across the electrode, R_{ext} is the external load (Ω-ohms) posed across the electrode and I_1 is the calculated current (A-amps) flowing across the circuit. Therefore, if for R_1 the voltage recorded under R_{ext}=1 **Ω**, is 0.317 V then,

$$I_1 = \frac{0.317\ V}{1\ \Omega} = 0.317\ A$$

Then the following table summarizes the example for the remaining electrodes assuming voltage is the value measured and external resistance is the known load.

Voltage V_i (V)	**Rext (Ω)**	**Current I_i (A)**
0.317	1	0.317
0.359	1	0.359
0.315	1	0.315
0.403	1	0.403

By Ohm's law, for the single R_1 electrode,

$$P_1 = V_1 \times I_1 = 0.317\ V \times 0.317\ A = 0.100489\ W$$

Where P_1 is the generated power (W-watts). These values are similarly calculated for the remaining three electrodes. The summary table is extended as following.

	Voltage V_i (V)	**Rext (Ω)**	**Current I_i (A)**	**Power P_i (W)**
R_1	0.317	1	0.317	0.100489
R_2	0.359	1	0.359	0.128881
R_3	0.315	1	0.315	0.099225
R_4	0.403	1	0.403	0.162409
MFC_{R1+R4}				0.491004

On a reactor level, the power *P* is the sum of the power generated from every electrode,

$$P = P_1 + P_2 + P_3 + P_4$$
$$= 0.100489\ W + 0.128881\ W + 0.099225\ W + 0.162409\ W$$
$$= 0.491004\ W$$

The total chemical oxygen demand consumption $COD_{consumed}$ was monitored on an overall reactor level with the use of tCODinfluent, tCODeffluent and volumetric flow rate. In the example used here, $tCOD_{influent}$=1870 mg/l and $tCOD_{effluent}$=195 mg/l, and the volumetric flow rate Q=1.5x10^{-7} m^3/s.

$$COD_{consumed} = Q \times (tCOD_{influent} - tCOD_{effluent})$$
$$= 1.5 \times 10^{-7} \frac{m^3}{s} \times \left(1870 \frac{mg}{l} - 195 \frac{mg}{l}\right) = 0.25125\ mg/s$$

Since it refers to the reactor as an overall, an assumption has to be made; the removal efficiency is equally distributed on each electrode, therefore, the tCOD removal efficiency referring to each electrode is,

$$COD_{consumed\mathbf{1}} = COD_{consumed\mathbf{2}} = COD_{consumed\mathbf{3}} = COD_{consumed\mathbf{4}}$$
$$= {COD_{consumed}}/{4} = {0.25125\ mg/s}/{4} = 0.062815\ mg/s$$

Coulombic efficiency *CE* (%) for single electrode R_1 is calculated based on *M* which is the molecular weight of the electron acceptor, which in the case of an open air cathode MFC is oxygen and therefore M=32, I_1 which is the current across the electrode R_1 previously calculated in Amps, *F* which is Faraday's constant and is 96,485.3565 C/mol, *b* which represents the amount of electrons exchanged per mole of electron acceptor and in the case of oxygen, b=4 is the number of number of electrons exchanged per mole of oxygen, and $COD_{consumed\mathbf{1}}$ is the COD consumed as calculated above. Therefore, for the single electrode R_1,

$$CE_1 = \frac{MI_1}{FbCOD_{consumed\mathbf{1}}}\% = \frac{32 \times 0.317\ A}{96485.3565\ \frac{C}{mol} \times 4 \times 0.062815\ \frac{mg}{s}}\%$$

$$= 41.84\ \%$$

On a reactor level coulombic efficiency is calculated by replacing the current of each electrode with the sum of the four electrodes and the COD consumed per electrode by the consumption for the overall reactor. The summary of the table is therefore expanded as following.

	Voltage V_i (V)	**Rext (Ω)**	**Current I_i (A)**	**Power P_i (W)**	**Coulombic efficiency CE_i (%)**
R_1	0.317	1	0.317	0.100489	41.84
R_2	0.359	1	0.359	0.128881	47.39
R_3	0.315	1	0.315	0.099225	41.58
R_4	0.403	1	0.403	0.162409	55.02
MFC_{R1+R4}				0.491004	46.00

Power density P_d (W/m^3) is also a parameter calculated on a reactor level based on the sum of the power produced from all four electrodes as previously calculated normalised over the volume of the anodic compartments which for the 122 L reactor is 48.7 L. Therefore, power density is calculated as following.

$$P_d = \frac{P_{MFC_{R1+R2}}}{V_{an}} = \frac{0.491004\ W}{48.7\ L} = 10.082218\ W/m^3$$

Normalised energy recovery is calculated on a reactor level too based on the sum of the power generated from the reactor and the volumetric flow rate (m^3/h) as,

$$NER = \frac{P_{MFC_{R1+R2}}}{Q} = \frac{0.491004\ W}{0.00054\ \frac{m^3}{h}} = 0.909267\ {}^{kWh}/{}_{m^3}$$

As it has been described in this study, the only source of energy consumption in the set-up was the consumption of the pump which is calculated as following.

$$\mathrm{P_{pumping}} = \frac{\mathrm{Q\times\gamma\times E}}{1000}$$

Where $P_{pumping}$ is the power requirement (kW), Q is the flow rate which in this case is 1.5 x 10^{-7} m^3/s, γ is 9800 N/m^3, and E is the hydraulic pressure head which for this reactor was at 1.20 m. Therefore, that is calculated as,

$$P_{pumping} = \frac{1.5 \times 10^{-7}\ \frac{m^3}{s}\ \times 9800\ \frac{N}{m^3}\ \times 1.20\ m}{1000} = 0.0033\ {}^{kWh}/{}_{m^3}$$

The energy balance (kWh/m^3) for the system is therefore, calculated by abstracting the power requirement of the pump from the energy produced from the MFC, referred to NER, and is calculated as,

$$Energy\ balance = NER - P_{pumping} = 0.909267\ {}^{kWh}/{}_{m^3} - 0.0033\ {}^{kWh}/{}_{m^3} = 0.905967\ {}^{kWh}/{}_{m^3}$$

The table of calculus is therefore finalised as following.

	Voltage V_i (V)	**Rext (Ω)**	**Current I_i (A)**	**Power P_i (W)**	**Coulombic efficiency CE_i (%)**	**Power density P_d (W/m^3)**	**NER (kWh/m^3)**	**$P_{pumping}$ (kWh/m^3)**	**Energy balance (kWh/m^3)**
R_1	0.317	1	0.317	0.100489	41.84				
R_2	0.359	1	0.359	0.128881	47.39				
R_3	0.315	1	0.315	0.099225	41.58				
R_4	0.403	1	0.403	0.162409	55.02				
MFC_{R1+R4}				0.491004	46.00	10.082218	0.909267	0.0033	0.905967

When the 122 L reactor is broken down into two identical sub-reactors the only difference in calculations is that the parameters referring to the sub-reactor level are the sum of two electrodes instead of four. Therefore, the principle of calculus described above applies throughout the study.