THE INFLUENCE OF CONFINING THE COMPRESSION ZONE IN THE DESIGN OF STRUCTURAL CONCRETE BEAMS

MOHAMED MOHAMED ZIARA, MSc

THESIS SUBMITTED FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

HERIOT-WATT UNIVERSITY
DEPARTMENT OF CIVIL AND OFFSHORE ENGINEERING
JULY 1993

This copy of the thesis has been supplied on condition that anyone who consults it is understood to recognize that the copyright rests with its author and that no quotation from the thesis and no information derived from it may be published without the prior written consent of the author or the University (as may be appropriate).
DECLARATION

THIS THESIS HAS BEEN COMPOSED BY MYSELF AND, EXCEPT WHERE STATED, THE WORK CONTAINED IS MY OWN.

MOHAMED ZIARA

JULY 1993
Contents

ACKNOWLEDGEMENT xxiv

ABSTRACT xxv

1 INTRODUCTION 1

1.1 BACKGROUND ........................................ 1

   1.1.1 The Riddle of the Shear Problem ................. 3

   1.1.2 Influence of Confinement on the Flexural Behaviour
         of Beams ....................................... 6

1.2 UNIFIED DESIGN APPROACH ........................... 6

1.3 OBJECTIVES .......................................... 8

1.4 RESEARCH PROGRAMME ................................. 9

1.5 OUTLINE OF THESIS ................................. 11

2 LITERATURE REVIEW OF THE BEHAVIOUR OF BEAMS UN-
   DER TRANSVERSE LOADING ............................ 14

   2.1 INTRODUCTION .................................... 14

   2.2 BEAM CLASSIFICATION ............................ 14
2.2.1 Type I (Long Beams) .............................................. 15
2.2.2 Type II (Normal Beams of Intermediate Length) .... 15
2.2.3 Type III (Short Beams) .......................................... 16
2.2.4 Type IV (Deep Beams) ........................................... 16

2.3 MODES OF FAILURE .................................................. 16
   2.3.1 Flexural Failure ................................................ 16
   2.3.2 Diagonal Failure ................................................ 17
   2.3.3 Deep-Beam Failure ............................................. 20
   2.3.4 Conclusions ..................................................... 21

2.4 BEHAVIOUR OF BEAMS UNDER THE COMBINED ACTION OF SHEAR AND BENDING MOMENT ............................................. 21
   2.4.1 Introduction ..................................................... 21
   2.4.2 Mechanisms of Shear Transfer ................................ 22
   2.4.3 Contribution of Shear Transfer Mechanisms to Shear Resistance ......................................................... 27
   2.4.4 Discussion and Conclusions ..................................... 29

2.5 SHEAR STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT ................................................................. 30
   2.5.1 Introduction ..................................................... 30
   2.5.2 Statistically Developed Models ................................ 30
   2.5.3 Physical Models .................................................. 33
   2.5.4 Discussion and Conclusions ...................................... 42
2.6 SHEAR STRENGTH OF BEAMS WITH WEB REINFORCEMENT

2.6.1 Introduction ........................................ 44
2.6.2 Truss Analogy ......................................... 44
2.6.3 Strut-and-Tie Models ................................. 51
2.6.4 Diagonal Compression Field Theory ................. 56
2.6.5 Modified Compression Field Theory .................. 60
2.6.6 Plasticity Theory Model .............................. 63
2.6.7 Equilibrium Analysis .................................. 64
2.6.8 Arch Action Theory .................................... 64
2.6.9 Compressive Force Path Concept ...................... 65

2.7 BEHAVIOUR OF BEAMS IN FLEXURE .................... 71

2.7.1 General .............................................. 71
2.7.2 Multiaxial Stress Behaviour ............................ 72
2.7.3 Over-Reinforced Beams ............................... 74
2.7.4 Conclusions .......................................... 75

2.8 GENERAL REMARKS AND CONCLUSIONS .................. 76

2.8.1 General .............................................. 76
2.8.2 Flexure Behaviour of Beams ......................... 77
2.8.3 Behaviour of Beams Under the Combined Action of Shear and Bending ......................... 77
2.8.4 Unified Design Approach ............................. 79
3 NEW APPROACH TO THE DESIGN OF BEAMS UNDER TRANSVERSE LOADING

3.1 INTRODUCTION .................................................. 92

3.2 REALISTIC DIAGONAL FAILURE MECHANISM: Compressive Force Path (CFP) Concept ....................... 93

3.3 STRUCTURAL BEHAVIOUR OF BEAMS FAILING BY DIAGONAL CRACKING .................................. 93

3.3.1 Beams without Web Reinforcement ....................... 94

3.3.2 Beams Traditionally Detailed for Shear .................. 96

3.3.3 Beams Specially Detailed for Shear: Test Series ‘1’ .... 97

3.3.4 Beams Specially Detailed for Shear: Beam Tests Conducted at Birzeit University ................... 101

3.3.5 Beams Specially Detailed for Shear: Beams Tested by Kotsovos ............................................. 102

3.3.6 Structural Behaviour of Beams under Transverse Loading ...................................................... 105

3.3.7 Conclusions ...................................................... 106

3.4 NEW DETAILING ARRANGEMENT FOR BEAMS ................................................................. 107

3.4.1 Introduction ...................................................... 107

3.4.2 Proposed Detailing for Preventing Diagonal Failures ......................................................... 108

3.4.3 Proposed Detailing Arrangement for the Evaluation of the Effect of Confinement on Flexural Capacity ... 109
3.5 TEST SERIES 'A': Applicability of the proposed detailing approach .............................................. 110
3.5.1 Description of Test Beams ............................................. 110
3.5.2 Experimental Work .................................................. 111
3.5.3 Test Results .......................................................... 112
3.5.4 Discussion of Test Results ......................................... 113
3.5.5 Summary and Conclusions ........................................ 119

3.6 THEORETICAL BASIS OF FLEXURE-SHEAR INTER-
ACTION DESIGN MODEL ............................................. 120
3.6.1 Introduction ......................................................... 120
3.6.2 Main Features of the Proposed Model ......................... 121

3.7 ADVANTAGES OF THE PROPOSED DESIGN APPROACH 123

3.8 SUMMARY ............................................................ 128

4 IMPLEMENTATION OF THE FLEXURE-SHEAR INTERACTION
MODEL IN THE DESIGN OF BEAMS 141
4.1 INTRODUCTION ....................................................... 141
4.2 EVALUATION OF THE SHEAR EFFECT ON REDUC-
ING THE CONCRETE COMPRESSIVE STRENGTH (CON-
FINEMENT REQUIREMENTS) ......................................... 141
4.2.1 Bending Moments ................................................. 141
4.2.2 Shear Forces ..................................................... 143
4.2.3 Variable Shear Forces .......................................... 143
4.2.4 Enhanced Flexural Capacity (Flexure-Shear Interaction) ........................................ 144

4.3 LEG CAPACITY ($M_l$) ......................................................... 144

4.3.1 Introduction ................................................................. 144

4.3.2 Beam Tests Required for the Evaluation of the Horizontal Leg Capacity ($M_{lh}$) ................................. 145

4.3.3 Beam Tests Required for the Evaluation of the Inclined Leg Capacity ($M_{li}$) ......................... 145

4.4 TEST SERIES 'B': Evaluation of the leg capacities .............. 146

4.4.1 Description of Test Beams ............................................. 146

4.4.2 Experimental Work ...................................................... 147

4.4.3 Test Results ................................................................. 147

4.4.4 Discussion of Test Results ............................................. 148

4.4.5 Relative Flexural Capacity of the Inclined Leg ($\frac{M_{li}}{M_l}$) ........................................ 151

4.4.6 Relative Flexural Capacity of the Horizontal Leg ($\frac{M_{lh}}{M_l}$) ................................. 152

4.4.7 Conclusions ................................................................. 153

4.5 GENERAL METHOD FOR THE EVALUATION OF THE LEG CAPACITIES .................. 153

4.6 Ultimate Flexural Capacity of Beams ($M_u$) ....................... 156

4.7 PROVISIONS OF THE CONFINEMENT REQUIREMENTS 158

4.7.1 Introduction ................................................................. 158

4.7.2 Types of Confinement ................................................... 159

4.7.3 Confinement Models for Circular Sections .................... 162
4.7.4 Confinement Models for Rectangular Sections .... 164
4.7.5 Influence of Strain Gradient .................. 165
4.7.6 Summary and Conclusions ...................... 167
4.7.7 Confinement Model for Rectangular Beams .... 168
4.7.8 Equivalent Concrete Compressive Block ......... 171
4.7.9 Proposed Modifications to the Confinement Model . 172
4.7.10 General Remarks ............................. 174

4.8 IMPLEMENTATION OF THE FLEXURE-SHEAR INTER-
ACTION DESIGN MODEL ........................... 175
4.8.1 Analysis Steps ............................... 175
4.8.2 Design Steps ................................. 176

4.9 TEST SERIES ‘C’: Verification of the proposed flexure-shear interaction design model .............. 177
4.9.1 Description of Test Beams ..................... 177
4.9.2 Design of Test Beams ......................... 179
4.9.3 Experimental Work ............................ 182
4.9.4 Test Results ................................ 182
4.9.5 Discussion of Test Results ................... 183
4.9.6 Serviceability ................................ 190
4.9.7 Conclusions .................................. 193

4.10 SUMMARY ..................................... 195

5 FACTORS AFFECTING DIAGONAL FAILURES IN BEAMS 218
5.1 INTRODUCTION ................................. 218
5.2 FACTORS AFFECTING THE LOAD CARRYING CAPACITY OF BEAMS AT DIAGONAL FAILURE

5.2.1 Shear Span to Depth Ratio ($\frac{a}{d}$) .......................... 218
5.2.2 Longitudinal Reinforcement Ratio ($\rho$) .......................... 219
5.2.3 Yield Strength of Longitudinal Reinforcement .................. 220
5.2.4 Bond Characteristics ............................................. 220
5.2.5 Shape of the Cross Section ........................................ 221
5.2.6 Axial and Prestressing Loads .................................... 222
5.2.7 Web Reinforcement .................................................. 223
5.2.8 Implications on the Proposed Flexure-Shear Interaction Design Model ........................................ 224

5.3 SIZE EFFECTS ......................................................... 225

5.3.1 Introduction ......................................................... 225
5.3.2 Beams Without Web Reinforcement ................................. 226
5.3.3 Beams With Web Reinforcement .................................... 228
5.3.4 Discussion and Conclusions ......................................... 228
5.3.5 Proposed Design Method and Size Effects ......................... 231

5.4 TEST SERIES 'D': Large beam ....................................... 231

5.4.1 Test Beam .......................................................... 231
5.4.2 Discussion of Test Results ......................................... 233

5.5 HIGH STRENGTH CONCRETE (HSC) ................................. 234

5.5.1 Introduction ........................................................ 234
5.5.2 Behaviour of HSC Beams ........................................... 235
5.5.3 Implication of the Concrete Strength on the Proposed Design Model .............................................. 240

5.6 TEST Series 'E': Validation of the flexure-shear interaction design model for high-strength concrete beams ...................... 242
5.6.1 Description of Test Beams ................................................. 242
5.6.2 Design of Test Beams .......................................................... 243
5.6.3 Experimental Work ............................................................ 244
5.6.4 Test Results ................................................................. 245
5.6.5 Discussion of Test Results .................................................. 245
5.6.6 Conclusions ................................................................. 249

5.7 SUMMARY ................................................................. 250

6 FLEXURAL BEHAVIOUR OF BEAMS RESULTING FROM CONFINEMENT .......................................................... 269
6.1 INTRODUCTION ................................................................. 269
6.2 EVALUATION OF FLEXURAL CAPACITY DUE TO CONFINEMENT .......................................................... 270
6.2.1 General ................................................................. 270
6.2.2 Confinement Enhancement Factor ($K_s$) ...................... 270
6.2.3 Concrete Compressive Strain ($\varepsilon_c$) ......................... 271
6.2.4 Modified Concrete Compression Block ......................... 272
6.3 FLEXURAL CAPACITY OF UNDER-REINFORCED BEAMS 273
6.4 FLEXURAL CAPACITY OF OVER-REINFORCED BEAMS 275
6.4.1 New Balanced-Failure Longitudinal Reinforcement Ratio (\(\rho_b^l\)) .................................................. 275
6.4.2 Relationship Between \(\rho_b^l\) and \(\rho_b\) .................................................. 277
6.4.3 Design Method for Over-Reinforced Beams ................. 279
6.5 TEST SERIES 'F': Verification of the design method for over-reinforced beams .................................................. 282
   6.5.1 Design and Description of Test Beams ......................... 282
   6.5.2 Experimental Work .................................................. 283
   6.5.3 Test Results .................................................. 284
   6.5.4 Discussion of Results .................................................. 286
   6.5.5 Practical Applications ........................................ 290
6.6 SUMMARY .................................................. 291

7 BEAMS WITH CIRCULAR CROSS SECTIONS ......................... 307
7.1 INTRODUCTION .................................................. 307
7.2 PREVIOUS RESEARCH ON BEAMS WITH CIRCULAR CROSS SECTIONS .................................................. 309
   7.2.1 Capon and de Cossio (Mexico) ......................... 309
   7.2.2 Khalifa and Collins (Canada) ......................... 310
   7.2.3 Yan, Masahide, and Kenji (Japan) ......................... 311
   7.2.4 Ghee, Priestley, and Paulay (New Zealand) ......................... 312
   7.2.5 Clarke and Birjandi (UK) ......................... 313
   7.2.6 Summary and Conclusions ........................................ 314
7.3 AIMS OF THE PROPOSED TEST PROGRAMME ......................... 315
7.4 TEST SERIES 'G': Influence of confinement on the behaviour of beams with circular cross sections

7.4.1 Description of Test Beams

7.4.2 Experimental Work

7.4.3 Test Results

7.4.4 Discussion of Test Results

7.4.5 Contribution of Compression Zone to Shear Capacity

7.4.6 Conclusions

7.5 SUMMARY

8 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

8.1 CONCLUSIONS

8.1.1 Current Design Approaches to Structural Concrete Beams

8.1.2 Flexural Behaviour of Beams with Confinement

8.1.3 Flexure-Shear Interaction Design Model

8.1.4 Miscellaneous

8.2 RECOMMENDATIONS FOR FUTURE WORK

APPENDICES

A EXPERIMENTAL PROCEDURES

A.1 MATERIALS

A.1.1 Cement
A.6.1 Loads ................................................................. 361
A.6.2 Deflection ............................................................. 362

B CONFINEMENT REQUIREMENTS TO PREVENT DIAGONAL FAILURES 371

B.1 EVALUATION OF $K_r$ FOR THE BEAMS IN TEST SERIES ‘A’ ................................................... 371
B.1.1 Prototype Example: Beam Type NA2-1 ......................... 371
B.1.2 Remaining Beams ..................................................... 373

B.2 DESIGN OF THE BEAMS IN TEST SERIES ‘C’ ................. 373
B.2.1 Prototype Example: Beam Type B1.5 .......................... 373
B.2.2 Design of the Remaining Beams ................................. 377

C FLEXURAL CAPACITY RESULTING FROM CONFINEMENT 381

C.1 PREDICTION OF THE FLEXURAL CAPACITY OF THE BEAMS IN TEST SERIES ‘A’ ......................... 381
C.1.1 Prototype Example: Beam NA2-1 ............................... 381
C.1.2 Calculation of the Flexural Capacity of the Beams in Test Series ‘A’. ........................................... 384

C.2 DERIVATION OF $\rho_b'$ ................................................. 384

D BEAMS WITH A CIRCULAR CROSS SECTION 389

D.1 TRADITIONAL BEAM DESIGN ........................................ 389
D.1.1 Flexural Design ..................................................... 389
D.1.2 Design for Shear ................................................... 393
D.2 DESIGN CALCULATIONS FOR THE BEAMS IN TEST

SERIES ‘G’ .......................................................... 395

D.2.1 Assumed Geometrical Properties .......................... 395
D.2.2 Assumed Material Properties ............................... 396
D.2.3 Flexural Capacity ........................................... 396
D.2.4 Design for Shear ............................................. 398
D.2.5 Anchorage Length ........................................... 398

D.3 AMOUNT OF CONFINEMENT REQUIRED TO PREVENT
  DIAGONAL FAILURES ........................................... 398

D.4 FLEXURAL CAPACITY ......................................... 400

BIBLIOGRAPHY .................................................. 402
List of Tables

2.1 Percentage Contributions to Shear Resistance. .................. 81

3.1 Concrete Mix Constituents used in Test Series 'A'. .................. 130

3.2 Summarised Results from Test Series '1' and Test Series 'A'. .... 130

3.3 Comparison between Test Results. .............................. 131

3.4 Summary of Crack Width and Deflection Measurements. .......... 132

4.1 Notation used in the Proposed Design Approach. .................. 196

4.2 Results from the Beams in Test Series '1' and Test Series 'B'. .... 197

4.3 Relative Flexural Capacities of the beams in Test Series '1' and Test Series 'B'. .................................................. 198

