Appendices

A. Derivation of undeformed chip thickness using machining parameters of SPDT

Appendices A is meant to demonstrate the mathematical relations obtained by applying the principles of geometry to derive the critical machining parameters during nanometric cutting.

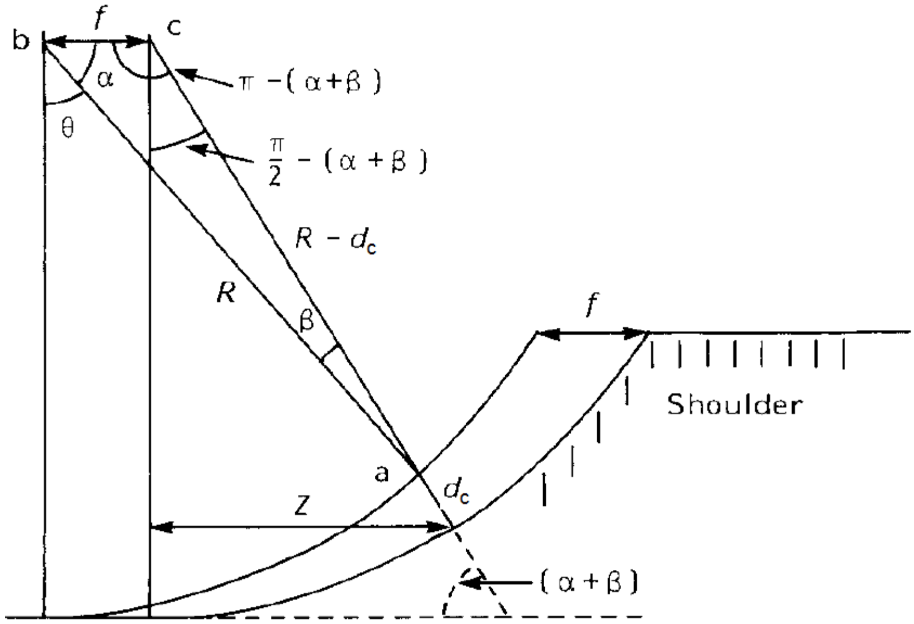


Figure A1: Tool-workpiece interface during SPDT with a round nose tool [23]

Applying sine law in Δ abc of figure A1, we can have:

$$\frac{\sin\{\pi-(\alpha+\beta)\}}{R}=\frac{\sin\beta}{f}=\frac{\sin\alpha}{R-dc}$$

$$\frac{\sin\{\pi-(\alpha+\beta)\}}{R}=\frac{\sin\beta}{f}$$

$$\frac{f\sin(\alpha+\beta)}{R}=\sin\beta$$

Expanding sin (α+β) and dividing by cos β on both sides gives the following:

$$\frac{f\sin\alpha}{R}=\tan\beta\left(1-\frac{f}{R}\cos\alpha\right)$$

$$\tan\beta=\frac{f/R(\sin\alpha)}{\left(1-\frac{f}{R}\cos\alpha\right)}$$

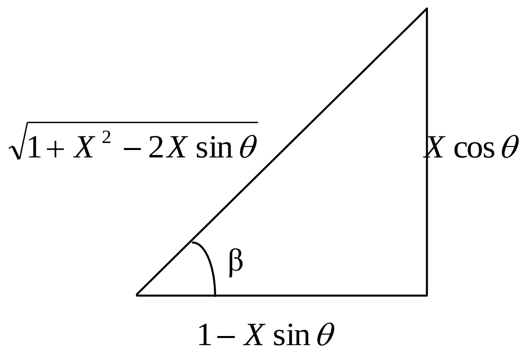
Since, $\alpha = 90-\theta$ $\Rightarrow \sin\alpha = \cos\theta$ and $\cos\alpha = \sin\theta$

$$\tan\beta = \frac{f/R(\cos\theta)}{\left(1-\frac{f}{R}\sin\theta\right)}$$

Let X = f/R

Now, f/R < < < 1 $\tan\beta = X\cos\theta$

The above equation can be presented in the form of triangle as:



Using (1) $\dfrac{\sin\beta}{f} = \dfrac{\sin\alpha}{R-dc}$

$$dc = R - \frac{f\cos\theta}{\sin\beta}$$

Substituting value of sin β and replacing X

$$dc = R - \frac{f\cos\theta\sqrt{1+X^2-2X\sin\theta}}{X\cos\theta} = R - R\sqrt{1+f^2/R^2-2\,{}^{f}\!/\!_{R}\sin\theta}$$

Now, f/R < < < 1

$$dc = R - R\sqrt{1-2\,{}^{f}\!/\!_{R}\sin\theta}$$

Applying Taylor's expansion and by neglecting higher order terms:

since θ is extremely small

$$dc = f\sin\theta = f\theta$$

Expression for Z:

$$\frac{Z}{R} = \cos(\alpha+\beta)$$

Expanding $\cos(\alpha+\beta)=\cos\alpha\sin\beta-\cos\beta\sin\alpha$ and replacing values from Δ

$$Z=\frac{R(\sin\theta-X)}{\sqrt{1+X^2-2X\sin\theta}}$$

Now, f/R < < < 1 and θ is small

$$Z=R\left(\sin\theta-\frac{f}{R}\right)$$
$$Z=R\left(\frac{dc}{f}-\frac{f}{R}\right)=\frac{Rd_c}{f}-f$$
$$dc=\frac{f(z+f)}{R}$$

