APPENDIX I DERIVATION OF CRITICAL VALUES FOR TESTS OF HYPOTHESES CONCERNING CORRELATION COEFFICIENTS WHERE SAMPLE SIZES ARE SMALL

The information below is summarised off the internet from the University of New England Website

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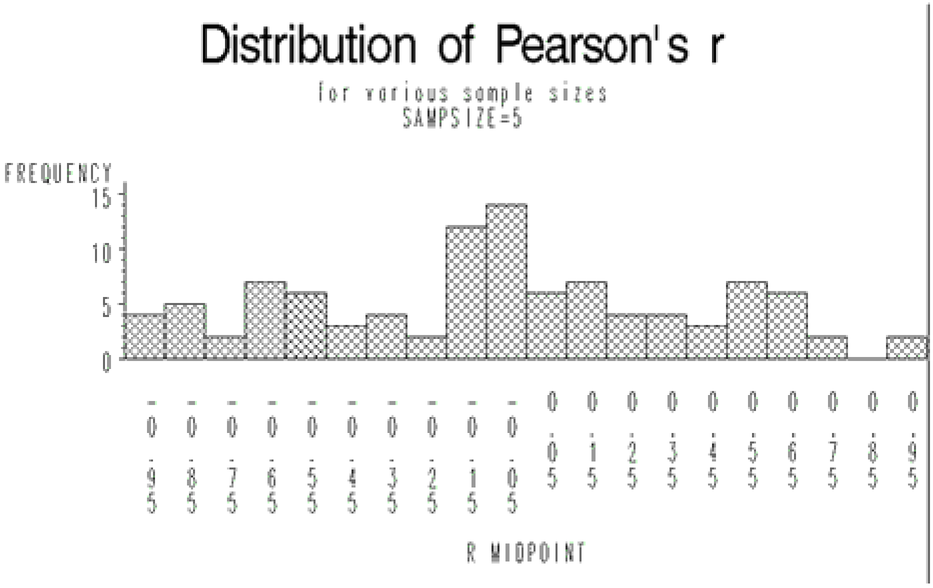
http://www.une.edu.au/WebStat/unit_materials/c6_common_statistical_tests/test_signif_pearson.html

Testing the significance of Pearson's r

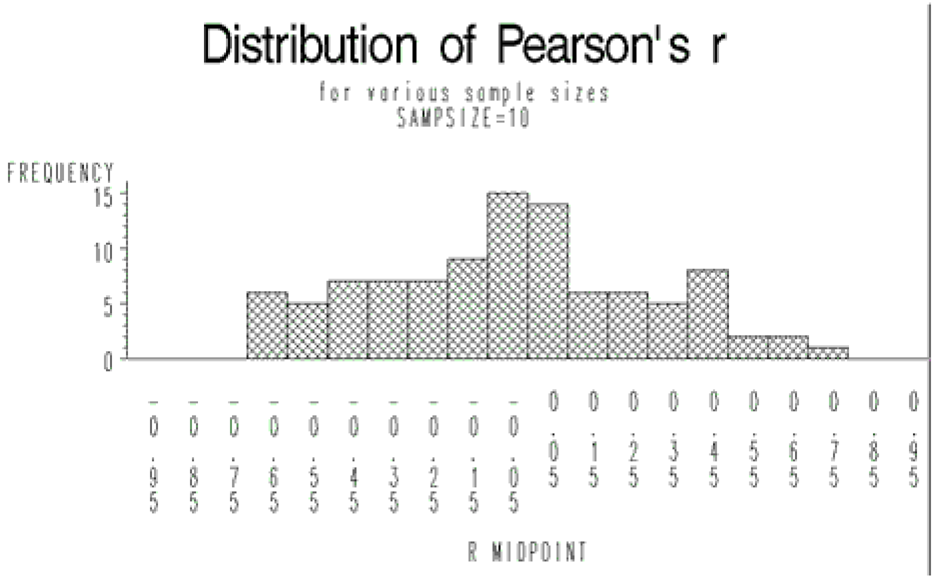
Pearson's r is a useful descriptor of the degree of linear association between two variables. When it is near zero, there is no correlation, but as it approaches -1 or +1 there is a strong negative or positive relationship between the variables (respectively). This article describes how to establish with a degree of certainty that a correlation is sufficiently different to zero to assert that a real relationship exists.

An estimate of how much variation in r can be expected by random chance is required. This can be achieved by constructing a sampling distribution for r and determining its standard error. All variables are correlated to some extent; rarely will a correlation be exactly zero.