4.4 Evaluation of the Confinement Requirements ($K_c$) for the Prevention of Diagonal Failure of the Beams in Test Series 'A'. ................. 199

4.5 Predicted Results for the Prevention of Diagonal Failure in the Beams in Test Series 'C'. .................................................. 200

4.6 Results from Test Series 'C'. ........................................ 201

4.7 Flexural Capacities of the Beams in Test Series 'C'. ............... 201
4.8 Crack Width and Deflection Measurements for the Beams in Test Series ‘C’. ................................. 202

4.9 Strains in the Stirrups in Beam Types B1.8 and B1.8(T). ............................. 203

5.1 Concrete Mix Constituents used in Test Series ‘D’. .................. 252

5.2 Concrete Compressive Strength obtained from the beam in Test Series ‘D’. ................................. 253

5.3 Summary of the Crack Width and Deflection Measurements at Different Load Levels found in the Beam in Test Series ‘D’. ............................. 253

5.4 Concrete Mix Constituents used in Test Series ‘E’ (High strength concrete). ................................. 254

5.5 Details of Stirrup Configurations required to prevent Diagonal Failure in the HSC Beams in Test Series ‘E’. ................................. 255

5.6 Summarised Results from the Beams in Test Series ‘E’. .................. 256

5.7 Actual Flexural Capacities of the Beams in Test Series ‘E’. .................. 256

5.8 Summary of the Crack Width and Deflection Measurements at Different Load Levels for the Beams in Test Series ‘E’. ............................. 257

5.9 Strains in the Stirrups in Beam Types B2.78 and B2.78(T). .................. 258

5.10 Concrete Strains obtained from Beam Type A1.86. ................................. 259

6.1 Flexural Capacities of the Beams in Test Series ‘A’. ................................. 293

6.2 Longitudinal Reinforcement Ratios for the Beams in Test Series ‘A’. ................................. 294

6.3 Concrete Mix Constituents used in Test Series ‘F’. ................................. 295

6.4 Summarised Results obtained from the Beams in Test Series ‘F’. ................................. 296
6.5 Evaluation of the Longitudinal Reinforcement Ratios for the Beams in Test Series ‘F’ ................................. 297

6.6 Comparison between the Measured and Predicted Flexural Capacities for the Beams in Test Series ‘F’. ................................. 298

6.7 Summary of the Crack Width and Deflection Measurements at Different Load Levels for the Beams in Test Series ‘F’. ................................. 299

6.8 Typical Concrete Strains obtained from the Beams in Test Series ‘F’. 299

6.9 Strains in the Reinforcement Bars at various Loading Levels obtained from the Beams in Test Series ‘F’. ................................. 300

7.1 Concrete Mix Constituents used in Test Series ‘G’. ................................. 329

7.2 Test Results. ................................. 330

7.3 Crack History. ................................. 331

7.4 Enhanced Flexural Capacities. ................................. 332

A.1 Sieve Analysis of Fine Aggregate. ................................. 363

A.2 Tensile Strength of Reinforcing Steel Bars. ................................. 364

A.3 Test Records for Beam Type NB3-1. ................................. 365

A.4 Processed Test Results for Beam Type NB3-1. ................................. 366

B.1 Confinement Requirements to Prevent Diagonal Failures in the Beams in Test Series ‘A’. ................................. 378

B.2 Prevention of Diagonal Failures in the Beams in Test Series ‘C’. ................................. 378

B.3 Strains in the Stirrups in Beam Types B1.8 and B1.8(T) in Test Series ‘C’. ................................. 379
C.1 Confined Flexural Capacities of the Beams in Test Series 'A' . . . . . 387

D.1 Enhanced Flexural Capacities . . . . . . . . . . . . . . . . . . . . . . 401
List of Figures

2.1 Classification of Beams Based on Kani's Valley. .................. 82
2.2 Flexural Failure Mode. ........................................... 82
2.3 (a) Types of Diagonal Cracking. (b) Modes of Diagonal Failure. . . 83
2.4 Deep-Beam Failures. (a) Arch Action. (b) Types of Failures. ........ 83
2.5 Mechanism of Aggregate Interlock. (a) Shear-Friction Hypothesis.
(b) Formation of Truss Action. (c) Partial Lateral Restraint. (d) Full
Lateral Restraint. ...................................................... 84
2.6 Traditional Concepts of: (a) Mechanisms of Shear Transfer. (b) Effect
of Web Reinforcement on Shear Capacity. ............................. 84
2.7 Behaviour of Concrete-Cantilever. (a) Longitudinal Reinforcement
without Bond. (b) Ideal Concrete Cantilever. .......................... 85
2.8 Kani's Hypothesis: (a) Arch Action. (b) Process of Transformation
into a Tied Arch. ......................................................... 85
2.9 Behaviour of the Modified Concrete-Tooth. .......................... 86
2.10 Modes of Failure. (a) Axial Compression Failure. (a') Column Fail-
ure. (b) Flexural Failure. (c) Diagonal Failure. ........................ 86
2.11 Mechanisms of Failure Based on the Theory of Plasticity. (a) Upper-Bound Solution. (b) Lower-Bound Solution. (c) Beam with Web Reinforcement. 87

2.12 Truss Analogy. (a) Original Truss Analogy. (b) Improved Truss Analogy. (c) Modified General Truss Analogy. 87

2.13 Basic Types of Compression Fields. (a) The Fan. (b) The Bottle. (c) The Prism. 88

2.14 Compression Field Theory. (a) Free-Body Diagram of a Beam Section. (b) Compatibility Conditions for Average Strains in Concrete. 88

2.15 (a) Compressive Force Path. (b) Effect of Bond Failure. (c) Equilibrium Conditions at Force Changing Direction. (d) Equilibrium Condition for Type III beams. 89

2.16 Strain Profiles at the Flexural Strength of a Section. 90

2.17 Uniaxial Stress-Strain Relationship. (a) Typical Curves. (b) Effects of Boundary Restraints. 90

2.18 Longitudinal Strain-Transverse Strain Relationships For a Uniaxial Compression Test and for Flexure. 91

3.1 Causes of Diagonal Failures at Different Regions along the CFP. 133

3.2 Failure of Beams without Web Reinforcement. (a) Type II Beams. (b) Type III Beams. 133

3.3 Failure of Beams Traditionally Detailed for Shear. (a) Type II Beams. (b) Type III Beams. 134

3.4 Test Series '1'. (a) Type II Beams. (b) Type III Beams. 134
3.5 Behaviour of the Beams in Test Series ‘1’ based on Kani’s Valley.
   (a) Kani’s Valley. (b) Transformation from Type II to Type III Beam
   Behaviour. ................................. 135

3.6 Proposed Detailing Arrangement for Beams. (a) To Prevent Diagonal
   Failure. (b) Study of the Effect of Confinement on the Flexural
   Capacity of Beams. ............................. 136

3.7 Detailing of the Beams in Test Series ‘A’. ................................. 137

3.8 Crack Patterns and Failure Modes for the Beams in Test Series ‘A’. 138

3.9 Load-Midspan Deflection Curves for the Beams in Test Series ‘A’. 139

3.10 Concrete Compression Frame. ................................. 140

4.1 Details of the Beams in Test Series ‘B’. ................................. 204

4.2 Load-Deflection Curves for the Beams in Test Series ‘B’. ................................. 205

4.3 Relative Flexural Capacity Curve (Valley) for the Inclined Leg. 206

4.4 Relative Flexural Capacity Curve (Valley) for the Horizontal Leg. 206

4.5 Mode of Failure of Beam Type D4 in Test Series ‘B’. ................................. 207

4.6 Confinement Model. ................................. 208

4.7 Concrete Confined Area for the Prevention of Diagonal Failure. (a) Hor-
   izontal leg and Inclined Leg Regions Away from the Supports. (b) In-
  clined Leg Near to the Supports. ................................. 208

4.8 Details of Type A Beams in Test Series ‘C’. ................................. 209

4.9 Details of Type B Beams in Test Series ‘C’. ................................. 210

4.10 Details of Type C Beams in Test Series ‘C’. ................................. 211

4.11 Location of the Strain Gauges used in Beam Types B1.8 and B1.8(T). 212
4.12 Load-Deflection Curves for Type A Beams in Test Series ‘C’. . . . . . . 213
4.13 Load-Deflection Curves for Type B Beams in Test Series ‘C’. . . . . . 214
4.14 Load-Deflection Curves for Type C Beams in Test Series ‘C’. . . . . . 215
4.15 Crack Patterns after Failure for all of the Beams in Test Series ‘C’. . . 216
4.16 Beam Types A2, B1.8, and C1.5 in Test Series ‘C’ after Failure. . . . 217

5.1 Details of the Beam in Test Series ‘D’. . . . . . . . . . . . . . . . . . . . . . 260
5.2 Load-Deflection Curve obtained from the Beam in Test Series ‘D’. . . 260
5.3 Shape of the Beam in Test Series ‘D’ after Failure. . . . . . . . . . . . 261
5.4 Details of the Beams in Test Series ‘E’. . . . . . . . . . . . . . . . . . . . . . 262
5.5 Locations of the Strain Gauges used in Beam Types B2.78 and B2.78(T).263
5.6 Location of the Neutral Axis in the HSC Beam Type B1.86. . . . . . . 263
5.7 Load-Deflection Curves obtained from Beam Type A in Test Series ‘E’.264
5.8 Load-Deflection Curves obtained from Beam Type B in Test Series ‘E’.265
5.9 Load-Deflection Curves obtained from Beam Type C in Test Series ‘E’.266
5.10 Crack Patterns after Failure for all of the Beams in Test Series ‘E’. . . 267
5.11 Shape of Beam Types B2.78 and C1.86 in Test Series ‘E’ after Failure.268

6.1 Flow Chart showing the Design Steps for Over-Reinforced Beams. . . 301
6.2 Detailed of the Beams in Test Series ‘F’. . . . . . . . . . . . . . . . . . . . . . 302
6.3 Locations of the Strain Gauges and the Demec Buttons used in the
   Beams in Test Series ‘F’. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 303
6.4 Load-Deflection Curves obtained from the Beams in Test Series ‘F’. . 304
6.5 Shape of Failure in Beam Type C3.2-3 in Test Series ‘F’. . . . . . . 305
6.6 Development of Cracks in Beam Type C2 in Test Series 'F'........ 306

7.1 Detailing of the Beams........................................... 333
7.2 Cross-Section of the Beams...................................... 334
7.3 Loading Arrangement............................................. 334
7.4 Crack Patterns and Failure Modes................................. 335
7.5 Load-Midspan Deflection Curves for: (a) Beam Types B1,B2,B3&B4.
(b) Beam Types B1,B5&B6. (c) All Beams in Test Series 'G'........ 336

A.1 Typical Stress-Strain Curves for Steel Reinforcement Bars. (a) 8mm
Nominal Diameter Bars. (b) 25mm Nominal Diameter Bars............. 367
A.2 Typical Steel Bar of Nominal Diameter of 25mm Used in a Tensile
Test............................................................................. 368
A.3 Crack Patterns at Various Load Levels for Beam Type B5 in Test
Series 'G'..................................................................... 369
A.4 Typical Steel Cages inside shutters................................. 370

B.1 Cross Section Geometry for Beam Type NA2-1..................... 380
B.2 Cross Section Geometry for Beam Type B1.5. (a) The Horizontal
Leg. (b) The Inclined Leg near the Horizontal Leg. (c) Near the
Supports....................................................................... 380

C.1 Modified Stress and Strain Diagrams Under Balanced-Failure Condi-
tions............................................................................. 388
ACKNOWLEDGEMENTS

I would like to thank my supervisor Mr. D. Haldane for his valuable guidance and support throughout this project and for his patience in correcting the manuscript. His kindness is gratefully acknowledged.

I would also like to thank Dr. A. Kuttab (Birzeit University) for the many stimulating and thought provoking discussions we have had. I also want to thank Professor M. Kotsovos (University of Athens) for his valuable comments and advice on the design approach for beams with circular cross sections included in this thesis. I also would like to thank Dr. X. Su (Heriot-Watt University) for allowing me to use results from tests conducted by him and for his discussions throughout this work. My thanks are also extended to my colleagues and friends who have contributed much, both directly and indirectly, to this thesis. My thanks are due too to the technical staff of the Department of Civil and Off-shore Engineering at Heriot-Watt University.

I acknowledge with thanks the financial support given to me by the Science and Engineering Research Council while at Heriot-Watt University.

My gratitude must be extended to Birzeit University (West Bank) for allowing me to participate in this project.

Finally, I would like to thank my wife, Amal and my children; Roba, Rana, Rami, and Raif as well as my parents for their patience and encouragement throughout the period of my study.
ABSTRACT

The programme of research described in this thesis has investigated the effect of confinement of the compression regions on the behaviour of structural concrete beams up to failure under the action of static loadings.

A flexure-shear interaction design model and a corresponding approach to detailing to prevent diagonal failures in beams have been developed based on the actual physical behaviour of beams. This has been achieved by confining only the compression regions in the beam structure with closed stirrups. The increase in concrete strength resulting from the presence of the confining stirrups was aimed at offsetting the reduction in the flexural capacity of beams due to the presence of shear forces. A method for the evaluation of the flexural capacity of beams in which confinement is present has also been proposed. The approach was extended to the design of over-reinforced beams to enable them to behave in a ductile manner.

A test programme comprising a total of fifty three simply supported beams was used for the development (twelve beams) and for the verification (forty one beams) of the proposed design models. The results obtained confirmed both the applicability and the effectiveness of all of the design concepts and detailing approaches which have been put forward in this research programme. It is believed that the concepts which have been put forward could form a basis for the development of a rational unified approach to the design of structural concrete members.
Chapter 1

INTRODUCTION

1.1 BACKGROUND

The approaches which are currently used for the design of structural concrete beams under the combined actions of shear and bending moment are widely recognised as being inadequate. A number of theories and analytical models have been put forward in an attempt to explain the shear mechanism and thus predict the shear strength of structural concrete members. Consequently, several design models have been developed for the prevention of diagonal failures in beams. None of these approaches, however, have been widely accepted. To date, there is still a lack of understanding with respect to the diagonal failure mechanism. The behavioural mechanisms which have been assumed in many cases are in disagreement with the observed physical behaviour of beams. The majority of the current design methods assume that two mutually exclusive behavioural mechanisms exist simultaneously in a beam in order to resist the applied shear loadings i.e. contributions from concrete in the form of beam and beam/arch actions and from the stirrups in the form of truss action. These methods also ignore the interaction between the actions of shear and flexure in the design of a beam in which they act simultaneously.
The lack of understanding of the mechanical characteristics of concrete at the material level has also contributed to the difficulty in finding a unified rational design approach for structural concrete beams. It is recognised that all types of beam failures occur as a result of the development of transverse tensile stresses in the concrete compression regions within the beam structure. It could therefore be argued that the confinement of these regions with closed stirrups would delay failure and thus enhance the behaviour of beams under such loadings.

The enhancing influence of confining stirrups on the strength and ductility of structural concrete members has been recognised in the literature. However, only a few attempts have been made to utilise this enhancement in the design of structural concrete beams under static loading conditions. This programme of research therefore represents an attempt to utilise confinement stirrups for the prevention of diagonal failures in beams and thus lead to an enhancement in their flexural behaviour. The design approaches which have been developed are based on a thorough understanding of the actual structural behaviour of beams and the actual mechanical characteristics of concrete. They also took into consideration the interaction relationship which exists between shear and flexure and concluded that in effect only one behavioural mechanism exists within the beam structure when subjected to such loadings.

In order to illustrate the significance of this programme of research the issues addressed are briefly discussed in sections 1.1.1 and 1.1.2.
1.1.1 The Riddle of the Shear Problem

During the last 100 years, a large number of research programmes have been undertaken to investigate the behaviour of structural concrete members under the action of shear loadings. Despite the enormous amount of data available from related theoretical and laboratory based research work the prediction of the behaviour of structural concrete beams under transverse loading is still a formidable task. It is only currently possible to determine with any degree of accuracy the ultimate capacity and deformation of beams under pure bending moment. Unfortunately, in practice the majority of beams are subjected to a combination of bending moment and shear. A wide range of analytical models have been proposed for beams under such loadings[1, 2, 3, 4, 5]. In practical design applications; as distinct from general theories, there is still no accurate approach for predicting the load carrying capacity of structural concrete beams under transverse loading. The truss analogy[6, 7] which was originally developed almost a century ago, forms the basis of the design procedures which are present in most current Codes of Practice[8, 9, 10, 11]. This shows that the problem is far from being solved and that there is still a need for the development of a more rigorous approach.

