Appendix-B

Validation to Cinco-Ley's solution

(Mass-in / Time) – (Mass-out / Time) = (Mass-Accumulation / Time)

$$(q_x \cdot \rho_x) - (q_{x+\Delta x} \cdot \rho_{x+\Delta x}) + 2u\,(h \cdot \Delta x) \cdot \rho_y = \; q_{\Delta x} \cdot \rho_{\Delta x}$$

Multiply by Δt:

$$(q_x \cdot \Delta t) + (2u\,) \cdot (h \cdot \Delta x) \cdot \Delta t - (q_{x+\Delta x} \cdot \Delta t) = \; q_{\Delta x} \cdot \Delta t$$

$$(q_x \cdot \Delta t) - (q_{x+\Delta x} \cdot \Delta t) = \Delta v$$

Introduce the compressibility equation to MBE:

Where: $C = -\frac{1}{v}\,\frac{\Delta v}{\Delta p} \;\rightarrow\; \Delta v = cv\,\Delta p$

$$(q_x - q_{x+\Delta x}) \cdot (\Delta t) + (2u\,) \cdot (h \cdot \Delta x) \cdot \Delta t = cv\,\Delta p = c \cdot (w_f \cdot h \cdot \Delta x \cdot \varphi) \cdot (p_{t+\Delta t} - p_t)$$

Divide by $(\Delta x \cdot \Delta t)$ and take limit as $\Delta x \rightarrow 0$ & $\Delta t \rightarrow 0$:

$$-\lim_{\Delta x \to 0}\left(\frac{q_{x+\Delta x} - q_x}{\Delta x}\right) + (2u\,) \cdot (h) = c \cdot (w_f \cdot h \cdot \varphi) \lim_{\Delta t \to 0}\left(\frac{p_{t+\Delta t} - p_t}{\Delta t}\right)$$

Equation 1

$$-\frac{dq}{dx} + (2u) \cdot (h) = c \cdot (w_f \cdot h \cdot \varphi)\frac{dp}{dt} \tag{1}$$

Introduce Darcy's law:

$$q = -\frac{k_f A}{\mu} \cdot \frac{dp_f}{dx},\; along\ fracture$$

Differentiate both sides w.r.t. x:

$$\frac{d}{dx} \cdot q = -\frac{k_f A}{\mu} \cdot \left(\frac{d}{dx} \cdot \frac{dp_f}{dx}\right)$$

Equation 2

$$\frac{dq}{dx} = -\frac{k_f A}{\mu} \cdot \left(\frac{d^2 p_f}{dx^2}\right) \tag{2}$$

Substitute (39) in (38):

$$\frac{k_f A}{\mu} \cdot \left(\frac{d^2 p_f}{dx^2}\right) + (2u) \cdot (h) = c \cdot (w_f \cdot h \cdot \varphi)\frac{dp_f}{dt}$$

Since, $A = w_f \cdot h$, then:

$$\frac{k_f}{\mu} \cdot w_f\left(\frac{d^2 p_f}{dx^2}\right) + (2u) = c \cdot (w_f \cdot \varphi)\frac{dp_f}{dt}$$

Re-arrange:

Two dimensional diffusivity equations for fluid flowing in fracture plane per unit area

$$\frac{\partial^2 p_f}{\partial x^2} + (2u) \cdot \frac{\mu}{w_f \cdot k_f} = \frac{c\mu\varphi}{k_f} \cdot \left(\frac{\partial p_f}{\partial t}\right)$$

Or

Equation 3

$$\frac{\partial^2 p_f}{\partial x^2} + (2u) \cdot \frac{\mu}{w_f \cdot k_f} = {}^1\!/_{\eta_f} \cdot \left(\frac{\partial p_f}{\partial t}\right) \tag{3}$$

Now, solve for (u') from Darcy's equation:

Equation 4

$$\mathrm{u}' = \frac{\mathrm{q}}{\mathrm{A}} = -\frac{\mathrm{k}}{\mu} \cdot \left.\frac{\mathrm{dp}}{\mathrm{dy}}\right|_{\mathrm{y}\to 0}, \tag{4}$$

Substitute (41) into equation (40) and re-arrange:

Equation 5

$$\frac{\partial^2 \boldsymbol{p_f}}{\partial \boldsymbol{x}^2} + \frac{\mathbf{2}\boldsymbol{k}}{\boldsymbol{k_f} \cdot \boldsymbol{w_f}} \left.\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{y}}\right|_{\boldsymbol{y}=\mathbf{0}} = \mathbf{1}/\boldsymbol{\eta_f} \cdot \left(\frac{\partial \boldsymbol{p_f}}{\partial \boldsymbol{t}}\right) \tag{5}$$

Where: ${}^1\!/_{\eta_f} = C \cdot \frac{c_{tf} \cdot \varphi_f \cdot \mu}{k_f}$, $C = 0.000264$

Substituting to dimensionless variables Equation 5, as detailed above, will produce Cinco-Ley's Finite Conductivity Fracture Solution Cinco et al. (1978)) for:

- **<u>Symmetric</u>** system on both sides of the fracture $k_1 = k_2$,
- Accounts for **<u>linear</u>** flow along **<u>y-plane</u>** only:

Equation 6

$$\frac{\partial^2 \boldsymbol{p_{Df}}}{\partial \boldsymbol{x_D}^2} + \frac{\mathbf{2}}{(\boldsymbol{k_f} \cdot \boldsymbol{w_f})_{\boldsymbol{D}}} \left.\frac{\partial \boldsymbol{p_D}}{\partial \boldsymbol{y_D}}\right|_{\boldsymbol{y_D}=\mathbf{0}} = \mathbf{1}/\boldsymbol{\eta_{fD}} \cdot \left(\frac{\partial \boldsymbol{p_{Df}}}{\partial \boldsymbol{t_{Dxf}}}\right) \tag{6}$$

Where: $\left(k_f\, w_f\right)_D = \frac{k_f\ w_f}{k_r\, x_f}$

While the FracFault-model assumes:

- **<u>Asymmetric</u>** reservoir $k_1 \neq k_2$,
- Accounts for flow on **<u>x-y plane</u>**:

$$\frac{\partial^2 \boldsymbol{p_{Df}}}{\partial \boldsymbol{x_D}^2} + \frac{\mathbf{1}}{\boldsymbol{F_{CDf}}} \left[(\boldsymbol{k_{D2}}) \cdot \left.\frac{\partial \boldsymbol{p_{D2}}}{\partial \boldsymbol{y_D}}\right|_{\boldsymbol{y_D}=\mathbf{0}} - (\boldsymbol{k_{D1}}) \cdot \left.\frac{\partial \boldsymbol{p_{D1}}}{\partial \boldsymbol{y_D}}\right|_{\boldsymbol{y_D}=\mathbf{0}} \right] = \mathbf{1}/\boldsymbol{\eta_{Df}} \cdot \frac{\partial \boldsymbol{p_{Df}}}{\partial \boldsymbol{t_{Df}}}$$

Where: $\left(k_f\, w_f\right)_D = \frac{k_f\ w_f}{k_r\, r_w}$