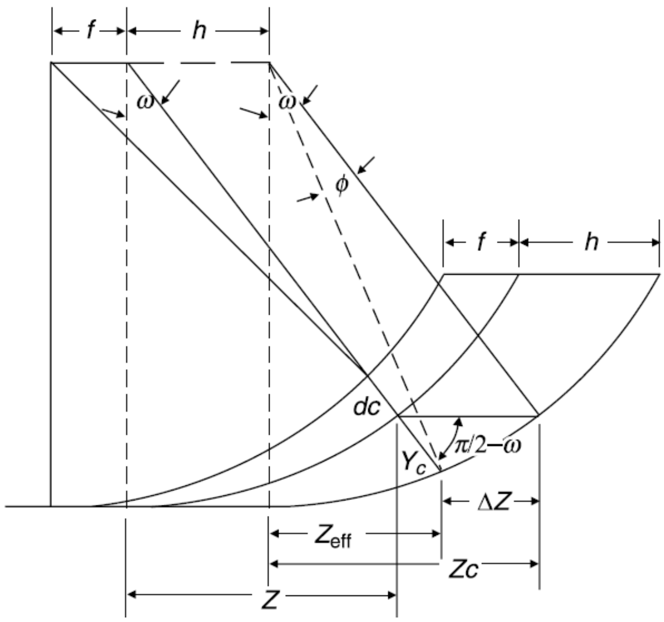


Figure A2: Geometry to derive transition point

Since, Yc is the sub-surface damage which is proportional with the feed rate which occurs at the distance Zeff where (figure A2),

$$Z_{eff}=Z_c-\Delta Z$$

$$Z_{eff}=\sqrt{Z_c^{\,2}-2RY_c}$$

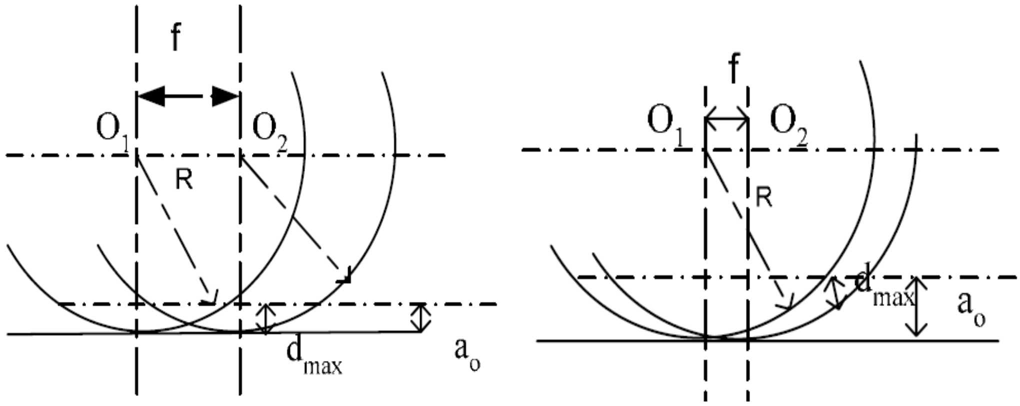
$$Z_{eff}^{\,2}=Z_c^{\,2}-2RY_c$$
$$Z_{eff}^{\,2}=\left(\frac{Rd_c}{f}-f\right)^2-2RY_c$$
$$Z_{eff}^{\,2}-f^2=R^2\left(\frac{dc^2}{f^2}-\frac{2d_c}{R}-\frac{2Y_c}{R}\right)$$

$$\frac{Z_{eff}^{\ 2} - f^2}{R^2} = \frac{dc^2}{f^2} - \frac{2}{R}(d_c - Y_c)$$

In order to evaluate critical feed rate, the process limits would be such that $Z_{eff}=0$

$$f_{\max} = dc\sqrt{\frac{R}{2(d_c + Y_c)}}$$ by assuming that $\left(\frac{d_c}{d_c + Y_c}\right)^2 <<<1$

The maximum undeformed chip thickness, dmax, (effective depth of cut) can be calculated according to the cutting tool geometry and cutting conditions as shown in figure 3.



(a): Large feed rate $\sqrt{(2Ra_0 - a_0^{\ 2})} \le f$ (b): Small feed rate $\sqrt{(2Ra_0 - a_0^{\ 2})} > f$

Figure A3: Schematic for maximum undeformed chip thickness [301]

Here, a_0 is the depth of cut; R is the nose radius; O1 and O2 are the centres of two adjacent arc cutting edges, and the distance between O1 and O2 is the feed rate, f. The maximum undeformed chip thickness dmax for the two conditions as shown in figure A3 can be calculated as follows:

The maximum undeformed chip thickness for large feed rate while

$\sqrt{(2Ra_0 - a_0^{\ 2})} \le f$ can be expressed as:

$$d_{\max} = a_0$$

The maximum undeformed chip thickness for small feed rate while

$\sqrt{(2Ra_0 - a_0^{\ 2})} > f$ can be expressed as:

$$d_{\max} = R - \sqrt{R^2 + f^{\ 2} - 2f\sqrt{2Ra_0 - a_0^{\ 2}}}$$

When R≫f and R ≫a_0

Above equation can be finally simplified as:

$$d_{\max} \approx \frac{f}{R}\sqrt{2Ra_0 - a_0^{\ 2}} \approx f\sqrt{\frac{2a_0}{R}}$$