The ACI-ASCE Committee 326[12] report on “Shear and Diagonal Tension” stated that between 1899 and 1960, tests on some 2,500 beams and frames have been reported in the literature. In the same period over 450 papers on this subject have been published in different parts of the world. The introduction to that report closes with the following statement:
"The problems of shear and diagonal tension have not been fundamentally and conclusively solved .......," “Committee 326 wishes strongly to encourage further research work, not only to explore other areas of the problem, but to establish a basically rational theory for effects of shear and diagonal tension on the behavior of reinforced concrete members”. Later in the report, the Committee sums up the difficulties encountered in reaching a satisfactory design procedure for shear as follows:

“It is again emphasized that the design procedures are empirical because the fundamental nature of shear and diagonal tension strength is not yet clearly understood. Further basic research should be encouraged to determine the mechanism which results in shear failures of reinforced concrete members. With this knowledge it may then become possible to develop fully rational design procedures”.

In the ACI-ASCE Committee 426[13] report entitled “The Shear Strength of Reinforced Concrete Members” an attempt was made to establish the state-of-the-art by reviewing research results and design proposals from more than 300 published papers. The following comment was included in the introduction to that report:

"Despite the tremendous number of references on this subject, the question of shear strength is far from being settled. In some instances the explanations of behavior and the design concepts that are presented are somewhat speculative and may change as more information becomes available”.

In 1984 MacGregor[14] described the shear provisions contained within the ACI Code of Practice[8] as “empirical mumbo-jumbo”.
In 1987 Bažant and Sun[15] stated that: "The diagonal shear failure of reinforced concrete beams is classical yet formidable problem that has not been resolved to complete satisfaction despite several decades of study."

Traditionally, the riddle of the shear problem was attributed to the large number of various interrelated factors which were believed to influence diagonal failures. In addition, the complexity of the shear problem may be regarded as being due to the general lack of understanding of the mechanical characteristics of concrete. The assumed diagonal failure and shear transfer mechanisms which were not in agreement with observed structural behaviour have also prevented the development of a rational unified solution.

In conclusion, it is believed that a satisfactory solution to the shear problem can be achieved only through a better understanding of the mechanism of diagonal failures. Research efforts should be directed to investigate the overall structural behaviour of beams under the combined action of shear and bending moment. The design methods for preventing diagonal failures in beams should take into consideration flexure-shear interaction behaviour. They should also consider the enhancing effects of confining stirrups on the ductility and strength of compression concrete.
1.1.2 Influence of Confinement on the Flexural Behaviour of Beams

The majority of published research on confinement has been mainly related to seismic loading conditions. The ductility resulting from confinement was exploited to enable the ultimate flexural capacity of a column to be maintained under the large deformations experienced during earthquakes. On the other hand, the design approaches adopted by Codes of Practice require beams to be under-reinforced in order to prevent brittle compression failures under static loading conditions. When increased ductility due to confinement is provided these limitations are believed to be too restrictive.

The enhancing influence of confinement on the flexural behaviour of beams is widely acknowledged, however, no attempt has been made to utilise this enhancement in the design of beams under static loading conditions. In an attempt to utilise the enhancing influence of the confining stirrups, it is suggested that the longitudinal reinforcement ratio should be allowed to exceed the maximum value specified in Codes of Practice. The brittle compression failures which are characteristic of over-reinforced beams can be prevented by confining the compression concrete with closed stirrups.

1.2 Unified Design Approach

Structures which have shown signs of distress have indicated that the integrity of a structure is dependent on rational design concepts and detailing approaches which
give greater consideration to overall force paths and resisting elements. A more realistic view of structural concrete should put emphasis on the examination of overall structural behaviour, reappraising the assumed mechanisms of failure which do not agree with actual structural behaviour, and studying the flow of forces throughout the structural members.

The basic objective of a rational unified design approach is to eliminate distracting artificial barriers and concepts which compartmentalise the designer's thinking i.e. critical sections, and highly precise calculations which are neither possible nor necessary since they are based on wild guesses of actual stiffness and crude estimates of loadings. Instead, the development of unified design models should be based on highly transparent design-orientated analytical tools which balance accuracy against simplicity. In this regard, Bobrowski[16] commented: "It is a human characteristic to approach an ordinary problem in a simplistic or in a complicated way; however, the resolution of a complex problem in a simple way is a divine privilege".

A unified design approach should ensure reliable structural behaviour at service (working) load levels and also ensure sufficient ductility up to failure. The fail safe behaviour of structures should be the principal aim and brittle failures should be avoided. The performance requirements in the ultimate limit state should include sufficient strength, ductility, clear warning of impending failure, ductile fracture, and possible load redistribution. In the serviceability limit state they must include sufficient rigidity and controlled crack formation.
1.3 OBJECTIVES

This programme of research was aimed at contributing to the development of a unified design approach by offering more realistic design models which are in agreement with the actual physical behaviour of beams. A new design approach for structural concrete beams has been developed in which the enhancing effects of the confining stirrups on the ductility and strength of the concrete compression regions in the beam structure have been utilised. Particular emphasis has been placed on examining the performance of beams at both working and ultimate load levels.

The principal objectives of this programme of research are as follows:

1. To reach a more realistic understanding of the structural behaviour of beams under the action of lateral loadings.

2. To develop a new detailing approach for the stirrup reinforcement in a beam in order to prevent diagonal failures.

3. To develop a flexural-shear interaction design model for the prevention of diagonal failures in beams.

4. To extend the applicability of the flexure-shear interaction design model to address the factors which are traditionally believed to influence diagonal failures in beams including: the shear span to depth ratio, the longitudinal reinforcement ratio, the concrete strength, the shape of the cross section, size effects,
etc.

5. To propose a method for the evaluation of the flexural capacity of beams in which the compression regions have been confined.

6. To develop a method for the design of over-reinforced beams to ensure that they fail in a ductile manner when their compression regions have been confined with closely spaced closed stirrups.

7. To derive a new longitudinal reinforcement ratio ($\rho'_l$) based on confinement under balanced-failure conditions and to compare it with the case in which confinement is not present ($\rho_b$).

1.4 RESEARCH PROGRAMME

The component parts of the research programme are as follows:

1. A detailed examination of the mechanisms of diagonal failure given in the literature. The most realistic concept (the Compressive Force Path (CFP) concept) was subsequently identified and subjected to further development.

2. The theoretical basis of the flexure-shear interaction design model and the detailing approach for the prevention of diagonal failures have been developed with reference to the actual structural behaviour of beams.

3. The applicability of the new approach to detailing of the stirrups was verified experimentally. Eight beams were designed using the provisions of BS 8110[9]
and detailed in compliance with the proposed detailing arrangement.

4. Twelve tests which were required for the development of the flexural-shear interaction design model were carried out on beams. In addition, the results from more than twenty beams which were tested in the initial stages of the investigation as well as results found in literature were analysed.

5. The applicability of the flexure-shear interaction design model was verified experimentally using ten full-size beams.

6. The theoretical basis of the shear-flexure interaction design model was extended to account for the factors which were believed to influence diagonal failures in beams e.g. the shear span to depth ratio, the longitudinal reinforcement ratio, the concrete strength, the size effects, etc.

7. The applicability of the extended flexure-shear interaction design model was verified experimentally using one large-scale beam and seven beams made from high-strength concrete.

8. The design approach for over-reinforced beams has been developed using a theoretical model and the results obtained from tests on beams in which the flexural compression regions were confined with closely spaced closed stirrups. The new balanced-failure longitudinal reinforcement ratio \( (\rho'_b) \) has been based on the proposed approach for the evaluation of the flexural capacity of beams resulting from confinement. The relationship between \( (\rho'_b) \) and \( (\rho_b) \) has been derived theoretically and was found to be a function of the confinement characteristics.
9. The applicability of the design method to over-reinforced beams was verified experimentally using the results obtained from seven beams.

10. The applicability of the flexure-shear interaction design model and the corresponding detailing arrangement for beams with circular cross sections were investigated using test results obtained from eight beams with circular cross sections. The beams were detailed in such a way that different configurations of stirrups were used to confine the concrete compression zone. The proposed method for the evaluation of the flexural capacity resulting from confinement has been verified for beams with circular cross sections using the results obtained from the test programme.

1.5 OUTLINE OF THESIS

The thesis consists of the following eight Chapters:

- **Chapter 1**: Discusses the present trend towards the development of unified approaches to structural concrete design and practice. It outlines the significance, the objectives, and the organisation of the present research work.

- **Chapter 2**: Critically reviews previous research work into the behaviour of rectangular beams under lateral static loadings. Emphasis has been placed on shear transfer and the resulting failure mechanisms, and on the recently developed techniques which aim to solve the shear problem. Conclusions relating to the influence of previous work on this investigation have been outlined.
• **Chapter 3:** Includes the development of a new understanding of the actual structural behaviour of beams under transverse loading. Also, the theoretical basis of a *flexural-shear interaction design model* and a corresponding *detailing approach* for the prevention of diagonal failures in beams have been developed. Tests required for the investigation of the applicability of the proposed detailing arrangement (Test Series 'A') have been also included.

• **Chapter 4:** Includes the tests required for the development of the flexure-shear interaction design model (Test Series 'B'). This Chapter also describes the implementation of the model into the analysis and design of normal-size beams made with normal-strength concrete. In the proposed model, the determination of the amount of stirrups has been based on the confinement requirements. The available confinement models have therefore been reviewed and the most reliable model has been modified to make it applicable to beams subjected to only transverse loading. Experimental verification of the proposed model for the prevention of diagonal failures (Test Series 'C') has been included.

• **Chapter 5:** Extends the applicability of the proposed model to take into account the factors which are traditionally believed to influence diagonal failures in beams. Emphasis has been placed on beam size effects and the concrete compressive strength. Experimental verification using a large beam (Test Series 'D') and several high-strength concrete beams (Test Series 'E') have also been included.
• Chapter 6: Includes the development of methods for the evaluation of the flexural capacity of beams with confinement and for the design of over-reinforced beams utilising the beneficial effects of confinement. A new balanced-failure longitudinal reinforcement ratio ($\rho'_l$) which results from confinement has also been derived. Experimental verification of the design method for over-reinforced beams has been included (Test Series 'F').

• Chapter 7: Critically reviews previous research work into the behaviour of beams with circular cross sections. The need for fundamental tests on beams with circular cross sections has been outlined. The effect of confining the compression zone on the load carrying capacity and ductility of beams with circular cross sections has been investigated experimentally (Test Series 'G').

• Chapter 8: Outlines the main conclusions obtained from the investigation and suggests possible areas of future work in which the enhancing effects of confinement can be utilized in the design of structural concrete elements under static loading conditions and other related subjects.

A description of the experimental work and testing procedures which were common to all the test series are given in Appendix A. The description includes concrete and steel material characteristics, manufacture and preparation procedures of the test specimens, loading arrangement, procedures, measurements, test data, etc. In addition, the thesis includes three Appendices which contain the data and the calculations used in the design and analysis of the beams included in the programme of research.
Chapter 2

LITERATURE REVIEW OF THE BEHAVIOUR OF BEAMS UNDER TRANSVERSE LOADING

2.1 INTRODUCTION

The principal aim of this programme of research has been directed towards the development of simple analytical models for the prediction of the behaviour of beams under transverse static loadings. Accordingly, the most relevant published research work, Code provisions, and basic concepts are critically reviewed in this Chapter. The emphasis has been placed on the nature of shear strength and the approaches which have been used for the prevention of diagonal failures in beams with rectangular cross sections. In addition, the flexural behaviour of beams in general is briefly examined. The conclusions relating to the impact of previous work on the scope of this proposed programme of research are also outlined.

2.2 BEAM CLASSIFICATION

It has been accepted practice for investigations into diagonal failures to test rectangular beams without shear reinforcement under a four-point loading arrangement. The results of such tests have indicated that both the ultimate flexural capacity \( (M_u) \) and the mode of failure depend on the shear span to depth ratio \( (a/d) \). The
results from such investigations are shown in Figure 2.1. The relationship is commonly referred to as Kani’s Valley[17, 18]. It can be noted from Figure 2.1 that the longitudinal reinforcement ratio ($\rho$) only affects the transition point from one type of behaviour to another. A reduction in the value of $\rho$ tends to increase the relative ultimate flexural capacity ($\frac{M_u}{M_f}$) and to either decrease or increase the a/d ratio which marks the transition points for the different types of beams.

Beams are somewhat artificially divided into four groups[5, 17, 19] depending on their perceived failure mode. For comparison and discussion purposes only and with particular reference to Figure 2.1, the following beam classification has been adopted in this programme of research.

2.2.1 Type I (Long Beams)

Type I beams achieve their full flexural capacity ($M_u = M_f$) and fail in flexure. Long beams, in which a/d > 6.0 and the value of $\rho \simeq 1.8\%$, have a shear strength which is higher than their full flexural capacity.

2.2.2 Type II (Normal Beams of Intermediate Length)

In this case beams do not reach their full flexural capacity ($M_f$). Their ultimate flexural capacity ($M_u$) is equal to the diagonal cracking capacity. In normal beams where 2.5 < a/d ≤ 6 and the value of $\rho \simeq 1.8\%$ the ratio ($\frac{M_u}{M_f}$) decreases as the a/d ratio decreases to a minimum value which is dependent on the value of $\rho$. This value is the lowest point in Kani’s Valley.
2.2.3 Type III (Short Beams)

These beams do not reach their full flexural capacity. Short beams, in which $1 < a/d \leq 2.5$ and the value of $\rho \simeq 1.8\%$, have an ultimate shear capacity which is higher than the inclined cracking capacity. The ratio $(\frac{M_u}{M_f})$ increases as the $a/d$ ratio decreases until it reaches unity ($M_u = M_f$).

Beams of types II & III are commonly provided with shear reinforcement in order to ensure that they achieve their full flexural capacity.

2.2.4 Type IV (Deep Beams)

Deep beams, in which $a/d \leq 1$ and the value of $\rho \simeq 1.8\%$, have an ultimate shear capacity which is higher than their full flexural capacity.

2.3 MODES OF FAILURE

The different types of failure which occur in beams unreinforced for shear are described in this section. In order to avoid confusion a consistent approach to the terminology used in this section has been adopted throughout the thesis.

2.3.1 Flexural Failure

In long beams (type I), almost vertical cracks develop in the region of the maximum bending moments. Eventually, these cracks cause failure of the beams as shown in Figure 2.2. The failure is due to either of the following:
(a) excessive yielding of the longitudinal reinforcement, followed by crushing (splitting) of the compression concrete resulting in a ductile failure (under-reinforced beams);
(b) crushing (splitting) of the compression concrete above the flexural crack before yielding of the longitudinal reinforcement which is termed a brittle failure (over-reinforced beams).
These modes of failure are collectively referred to as a “Flexural Failure”.

2.3.2 Diagonal Failure

Shear distress and diagonal failure have been reported in almost all types of structural concrete members. These include, beams, corbels, shear walls, slabs, columns, beam-column junctions, construction and expansion joints, foundations, etc. It is recognised that the cracking pattern and the failure mode may be different for each type of member, but, it is believed that the actual mechanisms by which shear is transferred within members are similar regardless of their structural use.

In general, a diagonal failure occurs under a combination of shearing force and bending moment. Axial load, torsion, or a combination of both may also be present and contribute to failure. Diagonal cracks in webs of either reinforced or prestressed concrete beams may develop regardless of the existence of flexural cracks in their vicinity. Diagonal cracks which occur in beam webs which were previously uncracked due to flexural stresses are referred to as “Web-Shear Cracks”. An inclined crack originating from the tip of a flexural crack and effectively becoming an extension
of this crack is referred to as a "Flexural-Shear Crack" and the flexural crack as an "Initiating Crack". In addition to the two primary inclined cracks (the web-shear and the flexural-shear cracks), other cracks caused by either splitting stresses between the longitudinal reinforcement and the concrete, or by dowel action forces in the longitudinal bars, are referred to as "Secondary Cracks"[19, 20, 21]. The different types of inclined cracks are shown in Figure 2.3.a.

The web-shear cracks are only common in thin-web I-shaped prestressed beams with relatively large flanges[19]. Web-Shear cracks may also be found near a point of inflection and at bar cutoff points in reinforced concrete continuous beams subjected to axial tension[20].

Flexural-shear cracks are common in both reinforced and prestressed concrete beams[22]. In non-prestressed concrete beams, almost vertical flexural cracks are expected to develop under service loads. These cracks cause no distress to the beams until a critical combination of flexural and shear stresses develops near the internal extremity of one of the cracks. At this point the inclined crack forms. The rate of transformation of the initiating crack into a flexural-shear crack depends on the growth and height of the flexural cracks as well as on the magnitude of the shear stresses near the tips of the flexural cracks. The resulting failure modes are as follows:
(i) Diagonal-Tension Failure

Following the formation of flexural cracks in the type II beams one of the diagonal cracks which developed in the shear span continues to propagate through the beam until it becomes unstable. Eventually, the beam collapses as a result of splitting of the compression concrete at the tip of the crack as shown in Figure 2.3.b. This mode of failure is referred to as a "Diagonal – Tension Failure"[13, 23].

