Appendix B: Determination of Measurement Errors.

In order to measure error at the 95% confidence level, a set of equation associated with each property has been developed based on procedure outlined by Doebelin (1966). Only the general equation and procedure is presented here, the original reference should be consulted for mathematical proof.

The error of the quantity N, calculated from a known function $f(x_1, x_2, x_3 \ldots \ldots x_n)$: of n meausremente each with their own associated error, is found from:

$$\alpha_N = \left\{ \left| \frac{\partial f}{\partial x_1} \alpha_{x_1} \right| + \left| \frac{\partial f}{\partial x_2} \alpha_{x_2} \right| + \cdots + \left| \frac{\partial f}{\partial x_n} \alpha_{x_n} \right| \right\} \quad \text{(C.1)}$$

where α is the error on the value denoted by its subscript. The value of the error is usually obtained from equipment manuals (iso-accuracy charts, stated +/- errors), calibration (from standard deviation), practical experience and rule of thumb.

The expression for the property in terms of the individually measured components is formulated, followed by taking the partial derivatives with respect to each component in turn. In the following example, error from measuring the steady state nitrogen permeability may be determined using the expression:

$$k = \frac{4\mu Q_b P_b l}{\pi d^2 \Delta P P_m} \quad \text{(C.2)}$$

taking partial derivatives with respect to the various components results in the following:

$$\frac{\partial f}{\partial \mu} = \frac{4 Q_b P_b l}{\pi d^2 \Delta P P_m} \quad \text{(C.3)}$$

$$\frac{\partial f}{\partial Q_b} = \frac{4\mu P_b l}{\pi d^2 \Delta P P_m} \quad \text{(C.4)}$$

$$\frac{\partial f}{\partial P_b} = \frac{4 Q_b \mu l}{\pi d^2 \Delta P P_m} \quad \text{(C.5)}$$

$$\frac{\partial f}{\partial l} = \frac{4 Q_b \mu P_b}{\pi d^2 \Delta P P_m} \quad \text{(C.6)}$$

$$\frac{\partial k}{\partial d} = \frac{-4\mu Q_b P_b l}{\pi d^2 \Delta P P_m} \quad \text{(C.7)}$$

$$\frac{\partial k}{\partial \Delta P} = \frac{-4\mu Q_b P_b l}{\pi d^2 (\Delta P)^2 P_m} \quad (C.8)$$

$$\frac{\partial k}{\partial P_m} = \frac{-4\mu Q_b P_b l}{\pi d^2 \Delta P {P_m}^2} \quad (C.9)$$

As shown in expression C.1, each partial derivative is calculated and multiplied by the error associated with that measurement. The sum of terms gives the total measurement error. Since the errors are not usually considered to be absolute errors but summed using the root-sum square formula. Thus expression C.1 can be given as follows for st6eady state gas permeability:

$$\alpha_N = \sqrt{\left(\frac{\partial k}{\partial \mu}\alpha_\mu\right)^2 + \left(\frac{\partial k}{\partial Q_b}\alpha_{Q_b}\right)^2 + \left(\frac{\partial k}{\partial P_b}\alpha_{P_b}\right)^2 + \left(\frac{\partial k}{\partial l}\alpha_l\right)^2} + \cdots + \sqrt{\left(\frac{\partial k}{\partial d}\alpha_d\right)^2 + + \left(\frac{\partial k}{\partial \Delta P}\alpha_{\Delta P}\right)^2 + \left(\frac{\partial k}{\partial P_m}\alpha_{P_m}\right)^2} \quad (C.10)$$

Analysis of expressions such as C.10 can be used to determine which components contribute the most to the total error. Consider the errors associated with a sample with a gas permeability of 0.508 mD; the equivalent form of expression C.10 is:

$$\alpha_N = \sqrt{(9.1 * 10^{-8})^2 + (1.8 * 10^{-6})^2 + (4.3 * 10^{-8})^2 + (1.1 * 10^{-7})^2} + \cdot\cdot + \sqrt{(9.0 * 10^{-7})^2 + +(3.2 * 10^{-7})^2 + (1.9 * 10^{-7})^2} \quad (C.11)$$

It is evident that flowrate followed rather closely by the diameter contribute the most to the total error with values of $1.8 * 10^{-6}$ cc/sec and $9.0 * 10^{-7}$ m respectively. The contribution from the error associated the flowrate could be reduced by improving the equipment or the calibration technique. Errors due to the measurement of diameter is a function of its role in C.7, little can be done to reduce its magnitude