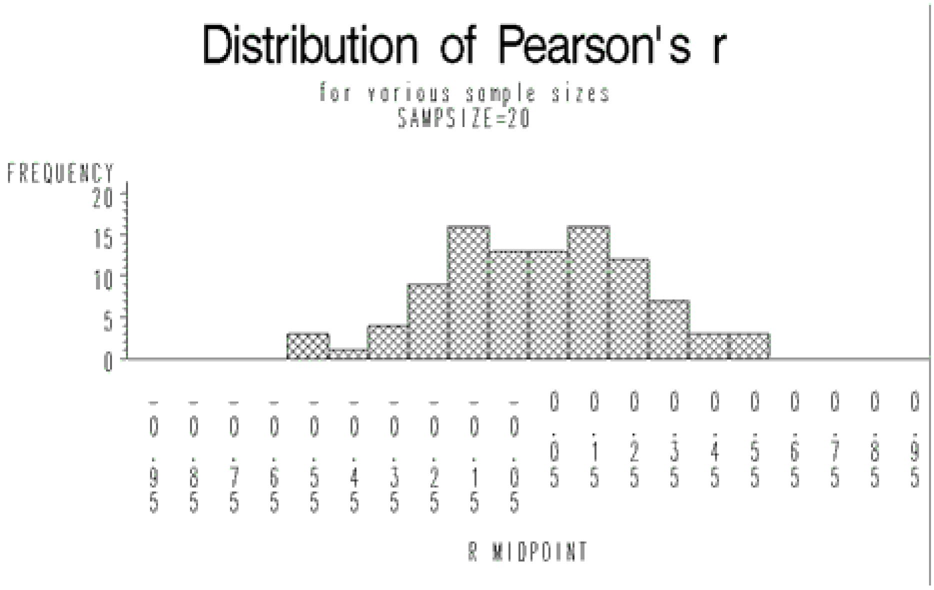
From the following illustrations, for small N's, r can vary markedly even when the null hypothesis is true (i.e., if chance is a reasonable explanation for the correlation). For larger sample sizes, the correlations will cluster more tightly around zero but there will still be a considerable range of values. The illustrations that follow show the distribution of the correlation coefficient between an X and a Y variable for 100 random samples simulated on a computer, using N's of different sizes. When N=5, we can see almost the full range from -1 to +1. This is less apparent for N=10, 20, 30 and so on, until when we have simulated 100 cases there is little variability around zero.



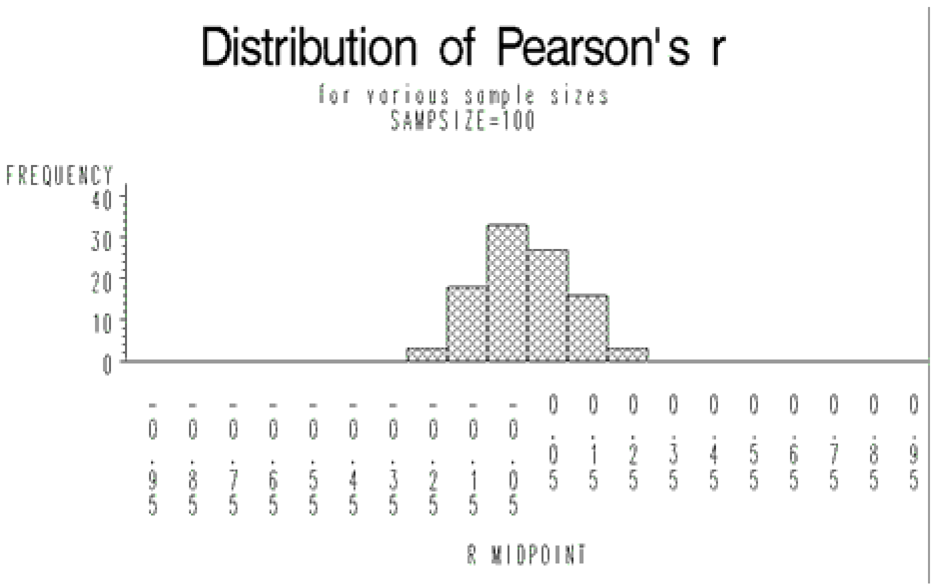
The distribution of correlations between two random variables when sample size = 5



The distribution of correlations between two random variables when sample size = 10.



The distribution of correlations between two random variables when sample size = 20.



The distribution of correlations between two random variables when sample size = 100.

From the above figures, as the sample size increases, the more the correlations tend to cluster around zero. Because just two random variables are correlated there should be no systematic or real relationship between the two. However just by chance some will appear to exhibit a real relationship.

In the above figures for r = -.65, with samples of size 5, 18 out of the 100 samples had a correlation equal to or more extreme than -.65 (4 at -.95, 5 at -.85, 2 at -.75, and 7 at -.65). In the Figure with samples of size 10, only 6 samples had a correlation of -.65 or more extreme. And in the Figures with samples of size 20 and 100, there were no samples having a correlation of -.65 or more extreme. So a correlation of -.65 is not an unusual score if the samples are only small. However, correlations of this size are quite rare when we use samples of size 20 or more.

The following table gives the significance levels for Pearson's correlation using different sample sizes.

Critical values for Pearson's r

Degrees of freedom=N-2 (N= number of pairs)	Level of significance for one-tailed test			
	.05	.025	.01	.005
	Level of significance for two-tailed test			
	.10	.05	.02	.01
1	.988	.997	.9995	.9999
2	.900	.950	.980	.990
3	.805	.878	.934	.959
4	.729	.811	.882	.917
5	.669	.754	.833	.874
6	.622	.707	.789	.834
7	.582	.666	.750	.798
8	.549	.632	.716	.765
9	.521	.602	.685	.735
10	.497	.576	.658	.708

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