(ii) Shear-Tension Failure

In short beams a curved diagonal crack forms in regions subjected to combined shear and bending moment actions which may also lead to the initiation of additional secondary cracks. The secondary cracks may propagate backwards along the longitudinal reinforcement resulting in a loss of bond and anchorage failure as shown in Figure 2.3.b. Eventually, the beam collapses as a result of splitting of the compression concrete. This mode of failure is referred to as a "Shear – Tension Failure"[24] (from reference [13]).

(iii) Shear-Compression Failure

Alternatively, a short beam may collapse as a result of splitting of the compression concrete above the tip of the diagonal crack but there is no accompanying anchorage failure as shown in Figure 2.3.b. This mode of failure is referred to as a "Shear – Compression Failure".
2.3.3 Deep-Beam Failure

In deep beams, after the occurrence of inclined cracking, it has been suggested that these beams behave as a tied-arch as shown in Figure 2.4.a. Five possible modes of failure have been suggested[25, 26, 27] as shown in Figure 2.4.b and described below:

(1) Anchorage Failure

"Anchorage failure" occurs near the support, and may be linked to splitting of the concrete due to dowel action.

(2) Bearing Failure

"Bearing failure" occurs at the supports, when the bearing stresses exceed the bearing capacity of the concrete.

(3) Flexural Failure

"Flexural failure" occurs due to either yielding of the steel reinforcement or fracture of the concrete near the top of the arch.

(4) Arch-Rib Failure Over the Support

"Arch-rib failure" occurs due to the presence of tension cracks over the support.

(5) Arch-Rib Failure Along the Diagonal Crack

"Arch-rib failure" may also occur due to cracking of the concrete along the diagonal cracks bordering the underside of the rib of the arch.
The structural behaviour of deep beams can be studied in isolation\cite{28, 29, 30, 31} as a special case. It is the fundamental structural behaviour of beams which is the principal concern of this programme of research and therefore the emphasis has been placed on the investigation of the behaviour of beam types I, II, and III.

2.3.4 Conclusions

The final collapse of beams undoubtedly occurs as a result of splitting of the compression concrete in spite of all the attempts to identify individual modes of failure many of which centre on the development of either diagonal cracks or bond failure. Therefore, it is believed that collapse of beams is related directly to the state of stress in the concrete compression zone. Consequently, it can be argued that enhancing the load carrying capacity and ductility of beams can be achieved by delaying the splitting of the compression concrete.

2.4 BEHAVIOUR OF BEAMS UNDER THE COMBINED ACTION OF SHEAR AND BENDING MOMENT

2.4.1 Introduction

While, no attempt have been made to review all the previously published work on this subject, basic concepts, Code provisions, and recently developed design approaches have been critically reviewed in this section.
2.4.2 Mechanisms of Shear Transfer

Traditionally, it is assumed that the behaviour and failure modes of beams subjected to shear loading are dependent on the method by which shear is transmitted from one plane to another. The majority of the Codes of Practice assume that shear is transferred through a beam by means of shear stress, aggregate interlock, dowel action, arch action, and shear reinforcement. The different mechanisms of shear transfer which have been assumed are briefly discussed below.

(a) Shear Transfer by Concrete Stress

Assuming concrete possesses no tensile strength in flexure, the maximum shear stress at the neutral axis in a beam subjected to a shear force \( V \) is given by equation (2.1).

\[
\nu = \frac{V}{b.jd} \approx \frac{V}{b.d}
\]  

(2.1)

where \( jd \) is the lever arm of the internal couple.

This equation which was developed by Mörsch[32, page:273] at the turn of this century has been widely used to date as a convenient “index” to measure diagonal tension stress even for cracked beams. The shear stress at failure in most beams is considerably less than the direct shear strength of the concrete. The real concern is with diagonal tension stress, resulting from the combination of shear and longitudinal flexural stresses[33].

At a point below the neutral axis which is subjected to shear stress \( (\nu) \) and
normal tensile stress \((f_t)\), the maximum principal tensile stress occurs on a diagonal plane and can be determined from equation (2.2).

\[
f_{t_{\text{max.}}} = \frac{1}{2} f_t + \sqrt{(\frac{f_t}{2})^2 + v^2}
\]  

(2.2)

The direction of the maximum principal tensile stress is found from equation (2.3).

\[
\alpha = \frac{1}{2} \tan^{-1} \left( \frac{v}{\frac{1}{2} f_t} \right)
\]  

(2.3)

where \(\alpha\) is approximately equal to 45 degrees assuming \(f_t\) is very small.

The maximum principal tensile stress \((f_{t_{\text{max.}}} )\) was linked to the inclined cracking of concrete. When the tensile stresses become excessive, diagonal cracks develop at approximately right angles to the compressive principal stress trajectories.

(b) Interface Shear Transfer

Interface shear, aggregate interlock, shear roughness, shear friction, or tangential shear transfer are different expressions which have been used by researchers to describe the transfer of shear forces along diagonal cracks.

If the shear plane was an existing crack, failure was assumed to involve slippage along the crack as well as movement at right angles to the direction of the crack[34]. In this case shear can be transferred only if either lateral reinforcement or lateral restraint is provided as shown in Figure 2.5.a. This type of shear transfer is referred to as the "shear-friction hypothesis"[8, 35]. Experimental studies on concrete push-off specimens[34, 36] have shown that shear stiffness and strength increase with
increasing reinforcement strength ($\rho f_y$).

If the shear plane is located in monolithic concrete, diagonal cracks normally form across that plane. In this case, it is assumed that failure involves truss action as shown in Figure 2.5.b.

Investigations into aggregate interlock have normally been carried out on pre-cracked specimens. The crack was either partially or totally prevented from widening as the shear forces were increased, as shown in Figure 2.5.c and 2.5.d respectively[37]. Other tests have been carried out on plate specimens cracked using a direct tensile force. The tensile force remained constant while in-plane shear forces were applied across the crack[38].

It is generally assumed that the aggregate interlock mechanism results from a combination of crushing and movement across the faces of the crack. Several models have been proposed to explain and/or predict aggregate interlock behaviour. One of the models[39] put forward two types of interlock.

1. The first is due to local roughness which is related to the interlocking of the fine aggregate particles (crushing action).
2. The other is the global roughness which is related to the interlocking of the coarse aggregate particles (sliding and overriding).

In this model, it was assumed that the predominant action results from the local roughness as long as the initial crack width is less than 0.25mm.
Other models have suggested that aggregate interlock is essentially related to frictional sliding of the two rigid surfaces which are effectively considered to have a saw-tooth shape[40] or to comprise of a series of parabolic segments[41].

A more recent model[42] considers concrete to be a two-phase material in which aggregates are modeled as rigid spheres distributed and embedded to various depths within a deformable rigid-plastic cement matrix. The shear forces are assumed to be resisted by a combination of crushing and sliding of the rigid spheres into and over the softer cement matrix. Contact and interaction between spheres projecting from opposite faces of the crack are neglected. This model was considered[43] to be more representative of the actual behaviour of concrete. However, applying it to reinforced concrete is not straightforward because of the interaction between aggregate interlock and dowel action.

It was concluded, based on test results[37, 38], that the shear stiffness and strength provided by aggregate interlock increase with increasing concrete compressive strength and the size of the aggregate in the matrix, and with decreasing crack width.

(c) Dowel Action

When longitudinal reinforcement crosses a crack, part of the shear force is resisted by dowel action. As a sequence of dowel action splitting cracks running along the bars
may occur as a result of increasing tension in the surrounding concrete combined with the wedging action due to the deformation of the bars. The occurrence of splitting cracks decreases the stiffness of concrete around the bar and decreases the shear strength which in turn reduces the possible contribution from actual dowel action.

(d) Arch Action

It has been argued that a load on a beam is transmitted to the supports through arch and beam actions[17]. The full strength of the two actions cannot be combined because of the assumed incompatibility of the deformation associated with the two mechanisms. It is assumed that there is a transition in the behaviour from beam to arch action[17]. Nevertheless, some of the recently developed models for the evaluation of shear strength are based on the assumption that both mechanisms take place simultaneously[44]. Kotsovos[45] has suggested that loads are transmitted to the supports along a compressive force path i.e. at the ultimate limit state beam action is insignificant.

(e) Shear Reinforcement

Traditionally, shear reinforcement is viewed as tension members in a conventional truss. Although this analogy is helpful in simplifying the design concept it was considered to oversimplify the solution because it does not consider the influence of web reinforcement on the other shear transfer mechanisms. Test results[46, 47, 48, 49] have shown that shear strengths can be up to 80% higher than that predicted by
the truss analogy because of the presence of stirrups.

The role of shear reinforcement has been very controversial among researchers. It was considered that in addition to their direct resistance to shear, they restrict the widening of cracks, maintain aggregate interlock, and increase dowel action[13]. Mphonde[50] argued that the increase in shear resistance was due among other things to the role of the stirrups in enhancing the concrete compressive strength resulting from confinement.

2.4.3 Contribution of Shear Transfer Mechanisms to Shear Resistance

In the design methods adopted by Codes of Practice it is postulated that all the types of shear transfer mechanism occur to widely varying extents in structural concrete members. The shear force \( V \) is assumed to be carried by the mechanisms shown in Figure 2.6.a and are related in equation (2.4) below.

\[
V = V_s + V_c \\
V = V_s + V_{c2} + V_d + V_{ay} 
\]

where:

\( V_s \)  is the resistance due to web reinforcement.

\( V_c \)  is the resistance due to other actions (excluding the shear reinforcement).

\( V_{c2} \)  is the resistance due to compression concrete.
$V_d$ is the resistance due to dowel action.

$V_{ay}$ is the resistance due to aggregate interface action.

A number of experimental investigations[51, 52, 53, 54] have been carried out on beams without shear reinforcement in order to assess the contribution from each of the above. It was concluded from these investigations that the contribution from $V_{cz}$, $V_{ay}$, and $V_d$ varied between 20% - 40%, 33% - 50%, and 15% - 25% respectively[19]. Etebar[36] proposed the relative contributions shown in Table 2.1.

The contribution from all of the internal mechanisms of shear transfer for beams with shear reinforcement is assumed[13] to be as shown in Figure 2.6.b. Figure 2.6.b indicates that before flexural cracking all the shear is carried by the concrete. After flexural cracking but before diagonal cracking has appeared the shear is resisted by $V_{cz}$, $V_{ay}$, and $V_d$. After the occurrence of inclined cracking the shear reinforcement contributes to the resistance of a section ($V_s$). When the stirrups have yielded, any additional shear force is assumed to be carried by the other shear transfer mechanisms. As the inclined cracks widen $V_{ay}$ is reduced and the contributions from $V_d$ and $V_{cz}$ have to increase until failure occurs.

Most ultimate load design procedures, which are based on these shear transfer mechanisms, divide the applied shear into two components. One component is assumed to be carried by the shear reinforcement ($V_s$) and the second component is carried by the other transfer mechanisms, collectively referred to as the concrete.
shear strength \((V_c)\). Empirical relationships\([8, 55]\) and/or tabulated values\([9]\) estimating the shear strength of concrete \((V_c)\) are incorporated in the different Codes of Practice.

2.4.4 Discussion and Conclusions

It is widely accepted that the main contributor to shear resistance in beams is aggregate interlock\([52, 56, 57, 58]\). The concept of aggregate interlock forms the basis of current Code provisions for shear design. Sliding along the crack interface must take place in order to mobilise this action. This concept is, however, incompatible with the observed behaviour of beams which have failed by diagonal cracking. In this case a crack propagates in the direction of the principal compressive stress and opens in an orthogonal direction\([59, 60]\). Kotsovos\([5]\) has argued that if there was a significant sliding movement along the crack interfaces, localised cracks would branch out in all directions along the crack. The occurrence of such crack branching has not been reported to date. Bobrowski\([16]\) stated that aggregate interlock and dowel action are only secondary mechanisms in beams. He emphasised that the principal aspect of a diagonal failure in beams is associated with the stress conditions in the compression zone.
2.5 SHEAR STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT

2.5.1 Introduction

The work described in this thesis is concerned amongst other things with the development of a design model for preventing diagonal failures. The model is based on the assumption that the load carrying capacity of beams without shear reinforcement is known. Some of the models put forward to predict the load carrying capacity of beams without shear reinforcement are, therefore, reviewed in the following sections.

2.5.2 Statistically Developed Models

Many empirical equations have been proposed as a consequence of the results obtained from numerous investigations. In these equations, factors which were believed to influence the shear strength were considered. The factors included concrete strength \( f_c \), concrete type (e.g. normal-weight and light-weight concrete), longitudinal reinforcement ratio \( \rho \), cross section dimensions, axial load level, and if, present prestressing force.

Equation (2.5) is included in BS 8110[9] for the prediction of the nominal concrete strength \( V_c \) for beams under the action of shear and flexural stresses.

\[
V_c = v_c bd = 0.79 \left( \frac{100A_s}{bd} \right)^{\frac{1}{3}} \left( \frac{400}{d} \right)^{\frac{1}{3}} bd^{-1} \gamma_m (N, mm)
\]

(2.5)

where:

\( \gamma_m \) is the partial safety factor = 1.25.
The relationship may be multiplied by \( \left( \frac{f_{cu}}{25} \right)^{\frac{1}{3}} \) when \( 25 \text{ MPa} < f_{cu} < 40 \text{ MPa} \), where \( f_{cu} \) is the cube compressive strength of the concrete.

BS 8110 also limits the applied shear stress \( (v = \frac{V}{bd}) \) to either \( 0.8\sqrt{f_{cu}} \) or 5 MPa, whichever is lower in order to prevent crushing of the web.

The ACI Code of Practice[8] puts forward either equation (2.6) or, as a more accurate form, equation (2.7).

\[
V_c = 0.17\sqrt{f'_c bd} (N, \text{mm}) \tag{2.6}
\]

\[
V_c = \left( 0.16\sqrt{f'_c} + 17.2\rho \frac{V_d}{M} \right) bd < 0.29\sqrt{f'_c bd} (N, \text{mm}) \tag{2.7}
\]

where \( f'_c \) is the cylinder compressive strength of the concrete.

Test results[61] have shown that the ACI equations were not conservative for low values of \( \rho \) and subsequently Rajagopolan and Ferguson[61] proposed the following equation:

\[
V_c = (0.066 + 8.3\rho)\sqrt{f'_c bd} (N, \text{mm}) \quad \rho < 0.0012 \tag{2.8}
\]

The CEB-FIP Model Code[10] gives equation (2.9).

\[
V_c = 0.092\sqrt{f_{cu}(1 + 50\rho)K bd} (N, \text{mm}) \tag{2.9}
\]
where:

\[ \rho < 0.02. \]

\[ K = 1.6 - d(m) > 1.0. \]

The Model Code also limits the applied shear stress to 30% the compressive strength of the web concrete in order to prevent crushing of the web.

Several relationships have been proposed by researchers. Equation (2.10) and equations (2.11-2.13) which were derived by Rafia[62] and by Zsutty[63, 64] respectively are the more widely accepted relationships. It was found [65] that the relationships put forward by Zsutty closely predicted the shear strength of beams even those made from high strength concrete.

\[
V_c = \alpha_u \frac{\rho^{\frac{1}{3}} f_{cu}^{\frac{1}{2}}}{d^{0.25}} \frac{7}{8} bd \quad (N, \text{mm})
\]

(2.10)

where:

\( \alpha_u \) is an empirical coefficient which is dependent on the \( \frac{a}{d} \) ratio. Its value can be found in reference [62].

Zsutty put forward the following equations:

For normal beams of intermediate length

(a) At diagonal cracking
\[ v_{cr} = 59 \sqrt{f_{c'} \rho d/a} \text{ (psi)} \]  (2.11)

(b) At ultimate capacity

\[ v_u = 63.4 \sqrt{f_{c'} \rho d/a} \text{ (psi)} \]  (2.12)

For Short Beams

\[ v_u = \left( \frac{2.5}{d/a} \right) \left[ 59 \sqrt{f_{c'} \rho \frac{d}{a}} \right] \text{ (psi)} \]  (2.13)

2.5.3 Physical Models

These can be grouped into two categories. The first is based on strength criteria and includes the concrete tooth, concrete arch, and strut and tie models. The second category is based on stability criteria and includes models which are derived from fracture mechanics considerations.

Several models have been proposed for each type of diagonal failure including secondary failure modes. The most important models for the principal failure modes are briefly reviewed below.

(a) The Comb-Like Structure Behaviour (Concrete-Tooth Models)

In an attempt to solve the riddle of the shear problem Kani[17] postulated that a reinforced concrete beam under increasing load and without shear reinforcement, transforms into a comb-like structure. The concrete blocks between the almost vertical flexural cracks represent the teeth and the compression concrete represents the
backbone of the concrete comb.

The diagonal failure mechanism of beams with effectively unbonded longitudinal reinforcement which was assumed is shown in Figure 2.7.a. In this case, the concrete body is mainly under diagonal compression (struts) and hence, diagonal failure cannot be expected to occur.

In the case where bond exists between the longitudinal bars and the concrete, the strut line tends to bend up to the right due to the steeper force resultant near the end support. In this case, the behaviour of the concrete tooth was compared to that of a short cantilever (corbel) subjected to the horizontal internal force ($\Delta T$) as shown in Figure 2.7.b.

The load carrying capacity of the beam at which the concrete tooth breaks ($M_{cr}$) was determined based on the simplified concrete tooth representation shown in Figure 2.7.b and given by equation (2.14).

$$M_{cr} = \frac{7}{8} \frac{f_t^2}{6} \frac{bd^2}{s} \frac{\Delta X}{d} a = M_o \frac{\Delta X}{s} \frac{a}{d}$$

(2.14)

Failure was assumed to occur when the tensile strength of the concrete at the root of the concrete cantilever was reached (Kani’s hypothesis).

On the basis of Kani’s hypothesis, the acceptance of beam theory “i.e. plane section remains plane after bending” is incorrect and leads to inconsistencies in the
shear strength theory.

When the concrete tooth fractures, the beam is assumed to be transformed into a tied arch as shown in Figure 2.8.a. The transformation may occur suddenly (for type II beams) or gradually (for type III beams).

The load carrying capacity of the remaining arch was given by equation (2.15) based on the conditions shown in Figure 2.8.b.

\[ M_{cr} = \frac{M_f}{0.9} \frac{d}{a} \]  \hspace{1cm} (2.15)

where \( M_f \) is the full flexural capacity of the beam \( \simeq \frac{7}{8} d A_s f_y \).

It was concluded, based on Kani's hypothesis and validated by his test results, that the load carrying capacity of a beam varies with respect to the \( a/d \) ratio (\( \alpha \)) as shown in Figure 2.1. The behaviour of beams was divided into three categories depending on the value of the \( a/d \) ratio. The different types of behaviour are as follows:

1. For small \( a/d \) ratios (\( \alpha \leq \alpha_{min} \)), the concrete tooth capacity is lower than the capacity of the remaining arch. In this region, transformation of a beam into an arch takes place gradually when the concrete tooth capacity has been exceeded.

2. For \( \alpha_{min} < \alpha < \alpha_{tr} \), the capacity of the arch is lower than the capacity of the concrete tooth. Therefore, transformation into an arch occurs suddenly.
In this case a beam fails suddenly when the concrete tooth capacity has been exceeded.

3. For $\alpha > \alpha_{tr}$, a beam reaches its full flexural strength i.e. a diagonal failure is not expected to occur.

The concrete-cantilever model was later modified by other researchers to account for dowel and aggregate interlock actions as shown in Figure 2.9. These actions were assumed to be initiated by tangential displacements along the crack faces.

In the model developed by Hamadi and Regan[66], the loading of the tooth was divided into two complementary systems. One system comprised partly of the bond force, aggregate interlock, and dowel action. The other system consisted of the remaining part of the bond force and the moment at the head of the tooth. The relationship given by equation (2.16), was subsequently put forward.

$$v_u = \frac{1}{2}(q_1 + \sqrt{q_1 + q_2}) \quad (N, \text{mm})$$

(2.16)

where:

$$q_1 = 1.75(E_c(10)^{-5} + r_d).$$

$$q_2 = 67.4(10)^{-7} K E_s a \frac{109.4}{bd}. $$

$$\tau_d = 4.12(d_b)^{0.5} b_n \sqrt{f_{cu}}. $$

$$b_n = b - \Sigma d_b. $$

$$K = 1.2 \text{ for gravel concrete.}$$
$d_b$ is the longitudinal bar diameter.

$E_s$ is the elastic modulus of the steel.

$E_c$ is the elastic modulus of the concrete.

Recently, Reineck[67, 68] developed a model for slender beams based on the modified concrete tooth concept. In this model, the state of the stress in the web was represented by a truss-model with concrete tensile struts. Reineck put forward equation (2.17) which takes into account size effects.

$$V_u = \frac{b d (0.4 f_t) + V_d}{(1 + 0.16 \frac{f_c}{f_t} \lambda (\frac{a}{d} - 1))}$$

where

$$\lambda = \frac{f_c}{E_s \rho} \frac{d}{\Delta_{nu}}.$$  

$V_d$ is the dowel resistance.

$\Delta_{nu}$ is the critical crack width.

$f_c$ is the concrete tensile strength.

**Appraisal of the Concrete-Tooth Models**

Detailed investigations have been carried out to evaluate the state of stress in the concrete cantilevers[52, 57]. The hypothesis based on concrete cantilever action does not explain why the critical diagonal crack invariably initiates near the tip of the flexural crack closest to the support. It also does not explain why failure always occurs in the support region of the cantilever (spalling of the flexural compression...
concrete) rather than in the concrete cantilever itself as is assumed in this model.

Bobrowski[16] considered that relating the ultimate strength of a member to the stress condition below the tip of the crack was a gross over-simplification. Instead, the real failure mechanism should be related to the actual state of stress which exists in the compression concrete. It should be emphasised that this failure condition exists in several structural concrete members as was pointed out by Bobrowski and is shown in Figure 2.10.

In the light of the comments above, it may be concluded that the concept of a comb-like structure does not offer a better understanding of the diagonal failure mechanism.

(b) Models Based on Arch Action (Strut and Tie Model Using Theory of Plasticity)

A number of models have been proposed, based on arch action[62]. Among these are the strut and tie models and the shear-compression theories.

The shear-compression theories[69, 70, 71] are based on the assumption that the end of the beam is supported in such a way that any rotation takes place about a plastic hinge at the tip of the critical inclined crack. This implies that the relative displacement is normal to the crack, which was considered to be correct only if there was no shear deformation[72].
The mathematical theory of plasticity has been applied to structural concrete beams by Nielsen and Braestrup[4, 73, 74]. The mechanism of diagonal failure was assumed to be as shown in Figure 2.11. A vertical deformation was assumed to occur in a yield line inclined to the axis of the beam at an angle equal to $\beta$ as shown in Figure 2.11.a. The concrete and the steel were assumed to have rigid and perfectly plastic material characteristics. The tensile strength of the concrete was ignored. It was also assumed that the longitudinal reinforcement did not yield and failure was due solely to the crushing of the concrete.

The upper and the lower bound solutions were identical in accordance with either the assumed failure mechanism or the stress field shown in Figure 2.11.a and Figure 2.11.b respectively.

The ultimate shear strength ($v_u = \frac{V}{b h}$) was given by equations (2.18) and (2.19).

\[
\frac{v_u}{f_c} = \frac{\nu}{f_c} \left( \sqrt{\frac{4\omega \nu - \omega}{\nu^2} + \left(\frac{a}{d}\right)^2 - \frac{a}{d}} \right) \quad \text{for } \omega \leq \frac{\nu}{2} \quad (2.18)
\]
\[
\frac{v_u}{f_c} = \frac{\nu}{f_c} \left( \sqrt{1 + \left(\frac{a}{d}\right)^2 - \frac{a}{d}} \right) \quad \text{for } \omega > \frac{\nu}{2} \quad (2.19)
\]

where:

$\nu$ is the effective concrete strength factor introduced to limit the usable concrete compressive strength.

$\omega = \frac{d_x f_y}{b h f_c}$. 
\( a \) is the distance between the edges of the support and the loading plates.

**Appraisal of Applying the Plasticity Theory to Structural Concrete**

In order to obtain good agreement with the test results, a complex expression for the effective concrete strength was necessary to account for all of the factors which were believed to affect the shear strength[62]. Nevertheless, Bázant[75] has criticised the application of the plasticity based approaches to brittle failures.

It was deduced[76] that the upper-bound solution put forward by Nielsen and Braestrup did not satisfy moment equilibrium for the rigid elements. Alternatively, Kemp and Al-Safi[76] proposed a rigid-plastic-upper-bound solution, which satisfied both force and moment equilibrium. In the solution, steel was assumed to yield at failure.

(c) **Fracture Mechanics Models**

Instability was used to determine failure in the models which have been developed from fracture mechanics. It was assumed that structures do not exceed their material strength level. The failure occurs when a critical crack length is reached. In this case, the energy necessary to create new surfaces is smaller than the energy released which results in unstable crack growth and a brittle failure. Hawkins, Mattock, and Wyss used this approach to propose equation (2.20) for a section uncracked in flexure[77].
\[ V_c = 2.87(bh)^{0.75} \sqrt{f_{cu}} \left( \frac{h}{2} < b < h \right) \quad (kN, \text{mm}) \quad (2.20) \]

where:

- \( h \) is the overall depth.
- \( b \) is the width.

For a section already cracked in flexure, they proposed to multiply equation (2.20) by a reduction factor \( \beta \) where:

\[ \beta = 1.07 - \frac{0.006 M}{\sqrt{\rho} \sqrt{h}} \leq 1.0. \]

Several models, based on fracture mechanics, have recently been proposed\[\text{75, 78, 79}\]. A combination of arch and beam actions and a gradual transition from strength criteria to instability criteria have been assumed in these models. Băzant and Kim\[\text{75}\] proposed equation (2.21) which also took into account size effects (Băzant size-effect law).

\[ v_u = K_1 \rho^p \left( (f'_c)^q + K_2 \sqrt{\rho} \left( \frac{3}{2} \right)^r \right) \left( 1 + \frac{d}{\lambda_o d_a} \right)^{-\frac{1}{2}} \quad (2.21) \]

Where \( K_1, K_2, \rho, q, \lambda_o, \) and \( r \) are empirical parameters obtained from the results of tests on geometrically similar specimens. The expression for the mean ultimate nominal shear strength given by equation (2.22) was put forward after a statistical comparison was made of the available test data.

\[ v_u = \frac{10 \sqrt{\rho}}{\sqrt{1 + \frac{d}{25d_a}}} \left( \sqrt{f'_c} + 3000 \sqrt{\frac{\rho}{(3/2)^5}} \right) \text{ psi} \quad (2.22) \]
Different forms of the size-effect law were proposed\[15, 78\] in order to account for the maximum aggregate size and the results obtained from several test programmes.

**Appraisal of Fracture Mechanics Models**

A comparison between experimental data and equation (2.20) highlighted the inaccuracy of the predicted average shear strength \( \frac{V_{\text{measured}}}{V_{\text{calculated}}} \) varied between 0.83 and 1.22\[80\].

The size-effect law was suggested for use in large structures and for ensuring the serviceability requirements of all beam sizes\[78\]. Nevertheless, it was deduced that the size-effect law model was over-conservative for extremely large beams\[79\]. In addition, in order to obtain good agreement with the test results six empirical parameters \((K_1, K_2, p, q, \lambda_0, \text{and } r)\) were introduced into the model. This reduces the model to one of an empirical form.

**2.5.4 Discussion and Conclusions**

There is a considerable difference of opinions among researchers regarding the diagonal failure mechanism of beams without web reinforcement.

- In the models which were based on the concrete-tooth action, the failure was assumed to be associated with the stress condition below the neutral axis. In the original model\[17\], the failure was assumed to occur when the tensile strength at the root of the concrete cantilever was reached (Kani's hypothesis). In the modified models\[66, 67, 68\] failure was assumed to be governed by
aggregate interlock, dowel action, and concrete tensile strength. Krefeld and Thurston[81] and Chana[82], however, have suggested that dowel action at the level of the longitudinal reinforcement initiates the failure of the beams.

- In the strut and tie models, failure was assumed to occur when the concrete compressive strength was reached[69, 71, 73, 83].

- Theorems based on fracture mechanics[84, 85, 86] are solely concerned with the instability of the diagonal crack and do not consider a crushing failure or splitting due to dowel action.

- Bobrowski[16] and Kotsovos[45] have deduced that failure takes place in the compression zone as a result of the development of a state of stress in which tensile stresses initiate failure of the beams.

Many other relationships have been proposed for predicting the nominal shear capacity of structural concrete beams[13, 23, 44, 69, 87, 88, 89, 90, 91, 92, 93, 94]. They are all found to be very similar to those reviewed above apart from the use of different coefficients.

It is believed that the nominal shear strength of beams is not a good indicator of the load carrying capacity of beams at diagonal failure since it is not an indicator of the actual state of stress which exists and results in the failure of beams. Nevertheless, the development of a theoretical model to evaluate the shear strength of beams without web reinforcement is not the principal aim of this programme of research. However, in order to develop the required analytical model to evaluate the
transverse reinforcement requirements it was necessary to review all the principal published models.

2.6 SHEAR STRENGTH OF BEAMS WITH WEB REINFORCEMENT

2.6.1 Introduction

Traditionally, in order to minimise the risk of having undesirable brittle diagonal failures, shear reinforcement is introduced into regions subjected to high shearing stresses. The basic philosophy of the current Codes of Practice is to ensure that stirrups restrain the growth of inclined cracking, increase ductility, and give adequate warning in situations in which diagonal cracking may result in a failure.

This section critically reviews some of the assumed shear mechanisms and the solution techniques for beams with web reinforcement which have been put forward in an attempt to clarify the shear problem.

2.6.2 Truss Analogy

(a) Truss Analogy-Original

In the pioneering work by Ritter (1899)[95] (from reference [13]) and Mörsch (1909)[7] (from reference [13]), it was postulated that after diagonal cracking a beam with web reinforcement can be replaced by an imaginary pin-connected truss as shown in Figure 2.12.a.
The web of the equivalent truss consisted of stirrups acting as vertical tension members and concrete struts running at 45 degrees parallel to the diagonal cracks. The concrete compression zone acts as a compression chord and the flexural reinforcement acts as a tension chord.

In the original truss analogy, the shear strength of the vertical stirrups \( (V) \) was calculated using equation (2.23) which was based on only equilibrium requirements i.e. compatibility of deformation was not considered.

\[
V = A_v f_v \frac{jd}{s_v}
\]  

(2.23)

where:

\( jd \) is the lever arm of the internal couple.

\( s_v \) is the spacing of the stirrups.

The contribution of the concrete to shear resistance was ignored in this relationship.

(b) Truss Analogy-Improved

The original truss analogy was considered to be over-conservative in estimating the shear reinforcement requirements because it ignored the shear forces which were assumed to be carried by concrete and by dowel action[13]. Also, actual crack patterns indicated that the diagonal cracks were not inclined at an angle of 45 degrees.
Talbot[96] (from reference [13]) in 1907 deduced from test results that shear strength is dependent on the concrete strength, the amount of flexural steel, the length of the beam, and the amount of shear reinforcement. He also indicated that the stirrup stresses were smaller than those predicted by the relationships based on the truss analogy approach and that part of the shear force must be carried by the concrete. Richard[97] (from reference [13]) reached a similar set of conclusions in 1927.

In attempt to improve the truss analogy, the changes in either the inclination of the compression chord or the slope of the concrete struts were proposed as shown in Figure 2.12.b (improved truss analogy)[56, 98].

(c) Truss Analogy-General Case

In another modification aimed at generalising the truss analogy approach only part of the shear force was assumed to be carried by the stirrups \( V_s \), while the remaining shear force \( V_c \) was assumed to be resisted by other actions. It was also recognised that concrete struts may form at an angle which was not equal to the assumed value of 45 degrees. The general case of the modified truss analogy is shown in Figure 2.12.c. The required amount of shear reinforcement \( A_v \) was determined from equation (2.24) which was based only on equilibrium requirements.

\[
A_v = \frac{v_s}{\sin \beta (\cot \theta + \cot \beta)} \frac{s_v b}{f_y v} \quad \text{(2.24)}
\]

where:
\[ v_s = v - v_c. \]

\( v_s \) is the shear stress carried by the stirrups = \( \frac{v}{bd} \).

\( v \) is the total shear stress = \( \frac{v}{bd} \).

\( v_c \) is the shear stress carried by the concrete = \( \frac{v}{bd} \).

\( f_{uv} \) is the yield strength of the stirrups.

\( s_v \) is the stirrup spacing.

\( \theta \) is the inclination of the diagonal struts.

\( \beta \) is the inclination of the transverse reinforcement.

In this relationship, it was assumed that all the stirrups had reached yield.

The compression stress in the concrete struts \( f_{cd} \) was found from equation (2.25).

\[ f_{cd} = \frac{v_s}{\sin^2(\cot \theta + \cot \beta)} \quad (2.25) \]

In the case where \( \beta = 90^\circ \) and \( \theta = 45^\circ \) (common truss analogy), the required amount of shear reinforcement \( (A_v) \), and the compression stress in the concrete strut \( (f_{cd}) \) were obtained from equations (2.26) and (2.27) respectively.

\[ A_v = \frac{v_s s_v b}{f_{uv}} \quad (2.26) \]

\[ f_{cd} = 2v_s \quad (2.27) \]

47
Most Codes of Practice assume that $\theta = 45$ degrees and that the contribution of the concrete is equal to the shear force carried by beams without shear reinforcement when diagonal cracking first appears. This estimate has been judged[13] from test results to be conservative. Also, the actual diagonal cracking process indicated that $\theta$ varies along the span of the beam. Steeper diagonal cracks were usually found to develop near the loading points.

The optimum angle of $\theta$ is about 38 degrees[32, page:296] based on strain energy considerations. The CEB-FIP 1990 Model Code[55] suggests that the flattest angle $\theta$ is equal to 18.4 degrees ($\cot \theta = 3$). The limiting ranges for $\theta$ in the 1984 Canadian[99] and the 1987 ACI draft[100] Codes of Practice are 15 degrees to 75 degrees and 25 degrees to 65 degrees respectively. Thürlimann[101] has suggested that in order to prevent excessive inclined crack widths, $\theta$ should not be less than 25.6 degrees.

When $\theta$ is less than 45 degrees, the number of stirrups which intersect a diagonal crack is more than those encountered when the inclination of the crack is assumed to be 45 degrees. This is frequently given as one of the reasons why design equations based on the truss analogy tend to be conservative.

Flat diagonal concrete struts and steep stirrups imply larger compression stresses in the concrete struts. In order to prevent crushing of the concrete struts, most Codes of Practice place a limit on the applied shear stresses e.g. BS 8110[9] limits
this value to either $0.8\sqrt{f_{cu}}$ or 5.0 MPa whichever is the minimum. However, other factors which will affect the compression stresses in the concrete struts are believed to exist. These include:

1. Secondary moments which are present because of the absence of true “pin joints”.

2. Stirrups transfer tensile stresses thus producing a state of biaxial stress in the concrete struts. It is known that the concrete compression strength is reduced significantly due to the existence of these transverse tensile stresses.

3. The compression force which is applied at the imaginary truss joint is far from being evenly distributed across the web.

4. Some inclined cracks might form at an angle considerably less than the assumed value of $\theta$ and as a result compression stresses in the concrete struts will tend to be much higher than predicted.

Different forms of the truss analogy have been proposed by a number of researchers[13, 62, 80, 100, 102, 103, 104, 105]. These models included the option to vary the angle of inclination of the diagonals. The design techniques used in some of these models were related to the ultimate and the serviceability limit state conditions.

(d) Discussion and Conclusions

Generally, the truss analogy is considered to be a powerful tool in the study of the behaviour of reinforced concrete beams with web reinforcement. It permits the determination of stirrup stresses, compression stresses in the concrete struts, and
shows the effect of varying the inclination of the stirrups upon these stresses.

The classical 45-degree truss model which was put forward by Mörsch has been adopted by most Codes of Practice as a basis for their shear and torsion design provisions. The original simple and straightforward approach was subsequently obscured by many empirical modifications such as those which exist in the design procedures for a section under either flexure and axial force or shear and torsion. As a result, rather than being simple and generally straightforward the approach has become complex, empirical, and too restricted.

This analogy was considered[13] to over-simplify the shear design problem. This is because, the influence of stirrups in the enhancement of aggregate interlock and dowel action was ignored. This reason was also given as an explanation for the conservative nature of the results obtained from the truss analogy.

The truss analogy cannot be applied to all static and geometric conditions including those involving discontinuities. This analogy would not give safe solutions when applied to beams failing due web-shear cracking since failure occurs before yielding of the stirrups. Kuttab and Haldane[106] have tested beams in which stirrups did not extend down the entire depth of the beams. As such, truss action could not possibly have developed yet the beams were found to have reached their full flexural strengths. The behaviour of such beams cannot be explained in terms of the assumptions put forward in the approach based on the truss analogy.
Despite these shortcomings and limitations, the truss analogy is still to date the only basis for designing reinforced concrete structures in many Codes of Practice[8, 9, 10, 11].

2.6.3 Strut-and-Tie Models

(a) General

The standard truss model was found not to be applicable to all types of members, particularly at static and geometric discontinuities. In these cases, approaches based on available test results, rules of thumb, and past experience were usually applied.

In order to apply a design concept to all parts of any structure a generalised form of the truss analogy was proposed in form of strut-and-tie-models[1, 107].

(b) Model Elements-Development

In the strut-and-tie model, the reinforced concrete is considered to carry loads through a set of compressive fields (struts) which are distributed and interconnected at nodes by either reinforcement bars (ties) or concrete tensile stress fields. The general form of the concrete strut is assumed to be in the shape of a prism, a bottle, or a fan as shown in Figure 2.13. The struts and ties are designed by orientating them along elastic stress fields and, if necessary, modifying them to conform to the prevailing practical considerations.
Nodes are considered to represent the intersection of three or more assumed linear stress fields which themselves represent either stress fields, or reinforcing bars. This implies that there is an abrupt change in direction of the force in the node region. For struts and ties which represent concentrated stress fields, the nodes are referred to as either singular or concentrated nodes. If they are representative of wide stress fields or several distributed reinforcing bars, the forces may be smeared (or spread). In this case they are called smeared (or continuous) nodes.

For design purposes, a beam is divided into "D"- and "B"-regions (near the loading points, openings, corners, or bends, and the rest of the beam respectively). Where "B" stands for beam, or Bernoulli, and "D" stands for discontinuity, disturbance, or detail. In B-regions, the Bernoulli hypothesis of plane stress distribution is assumed to be valid. These regions can easily be designed to a high degree of accuracy prior to cracking. After cracking, normally one of the truss models can be applied. In D-regions, the strain distribution is nonlinear. In the case where the concrete is not cracked, regions are designed using linear elastic analysis. After cracking, the strut and tie models are used. The boundary conditions for these regions are obtained using a general structural analysis approach and the results from the B-regions.

In D-regions the most appropriate load carrying model is suggested to be obtained assuming that loads are transferred along a pre-determined path with minimum forces and deformations. This assumed criteria of optimisation was modeled
using equation (2.28).

\[ \sum F_t L_t \epsilon_{m_i} = \text{minimum} \]  

(2.28)

where:

- \( F_t \) is the force in the strut or tie member.
- \( L_t \) is the length of the member.
- \( \epsilon_{m_i} \) is the main strain in the member.

(c) Compressive Strength of Cracked Concrete (\( f_{cd'} \))

The compressive strength of concrete (\( f_{cd'} \)) in compression fields or within nodes is assumed to be dependent on the actual state of stresses and the disturbance resulting from the cracks and the reinforcement. Collins[108] has suggested that \( f_{cd'} \) is a function of the strain perpendicular to the direction of the principal compressive stress. Kollegger and Mehlhorn[100] concluded from test results that \( f_{cd'} \) is more accurately described as a function of the transverse tensile stresses rather than the transverse tensile strain. The 1978 CEB Model Code[10] and the First Draft of the 1990 CEB-FIP Model Code[55] gave \( f_{cd'} \) as a function of the shape of the cracks. The existence of cracks which were not parallel to the compression stress fields is considered to be detrimental to the load carrying capacity of the member.

It was suggested[1] for practical purposes that the concrete compressive strength (\( f_{cd'} \)) in the concrete struts, and in the nodal zones should be taken as \( 1.0 f_{cd} \) for a
prismatic compressive field type. When tensile strains or tensile reinforcement result in cracks parallel to the compressive stresses, $f_{cd}$ was to be taken equal to $0.8 f_{cd}$. If the cracks or reinforcement were not parallel to the compressive stresses, $f_{cd}$ was taken as $0.6 f_{cd}$ and in the case where cracks were very wide, $f_{cd}$ was taken as $0.4 f_{cd}$.

Based on the CEB-FIP Model Code[10], $f_{cd}$ can be found from equation (2.29).

$$f_{cd} = \frac{0.85 f'_c}{\gamma_c}$$  \hspace{1cm} (2.29)

where: $\gamma_c = 1.5$ (partial safety factor).

It should be emphasised that in the strut and tie model, the increased strength due to the existence of the transverse compression stresses which may result from confinement is normally neglected. Also, it was considered that the cracks would not develop if the theory of elasticity was closely followed i.e. the inclination of the struts should not be at a too shallow angle.

(d) Model Elements-Dimensioning

In the case of the singular nodes, it was assumed that forces normally balance each other in the interior of the node throughout direct compressive stresses. For practical purposes, the anchorage and lap length requirements from Codes of Practice were suggested[1]. In the design of the nodes, the geometry of the node is determined by the applied forces. The concrete pressure is checked to ensure that it is within the allowable concrete compressive strength limits. This is automatically
satisfied if the stresses at the boundaries and the anchorage requirements are satisfied. If singular nodes are safe, it is not considered necessary to check smeared nodes.

In the design of a compression strut, it was assumed that fan and prismatic shaped stress fields do not develop transverse stresses and therefore the concrete uniaxial strength can be used. Transverse tensile stresses are assumed to develop in the case of bottle shaped compression stress fields. In this case, the required concrete compressive strength is obtained from the shape of the stress field[107]. In the case of tensile stress fields in uncracked concrete it has been suggested that the tensile strength of the concrete is used.

In relation to serviceability, it has been suggested that satisfying detailing provisions is normally better than using sophisticated crack calculation techniques. Nevertheless, the same model was assumed to be valid for the serviceability limit state.

(e) Appraisal of the Strut-and-Tie Model

The strut and tie model has proved to be a very powerful technique in the solution of geometric and static discontinuities[83, 109, 110, 111, 112, 113, 114, 115, 116]. A disadvantage of this model is that a different set of internal forces and hence member sizes is required for each loading case. As a result, multiple load cases must be considered separately and the use of different loading cases may require different
models. Such an approach is generally regarded as being too tedious for the design of conventional beams.

In addition, the model is dependent on the theory of elasticity (modified if necessary to comply with practical considerations) in order to develop the struts and ties which provide the internal structural system. It is also assumed that the structure will comply with such a system. The adoption of the theory of elasticity in ultimate load design approaches does not reflect the actual behaviour of the concrete. In general, structures do not behave in the way they are modeled. The reason for this is that modeling is usually based on concepts which do not give full consideration to important aspects of the behaviour of concrete. Relating the load carrying capacity of a beam to the strength of the cracked-concrete web implies an acceptance of the concepts of concrete softening and aggregate interlock mechanism. These concepts have been proved to be incorrect[16, 45, 117]. In this model, the confinement effect of the stirrups on the strength and the ductility of concrete have not been adequately considered.

2.6.4 Diagonal Compression Field Theory

(a) General

In 1929 Wanger[108] proposed a theory to predict the post-buckling shear resistance of thin metal beams. Wanger assumed that after buckling, metal would not resist compression and that shear would be carried by a diagonal tensile field. In 1978 Collins[108] investigated the applicability of this theory to structural concrete. He
assumed that after cracking, concrete cannot resist tension, and that shear would be carried by a diagonal compression field. The ultimate shear capacity of a member was assumed to be reached either when the longitudinal or the transverse steel reached yield, or when the average concrete compressive stress reached its limiting value.

The theory was first developed for rectangular sections with symmetrical arrangements of longitudinal reinforcement. The stirrups were assumed to be perpendicular to the beam axis and it was also assumed that the crack widths would be controlled. The effect of bending moment and local disturbances were neglected. The average stresses and strains were considered in the approach, Figure 2.14.a.

(b) Lateral Load Capacity of Beams

At ultimate load, an upper limit for shear capacity was set by assuming yielding of the longitudinal steel. The resulting ultimate shear strength \( (\upsilon_u) \) was given by the following equation:

\[
\upsilon_u \leq \sqrt{(\rho t f_{yt}) \left( \rho f_y + \rho_p f_{yp} + \frac{N}{b f_d} \right)} \leq \sqrt{f_{du} \rho t f_{yt} - (\rho t f_{yt})^2}
\]  

(2.30)

where:

\( f_{yt} \) is the yield strength of the transverse reinforcement.

\( f_{yp} \) is the yield strength of the prestressing tendons.

\( f_{du} \) is the average diagonal compressive stress in the beam.
\( N \) is the applied axial force.

\( \rho_t \) is the reinforcement ratio for the transverse steel and is equal to \( \frac{A_{st}}{b'P_p} \).

\( \rho_p \) is the reinforcement ratio of the prestressing steel.

A difficulty exists in determining the limiting value of the average diagonal compressive stress \( f_{du} \) in a beam, however, from the analysis of 153 shear tests on simple T-beams Nielsen and Braestrup\[4\] proposed that \( f_{du} = 0.72f'_c \). Collins put forward the following relationship to account for the strain state in the concrete:

\[
\frac{f_{du}}{f'_c} = \frac{3.6}{1 + \frac{2\epsilon_m}{\epsilon_o}}
\]

where:

\( \epsilon_m = \epsilon_t + \epsilon_l + 2\epsilon_d \).

\( \epsilon_t \) is the transverse strain in the concrete.

\( \epsilon_l \) is the longitudinal strain in the concrete.

\( \epsilon_d \) is the diagonal strain in the concrete.

\( \epsilon_o \) is the strain in concrete corresponding to the maximum compressive stress.

To determine the shear strength of a beam, the equations above were rearranged in the following form:

\[
\left[ 2 \left( 1 + \frac{1}{n\rho_{tt}} + t^2 \right) \left( \frac{1}{1 + t^2} \right) \left( \frac{v_u}{f'_c} \right)^2 \right] + \left[ 1 - \frac{2t}{n\rho_{tt}} \frac{\epsilon_o}{1 + \frac{1}{t^2}} \left( \frac{v_u}{f'_c} \right) - \frac{3.6t}{1 + t^2} \right] = 0.0
\]

(2.32)
where:

t represents \( \tan \alpha \) \( (\tan \alpha = \frac{e_\alpha}{v_u}) \).

\( n \) is the modular ratio \( = \frac{E_s}{E_c} \).

\( E_c \) is the modulus of elasticity of the concrete.

\( E_s \) is the modulus of elasticity of the steel.

\( \varepsilon^* \) is the strain parameter which indicates the intensity of the prestress and the axial load.

\( \rho_{tt} = \rho_t + \rho_p \).

(c) Discussion and Conclusions

The compression field theory attempted to outline a framework for developing a rational theory for evaluating not only the shear strength of all types of structural concrete elements, but also their overall load-deformation response. However, because of the ideal conditions considered, which rarely exist in practice, and because of the large number of assumptions required to develop this theory, it was very difficult for it to be adopted as a rational approach for the solution of the shear problem.
2.6.5 Modified Compression Field Theory

(a) Introduction

Vecchio and Collins[3, 118] proposed the modified compression field theory because of the perceived limitations in the original compression field theory. The modified theory studied the plane state of stress which influences the concrete compressive strength as well as the presence of the tensile stresses between cracks which had been ignored in the original approach.

(b) Modified Behavioural Characteristics of Structural Concrete

In the modified theory, it was assumed that the strain in the concrete was equal to that in the steel. The principal stress axes were assumed to coincide with the principal strain axes in the concrete. The relationships between the principal stresses and principal strains were evaluated for both tension and compression stresses using Mohr's circle, Figure 2.14.b. The principal compressive stress ($f_{c2}$) was given as a function of the compressive strain ($\varepsilon_2$), and the corresponding tensile strain ($\varepsilon_1$).

The average stress/strain relationship which was derived is given below:

$$ f_{c2} = f_{c2\text{max}} \left[ 2 \left( \frac{\varepsilon_2}{\varepsilon_0} \right) - \left( \frac{\varepsilon_2}{\varepsilon_0} \right)^2 \right] $$

(2.33)

where $\frac{f_{c2\text{max}}}{f_0} = \frac{1}{0.8 - 0.34 \frac{\varepsilon_2}{\varepsilon_0}} \leq 1.0.$

The inclination of the compression fields was given by the following equation.
\[ \tan^2 \alpha = \frac{\epsilon_l + \epsilon_d}{\epsilon_l + \epsilon_d} \] (2.34)

The full behavioural response of the structural concrete members in shear can be predicted using the compatibility condition given in equation (2.34), the equilibrium equation of the truss model, and the stress-strain relationship of the concrete and the steel.

(c) Generalisation of the Modified Compression Field Theory

In another attempt to simplify the design approach and to generalise the theoretical approach in order to make it applicable to any shape of cross section, Vecchio and Collins[119] proposed dividing the cross section into layers and treating the concrete and steel layers separately. The principle of plane sections remaining plane after bending was assumed. The equilibrium conditions included:

1. Balancing of vertical shear, moment, and normal forces.

2. Balancing of the horizontal shear.

In order to evaluate the shear strength using this approach it was necessary to estimate the longitudinal strain and the shear distribution. The longitudinal stresses in the steel were found directly from the strain in each layer. The longitudinal stresses in the concrete layers were found from the modified compression field theory. If the conditions of equilibrium were not satisfied, the assumed longitudinal strain was required to be modified. This procedure was repeated until equilibrium was achieved.
(d) **1984 Canadian Code of Practice**

Collins and Mitchell[2] outlined the new design procedure for shear which was included in the 1984-Canadian Code of Practice[99]. The approach was largely based on the modified compression field theory. In the proposed design procedure, the inclination of the diagonal compression was given an arbitrary value assumed to be constant along the length of the beam. The longitudinal and the transverse strains in the concrete were assumed to have a value equal to 0.002. For sections subjected to variable shear, the “staggering” concept[120, 121] in which a section is designed for a shear force less than that indicated by the shear diagram was adopted. The strut and tie models were recommended for the design of the D-regions.

(e) **Conclusions**

The compression field theory can be considered to be an attempt to promote a rational theory for solving the shear problem. It is based on equilibrium and compatibility considerations as well as material characteristics.

On the other hand, this theory is suited to conditions, where stress trajectories are parallel and the shear distribution is uniform. The traditional truss analogy, however, offers a simpler and an adequate solution to these conditions. The theory was not applied, to static and geometric discontinuities (D-regions). Instead the Canadian Code of Practice adopted strut and tie models. In addition, the design was based on the concept of critical sections for shear (sectional design) rather than considering the overall behaviour of the beam under load (member design).
The enhancing influence of stirrup confinement on the strength and the ductility of concrete was also not considered. The modified compression field theory cannot therefore be considered as a rigorous and straightforward approach which could be followed for the solution of different shear problems in a similar manner.

2.6.6 Plasticity Theory Model

The mathematical theory of plasticity[4, 73, 74, 122] was applied to beams with web reinforcement. In this approach, the shear resistance was obtained by equating the internal and the external work done in a beam under the assumed deformation pattern shown in Figure 2.11.c. The ultimate shear strength \( v_u \) was given by equations (2.35) and (2.36).

\[
\frac{v_u}{f'_c} = \sqrt{\lambda(K_o - \lambda)} \quad \text{for} \quad 0.0 < \lambda \leq \frac{K_o}{2} \quad (2.35)
\]
\[
\frac{v_u}{f'_c} = \frac{K_o}{2} \quad \text{for} \quad \lambda > \frac{K_o}{2} \quad (2.36)
\]

where:

\[ v_u = \frac{V}{bd'} \]

\( d' \) is the truss height (distance from the centre of the compression zone to the tensile reinforcement).

\[ \lambda = \frac{v_u f'_{yw}}{b_d x f'_c} \]

\( K_o \) is the effective concrete strength factor (concrete compressive strength = \( K_o f'_c \)).
Campbell[123] showed that the rigid plastic theory over-estimates the amount of reinforcement required for a balanced section when the elastic deformation was considered. It also over-estimates the shear capacity of a section which is over-reinforced in shear.

2.6.7 Equilibrium Analysis

It was suggested that the provision of shear reinforcement be based on the shear-compression theory[13] for short beams failing in shear-compression. In the analogy, equilibrium was satisfied by summing moments about a point in the compression zone above the tip of the inclined crack.

Regan[56] assumed that failure was caused by normal tensile stresses in the compression zone of the beam. These stresses were obtained using equilibrium and approximate compatibility equations. Regan admitted that the resulting equation for the ultimate shear strength was too complex and recommended the use of a graphical or other related types of solutions.

2.6.8 Arch Action Theory

The remaining tied arch theory which was developed by Kani[17] for beams unreinforced for shear was extended to beams with web reinforcement. The transverse loading was assumed to be carried by arch action. Kani[124] postulated that after cracking a beam was transformed into a number of tied arches hanging into the compression zone by stirrups. Only the outer arch was supported directly at the
supports. The purpose of stirrups, based on this theory, was to provide reactions for the internal concrete arches which support the compression zone, and not to carry the shear force as widely accepted and adopted in Codes of Practice. This theory was intended to be regarded as a rational approach, however, it is a qualitative and impractical theory[16].

2.6.9 Compressive Force Path Concept

(a) Introduction

The aim of the Compressive Force Path (CFP) concept was to promote a better understanding of the behaviour of reinforced concrete beams under transverse loading and to produce a more realistic explanation of the causes of diagonal failure. In this concept the applied loads were assumed to be carried to the supports along a compressive force path.

(b) Shape of the CFP

The shape of the CFP as shown in Figure 2.15.a is based on the diagonal failure mode. The failure mode characterising type II behaviour, is represented by a curved path comprising two intersecting and almost linear portions connected by a smooth transitional curve. The failure mode, characterising type III behaviour, is represented by an almost linear path connecting the load point to the support.

The CFP can be visualised as a flow of compressive stresses with varying cross-sections perpendicular to the path. The compressive force represents the resultant
of the stresses at each section[5]. In the case of a simple beam at the ultimate limit state, the shape of the path was considered to be bi-linear (horizontal and inclined legs) as shown in Figure 2.15.a. The horizontal projection of the inclined leg is approximately equal to 2d in the case of two point loading with $a/d > 2$, and also equal to 2d in the case of uniformly distributed loads with span to depth ratios ($l/d$) greater than 6.0. For smaller $a/d$ or $l/d$ ratios the point at which the force direction changes was assumed to coincide with the load points. In the case of uniformly distributed loads it was assumed that the load can be replaced by an equivalent two-point loading positioned at the third points along the span.

(c) Causes of Failure

In this approach, the failure was related to the development of transverse tensile stresses in the region of the path along which the loads were transmitted to the supports.

Some of the reasons for the development of the tensile stresses, Figure 2.15.a, are detailed below.

1. The change in the path direction produces a tensile force ($T$). This is to satisfy the equilibrium requirements at that location.

2. The variation in the intensity of the compressive stresses along the horizontal leg of the CFP which results in the development of tensile stresses ($t_1$).

The highest stress intensity exists at the point where the cross section of the
compression zone is smallest. The adjacent concrete provides restraint (confinement) at this section. A critical level of stress intensity would be reached before the stresses in the adjacent sections reach similar intensities. This critical stress level marks an abrupt and large dilation in the concrete which induces transverse tensile stresses in the surrounding concrete.

3. Large tensile stresses \( (t_2) \) develop perpendicular to the compressive stress trajectories in the region of the crack tip\([59, 60, 125]\).

4. Bond failure between the longitudinal reinforcement and the concrete results in changes in the compressive stress distribution in the zone between two consecutive flexural cracks, as shown in Figure 2.15.b. The rise of the neutral axis at the right-hand side of the crack which is required to maintain equilibrium after loss of bond can be noted from Figure 2.15.b. The change in the intensity of the compressive stress produces tensile stresses in the adjacent concrete region in a similar way to that discussed previously \( (t_1) \).

The behaviour of type II and type III beams is discussed below based on the compressive force path concept.

(d) Behaviour of Type II Beams

An analytical representation of the two applied actions (shear and bending) was derived empirically by Bobrowski and Bardhan-Roy\([87]\). This expression was modified slightly by Kotsovos\([126]\) as detailed below:
\[ M_u = 0.875 S d \left( 0.342 b_1 + 0.3 \frac{M_l}{d^2} \sqrt{\frac{Z}{S}} \right) \sqrt{\frac{16.66}{\rho f_y}} \quad (N, \text{mm}) \quad (2.37) \]

where:

- \( S \) is the distance of the cross-section from the support i.e. the shear span for a two-point loading system and \( 2d \) for uniformly distributed loading.
- \( Z \) is the lever arm.
- \( b_1 \) is the effective width.

Transverse reinforcement will be required if \( M_u \) is less than \( M_l \). The amount of transverse reinforcement required to resist the development of the tensile forces was determined as follows:

(a) A significant tensile force (\( T \)) develops in the region where the compressive force path changes its direction as shown in Figure 2.15. The action of the transverse reinforcement not only balances the vertical component (\( V \)) of the inclined compression, but also subjects the concrete block at that region to a compressive force (\( D \)), Figure 2.15.c. This force balances the shear force (\( V \)) acting at the right-hand side of the above concrete block. This action is considered to be activated only when the concrete carrying capacity is exceeded. The amount of transverse reinforcement required was determined[127] as follows:

\[ T_{sv} = V = V_{app} - V_c = A_{sv} f_{yy} \quad (2.38) \]
where:

$V_{app}$ is the applied shear force.

$V_c$ is the portion of the $V_{app}$ sustained by concrete alone = $M_u/S$.

$A_{sv}$ is the transverse reinforcement required to resist $T$.

(b) In the case of point loading, it was confirmed that it was possible that a significant tensile force developed within the horizontal leg of the CFP in the region of the point-load. In such a case, the shear capacity may be lost when bond failure occurs between two adjacent flexural or inclined cracks. For simplicity, the internal force condition, shown in Figure 2.15.b, was considered.

The required transverse reinforcement was determined as follows:

1. $\Delta Z = VX/2T; \quad V = V_{app} - V_c$.

2. $X' = 2(d - Z - \Delta Z) > 0.0; \quad$ for $X' < 0.0$, the cross section should be enlarged.

3. $\sigma'_c = C/b X'$.

4. For $f_{\text{triaxial}} = 0.8 f'_C$

   $\sigma'_c = 0.8 f'_C + 5 \sigma_{\text{confinement}}$.

   where $\sigma_{\text{confinement}} = \frac{\sigma'_c - 0.8 f'_C}{5}$

5. Assume $\sigma_t = -\sigma_{\text{confinement}}$.

6. $T_{sv} = \sigma_t b d$.

7. $A_{sv} = T_{sv}/f_{yy}$.
In this case, the transverse reinforcement is to be placed in the horizontal leg region starting at a distance equal to $d/2$ from the point at which the path changes direction.

(e) Behaviour of Type III Beams

In this type of beam, failure is associated with a large reduction in the depth of the neutral axis in the region of the tip of the main inclined crack, which in turn will lead to the development of tensile forces within the compression zone. As a result, a compressive-tensile state of stress exists. A failure can be prevented in this case by either, providing stirrups to carry the tensile stresses or by enhancing the concrete compressive strength. Since, calculating the tensile stresses is difficult it was proposed[128] to enhance the compressive strength by providing stirrups throughout the horizontal projection length of the inclined leg of the CFP. It was proposed that the required transverse reinforcement, in reference to Figure 2.15.d, would be determined using the following relationship:

$$T_{sv} = \frac{2(M_f - M_u)}{a} = A_{sv} f_{yu}$$  \hspace{1cm} (2.39)

It was emphasised[129] that shear reinforcement should be extended outside the shear span region for a distance equal to the depth of the neutral axis.

(f) Conclusions

It can be concluded that the compressive force path concept has the advantage of considering the overall behaviour of the beam (member design). It also offers a
realistic explanation based on a better understanding of concrete at the material level, of the causes of diagonal failure. The failure, in this context, is related to the actual state of stress in the compression zone of the beam structure (CFP) in which the transverse tensile stresses initiates the failure.

It is believed that an artificial classification of beams and the corresponding individual solution techniques have been imposed unnecessarily. In addition, an active confinement model was used for determining the stirrup requirements for type II beams, although, the confinement for such a case is a passive one. Moreover, the detailing arrangement for type II beams implies that stirrups may not be needed in the shear span near the supports. However, exploratory tests[130] which were conducted as part of this programme of research indicated that such detailing (absence of stirrups for a distance equal to 1.5d from the support) may result in a brittle diagonal failure.

2.7 BEHAVIOUR OF BEAMS IN FLEXURE

2.7.1 General

The currently accepted theory for evaluating the flexural capacity, equilibrium, and compatibility requirements of beams is based on the assumption that plane sections remain plane after bending. Concrete compressive and longitudinal reinforcement stresses are found by considering the uniaxial stress-strain relationships for the two materials. Transverse stresses in the concrete are assumed not to influence the be-
haviour of the beams and are therefore ignored. Figure 2.16 shows the strain profiles used to determine the flexural strength at a section in a beam. The longitudinal reinforcement ratio ($\rho$) is used to determine the type of failure. When $\rho < \rho_b$ (at the balanced-failure condition), $\varepsilon_s > \varepsilon_y$ and hence the longitudinal steel yields and a tension failure occurs. When $\rho > \rho_b$, $\varepsilon_s < \varepsilon_y$ and a brittle failure occurs in the concrete compression region without yielding of the longitudinal reinforcement.

2.7.2 Multiaxial Stress Behaviour

The stress-strain relationships for concrete under uniaxial loading are shown in Figure 2.17.a[129]. Figure 2.17.a shows the ascending and descending parts of the curves for the longitudinal strains developed. The transverse strains show an abrupt increase in value just before the load reaches its peak level. The volumetric strain ($\varepsilon_v = \Delta V/V$) relationship indicates that the peak load is reached when the volume reaches its minimum level at which point an abrupt increase in transverse strain is initiated.

To investigate the validity of using the uniaxial stress-strain relationship to describe the actual behaviour of concrete in the design of beams, the longitudinal and the transverse strains were measured in the region of the beams subjected to maximum bending[131]. The results from these measurements are shown in Figure 2.18. Figure 2.18 indicates that while the first part corresponding to the ascending portion of the curve is correct the other part corresponding to the descending portion is totally different.
It was concluded, from the above test results, that in the case of the uniaxial concrete compression tests, the descending portion of the longitudinal strain relationship does not exist. The appearance of this part was attributed to secondary effects resulting from the test machine i.e. due to the restraint between the specimen and the loading platens of the test machine. Kotsovos and Cheong[132] tested prisms under different boundary conditions. In order to minimise the friction between the specimens and the loading platens of the test machine the axial load was applied through loading plates smaller than the cross section of the specimen. The results from the tests confirmed that the true stress-strain relationship consists of only the ascending portion of the curve as shown in Figure 2.17.b.

It can be concluded from the above discussion that the uniaxial stress-strain characteristics cannot describe the post ultimate behaviour of the compression concrete in beams under bending action i.e. the descending portion does not exist. The ascending portion of the curve only partially describes the response of the specimens because it ignores the multiaxial state of stress which develops during the later stages of loading at about 90 percent of the failure load. Test results have indicated increases of up to 75 percent in the uniaxial concrete compressive strength in the compressive stresses in beams[131].

In cases where there are no stirrups a flexural failure may be related to the development of a multiaxial state of stress resulting from dilation of the concrete
in a localised region within the compression zone. This region coincides with the position of a deep flexural crack. The localised transverse expansion of the concrete is restrained by the concrete in the adjoining regions. This may be considered to have the same effect as confinement. At the same time, this restraint induces tensile stresses in the adjacent areas of concrete. These tensile stresses reduce significantly the compressive strength of the concrete and eventually initiate failure by splitting of the compression zone in the region between two flexural cracks. The crushing of the compression concrete appears to be a post-failure phenomenon which occurs as a result of the loss of the restraint in the adjacent concrete regions.

2.7.3 Over-Reinforced Beams

Most Codes of Practice limit the reinforcement ratio ($\rho$) to a value less than $\rho_b$ in order to provide warning of impending failure (e.g. in the ACI Code of Practice[8] $\rho \leq 0.75\rho_b$).

Test results[133] have indicated that confining the concrete compression regions with helical binding improves the ductility of beams. Furthermore, by utilising the enhanced ductility, it was possible for over-reinforced beams to fail in a ductile manner. Therefore, it can be concluded that design recommendations which exclude over-reinforced beams in practice, in order to avoid brittle failures, are too restrictive when confinement is present.
2.7.4 Conclusions

The evaluation of the multiaxial stress conditions which develop within the compression zone as the ultimate load is approached is difficult. In accordance with the approach to structural concrete which is based on simplicity rather than misleading precision, is considered sufficient for practical purposes to assess the flexural capacity on the basis of the rectangular stress block as specified by Codes of Practice.

It should be noted, however, that despite the presence of a multiaxial state of stress in concrete compression regions, Codes of Practice do not always insist on providing these regions with transverse reinforcement in order to restrain the development of the tensile stresses which significantly reduces the concrete compressive strength. The provision of transverse reinforcement in columns implies that compression concrete requires such reinforcement. In the case of a beam this requirement depends mainly on whether or not shear stresses are present. This inconsistency in Codes of Practice appears to have originated from the adoption of design principles based on an uniaxial state of stress. The introduction of confining stirrups in the compression zone in the beam structure will enhance the strength and ductility of the concrete which in turn prevents brittle failures which are characteristic of the behaviour of over-reinforced beams.
2.8 GENERAL REMARKS AND CONCLUSIONS

2.8.1 General

The literature review has highlighted the inadequacy of current design approaches for structural concrete beams subjected to the combined action of shear and bending moment. The design approaches assume an artificial separation between the two actions. A beam is first designed for flexure using the moment envelope and then it is designed for shear using the shear envelope (section design). The interaction between shear and bending is usually ignored, or at best dealt with empirically. In the design for shear these models assume that part of the shear is carried by the concrete ($V_c$) by beam and/or arch actions. The stirrups are assumed to carry the shear ($V_s$) in excess of the concrete capacity through truss action. It is believed that the assumption of the presence of different load resisting mechanisms is behind the confusion in the Code provisions for shear described by MacGregor[14] as "empirical mumbo jumbo". In addition, $V_c$ is normally related to the strength of the cracked concrete below the level of the neutral axis through the so called aggregate interlock action which, if indeed it exists, is to be regarded as only a secondary mechanism[16, 45]. The design methods adopted by the different Codes of Practice do not relate the failure of beams to the actual state of stress which exists in the region of the path along which loads are transmitted to the supports.

The following general conclusions regarding the behaviour of beams under transverse loadings have emerged from the literature review:
2.8.2  Flexure Behaviour of Beams

1. The flexural failure of beams occurs as a result of the development of a multiaxial state of stress resulting from the dilation of the concrete in a localised region within the compression zone. The evaluation of the multiaxial stress state in this region is difficult. The acceptance of the unified design approach which is based on simplicity rather than misleading precision, it is considered sufficient for practical purposes to assess the unconfined flexural capacity of beams using the rectangular stress block specified by the Codes of Practice.

2. Test results[133] have indicated that confining the compression concrete with closed stirrups improves the ductility of beams. Furthermore, it was possible to make over-reinforced beams fail in a ductile manner. Therefore, it is concluded that the limitations on the longitudinal reinforcement ratio ($\rho$) imposed by Codes of Practice are too restrictive when the compression concrete is confined with closed stirrups. In this case the concrete compression block used for the determination of the flexural capacity of beams must be related to the amount of confinement provided.

2.8.3  Behaviour of Beams Under the Combined Action of Shear and Bending

1. Behaviour of beams under the combined action of shear and bending is currently not well understood. An accurate mathematical model to represent shear in structural concrete is still elusive. Several models have been proposed for the solution of the shear problem. At the national level, a lot of prominence has been given to recently developed techniques. At the international
level, none of these techniques have been widely accepted. At present, there is an urgent need for unified and consistent rational analytical and design approaches for the entire range of structural concrete members. The difficulties in reaching a unified design approach arise from the following factors:

(a) A lack of understanding of concrete at the material level: The adoption of uniaxial stress-strain curves based on uniaxial compression tests do not represent the actual behaviour of the compression concrete in beams.

(b) The inability to define the actual mechanism of diagonal failure: Current accepted models for shear design, excluding perhaps those based on the CFP concept, link diagonal failure to the state of stress in the cracked concrete below the neutral axis. This implies that the load carrying capacity of beams depends on the residual strength of cracked concrete, assuming concrete exhibits strain-softening material characteristics. Concepts such as; critical sections for shear, "staggering" effects, and the mechanism of aggregate interlock, are usually considered in these proposed models. These models have proved not to agree with the actual physical behaviour of beams failing by diagonal cracking. Although failure starts to develop from diagonal cracks in the web, the failure occurs as a result of splitting of the compression concrete. This indicates that the stress state in the concrete compression region plays a more significant role in diagonal failures than that assumed in the traditional design approaches. Therefore, rational detailing and design procedures for preventing diagonal failures in beams should be based on the restraint of the tensile
stresses in the concrete compression regions of the structure of the beam.

2. Traditional design models for shear consider the nominal ultimate shear strength \(v_u = \frac{V}{b d}\) as an indicator of the load carrying capacity of beams at diagonal failure. The evaluation of \(v_u\) as given by Codes of Practice is based on results obtained from a large number of tests. The variation in \(v_u\) may reach 1500% depending on the value of \(\rho\) and the a/d ratio[17]. The relative flexural capacity of beams \(\left(\frac{M_u}{M_f}\right)\) represents a more realistic indicator of the load carrying capacity of beams at diagonal cracking and therefore is used in this research programme. The results from tests on beams have shown that all the values of \(M_u\) range between 50% to 100% of \(M_f[17]\).

3. Rational design models for preventing diagonal failures should account for the presence of shear on reducing the flexural capacity of beams (flexural-shear interaction design approach). The evaluation of the amount of stirrups needed to prevent diagonal failure should be related to the level of the applied bending moment which is usually larger than the flexural strength predicted from flexural theory.

2.8.4 Unified Design Approach

The aim of this programme of research is to develop a rational unified analytical model for the prediction of the behaviour of beams under the combined action of shear and bending moment resulting from the application of transverse static loadings.
In the following Chapters, physical models based on experimental results and theoretical approaches leading to analytical-predictive models have been developed for structural concrete beams.
Contributions to Shear Resistance

<table>
<thead>
<tr>
<th>Parameters Investigated</th>
<th>Long Beams</th>
<th>Deep Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Interlock</td>
<td>8.48%</td>
<td>7.12%</td>
</tr>
<tr>
<td></td>
<td>25-40%</td>
<td>5.42%</td>
</tr>
<tr>
<td></td>
<td>25-50%</td>
<td>7.13%</td>
</tr>
<tr>
<td>Dowel Action</td>
<td>22.42%</td>
<td>31.43%</td>
</tr>
<tr>
<td></td>
<td>40-60%</td>
<td>30-50%</td>
</tr>
<tr>
<td></td>
<td>32-55%</td>
<td>33-42%</td>
</tr>
<tr>
<td>Compression Zone</td>
<td>10-62%</td>
<td>45-65%</td>
</tr>
<tr>
<td></td>
<td>0-35%</td>
<td>2-65%</td>
</tr>
<tr>
<td></td>
<td>20-32%</td>
<td>45-60%</td>
</tr>
</tbody>
</table>

Table 2.1: Percentage Contributions to Shear Resistance.
Figure 2.1: Classification of Beams Based on Kani’s Valley.

Figure 2.2: Flexural Failure Mode.
Figure 2.3: (a) Types of Diagonal Cracking. (b) Modes of Diagonal Failure.

Figure 2.4: Deep-Beam Failures. (a) Arch Action. (b) Types of Failures.
Figure 2.5: Mechanism of Aggregate Interlock. (a) Shear-Friction Hypothesis. (b) Formation of Truss Action. (c) Partial Lateral Restraint. (d) Full Lateral Restraint.

Figure 2.6: Traditional Concepts of: (a) Mechanisms of Shear Transfer. (b) Effect of Web Reinforcement on Shear Capacity.
Figure 2.7: Behaviour of Concrete-Cantilever. (a) Longitudinal Reinforcement without Bond. (b) Ideal Concrete Cantilever.

(after Kani[17])

Figure 2.8: Kani's Hypothesis: (a) Arch Action. (b) Process of Transformation into a Tied Arch.

(after Kani[17])
Figure 2.9: Behaviour of the Modified Concrete-Tooth.

Figure 2.10: Modes of Failure. (a) Axial Compression Failure. (a') Column Failure.
(b) Flexural Failure. (c) Diagonal Failure.
Figure 2.11: Mechanisms of Failure Based on the Theory of Plasticity. (a) Upper-Bound Solution. (b) Lower-Bound Solution. (c) Beam with Web Reinforcement.

Figure 2.12: Truss Analogy. (a) Original Truss Analogy. (b) Improved Truss Analogy. (c) Modified General Truss Analogy.
Figure 2.13: Basic Types of Compression Fields. (a) The Fan. (b) The Bottle. (c) The Prism. 

(after Schlaich et al.[107])

Figure 2.14: Compression Field Theory. (a) Free-Body Diagram of a Beam Section. (b) Compatibility Conditions for Average Strains in Concrete.
Figure 2.15: (a) Compressive Force Path. (b) Effect of Bond Failure. (c) Equilibrium Conditions at Force Changing Direction. (d) Equilibrium Condition for Type III beams.
Figure 2.16: Strain Profiles at the Flexural Strength of a Section.

Figure 2.17: Uniaxial Stress-Strain Relationship. (a) Typical Curves. (b) Effects of Boundary Restraints.

(after Kotsovos[129])
Figure 2.18: Longitudinal Strain-Transverse Strain Relationships For a Uniaxial Compression Test and for Flexure.

(after Kotsovos[131])
Chapter 3

NEW APPROACH TO THE DESIGN OF BEAMS UNDER TRANSVERSE LOADING

3.1 INTRODUCTION

The main objective of the design of structural concrete beams is to achieve full flexural capacity and to prevent a brittle diagonal failure. This is normally achieved by providing transverse reinforcement in the critical regions of the shear spans. The actual structural behaviour of beams has shown that a diagonal failure starts from the development of cracks in the region below the neutral axis and extends up into the compression zone. Eventually, collapse occurs as a result of splitting of the compression concrete. A new design approach for the prevention of diagonal failures in beams utilising the effect of confinement of the concrete compression region has been put forward in this Chapter. The proposed design model is only concerned with the determination of the confinement requirements (the amount of stirrups required to prevent a diagonal failure). However, to develop the model it is essential that a better understanding of the actual structural behaviour of beams under transverse loading is put forward. This can be achieved by using more realistic concepts for the diagonal failure mechanisms.
3.2 REALISTIC DIAGONAL FAILURE MECHANISM: Compressive Force Path (CFP) Concept

It was concluded in Chapter 2 that the difficulty in the solution of the shear problem exists because of the difficulty in defining the actual mechanism of diagonal failure and also because of the lack of understanding of the true behaviour of concrete at the material level. It is realised that the most important fundamental characteristic of concrete is its splitting strength and not its compressive strength as implied in most Codes of Practice. In the Compressive Force Path (CFP) concept, the causes of diagonal failure are related to the actual state of stress in the region of the CFP. Failure occurs as a result of the development of secondary transverse tensile stresses in the concrete compression zone. It is interesting to note that in the design of beams with large openings, the compression regions (top chords) are considered to resist all of the applied shear[134, 135, 136, 137]. It was also noted[135] that when openings were placed in the tension region of beams, their ultimate capacity was not correspondingly reduced. These observations also show the important role that the concrete compression regions of the beam structure have in resisting lateral loadings. The CFP concept, therefore, offers a more realistic explanation of the causes of diagonal failures in beams.

3.3 STRUCTURAL BEHAVIOUR OF BEAMS FAILING BY DIAGONAL CRACKING

The structural behaviour of various types of beams is discussed in this section from a theoretical perspective and also using the results available from tests on beams. The
variables in the test beams included the shear span to depth ratio, detailing of the transverse reinforcement, and the amount of longitudinal reinforcement. The beams were tested under static loading using a typical four-point loading arrangement in which the mid-span region was subjected only to a constant bending moment and the two shear spans were subjected to a constant maximum shear and linearly varying bending moment.

In the following discussion, the possible failure mechanisms in the various regions of the load path are based on the CFP concept as shown in Figures 2.15 and 3.1. The classification of the beams has been based on Kani's Valley as shown in Figure 2.1.

3.3.1 Beams without Web Reinforcement

In type II beams, the diagonal failure, Figure 3.2.a, occurs as a result of the development of tensile stresses in the compression concrete in the horizontal leg region of the CFP inside the shear span (region 1 in Figure 3.1) and in the region where the compressive force changes direction (region 2 in Figure 3.1). The tensile stresses which develop in the inclined leg regions as a result of the presence of shear (region 3 in Figure 3.1) are not significant enough to cause a final tensile-compressive state of stress which may lead to failure. In this case the final state of stress is compression-compression as shown in Figure 3.2.a. It can be concluded that for this type of beam the stress conditions present in the horizontal leg region are more severe than those which exist in the inclined leg region. The load which is transmitted to the supports (at a constant moment level) decreases as the a/d ratio increases. As a result, the
relative ultimate flexural capacity at diagonal failure \( \left( \frac{M_u}{M_f} \right) \) increases. The trend is towards full flexural behaviour with increasing a/d and a decrease in the value of \( \rho \) i.e. decreasing the load carried at failure. On the other hand, as the a/d ratio is reduced and the value of \( \rho \) is increased, the load carried to the supports (under constant moment) increases. The \( \frac{M_y}{M_f} \) ratio thus decreases until it reaches a critical value which corresponds to the lowest point in Kani's Valley.

In the type III beams, a relatively large load needs to be transmitted to the supports in order for them to reach their full flexural capacity \( (M_f) \). The subsequent large shear force results in the development of tensile stresses, which influence the behaviour of the inclined leg regions (region 3 in Figure 3.1). A critical tensile-compressive state of stress develops in the concrete compression region at the tip of the diagonal crack as shown in Figure 3.2.b. The critical tensile-compressive stress state which develops in the inclined leg region eventually causes failure. In this case, the state of stress in the horizontal leg (regions 1 and 2 in Figure 3.1) appears to be less critical. The load transmitted through a beam increases (to reach the full flexural capacity) as the a/d ratio reduces which means that the critical tensile-compressive stress state will become more severe. This implies that contrary to experimental evidence the \( \frac{M_y}{M_f} \) ratio should have decreased. However, in the case of type III beams the \( \frac{M_y}{M_f} \) ratio increases as the a/d ratio decreases. This apparent contradiction can be illustrated by noting that as the a/d ratio decreases more load is transmitted directly to the support with no corresponding increase in the shear stresses. As a result, the flexural capacity at diagonal failure \( (M_u) \)
increases. The increase in the flexural capacity continues until it reaches the full flexural capacity. The trend as the a/d ratio decreases is therefore towards that of deep beam behaviour. For type III beams the effect of the longitudinal reinforcement ratio ($\rho$) is the same as for the type II beams i.e. as the value of $\rho$ increases the full moment capacity ($M_f$) increases which requires a relatively large load (shear) to be transferred to the supports in order to reach the full flexural capacity level. As a result, at diagonal failure, the $\frac{M_s}{M_f}$ ratio decreases.

### 3.3.2 Beams Traditionally Detailed for Shear

In the type II beams, the critical tensile stresses which develop in the horizontal leg (regions 1 and 2 in Figure 3.1) are restrained by the stirrups as shown in Figure 3.3.a. Other possible causes of failure which influence the inclined leg (region 3 in Figure 3.1) although less critical, are also restrained. The absence of confinement stirrups in the mid-span region does not prevent beams from reaching their full flexural capacity, however, it reduces their ductility.

In type III beams, the critical tensile-compressive stress state which exists in the inclined leg region inside the shear span is restrained. However, beams may fail by spalling of the compression concrete outside the shear span but close to the loading points as shown in Figure 3.3.b because of the absence of confinement at that location.
3.3.3 Beams Specially Detailed for Shear: Test Series ‘1’

The beams in Test Series ‘1’ were examined during the initial stages of this programme of research at Heriot-Watt University[130].

A total of eight beams with a longitudinal reinforcement ratio ($\rho$) of 1.8% were detailed to resist shear as shown in Figure 3.4. Four beams had a shear span to depth ratio (a/d) of 3.2 (type II beams) and the other four beams had an a/d ratio of 2.0 (type III beams). Two of the type II beams had a stirrup spacing of 120mm and the corresponding spacing in the other two beams was 60mm. Two of the type III beams had a stirrup spacing of 70mm and the other two had a spacing of 35mm.

In the type II test beams the critical tensile stresses which develop in the horizontal leg were restrained. Consequently, the ultimate load carrying capacity increased compared with that for beams without shear reinforcement. Some of the test beams were able to reach their full flexural capacity ($M_f$) and failed in a flexural mode. However, in the cases of the other beams, prevention of the diagonal failure in the horizontal leg region, did not prevent the critical diagonal crack from propagating deeply into the concrete compression zone in the inclined leg region where no stirrups had been provided. It appears that improving the resistance of the horizontal leg provided a stiffer path which enabled the load to be transmitted to the support along a new path. In the case of this path, the point at which the force changed direction may have moved nearer to the supports as shown in Figure 3.4.a. Hence, the behaviour of these beams was similar to that of the type III beams without web...
reinforcement, in which the stress condition at the tip of the inclined crack initiated a diagonal failure. To prevent this type of failure, stirrups are also required in the inclined leg region starting from the support. The diagonal failure in some of the test beams could have been attributed to the absence of such stirrups. The introduction of the short stirrups into the test beams starting from a distance of 1.5d from the support appears not to be sufficient to fully restrain the tensile stresses in the region where force appeared to have changed direction. Another stress condition which might have been attributed to the diagonal failure is the abrupt change in beam stiffness along the load path. The stiffness of the horizontal leg which was confined with short stirrups, was higher than that for the remaining parts of the beam. This should have resulted in a stress concentration in the region where the confinement was terminated and would have added to the critical stress condition in that region. It is believed that the uncertainties which were caused by the factors discussed above have contributed to the different modes of failure and variations in the flexural capacities. The failure modes varied randomly between shear, flexure, and shear-flexure. It is anticipated that as the a/d ratio is increased, the stress conditions in the inclined leg will become less critical. This is due to the smaller load that will be transmitted to the supports at the full flexural capacity of the beam. In such cases, beams are expected to reach their full flexural capacity if the a/d ratio was large enough.

In the type III test beams, Figure 3.4.b, the tensile stresses which develop in the inclined leg region were not restrained. The critical tensile-compressive state
of stress in the inclined leg region eventually caused failure. Collapse of the beams occurred as a result of spalling of the compression concrete above the tip of the critical crack near the applied load but inside the shear span where no confinement had been provided.

The provision of short stirrups as detailed in the beams in Test Series '1' appeared from the test results to have improved the load carrying capacity of the type II beams. However, providing the mid-span region with stirrups failed to clearly indicate how much improvement in the ductility had occurred in the beams which failed by diagonal cracking. On the other hand, in the case of the type III beams the short stirrups did not significantly influence the behaviour of the test beams.

**Behaviour of Series ‘1’ Beams Based on Kani’s Valley**

The test results obtained from the type II beams and the shape of the diagonal cracks which caused failure of these beams, lead to the conclusion that the behaviour of the beams had been changed from type II to type III beam behaviour with a new theoretical shear span equal to $a'$, Figure 3.5.b. The new imaginary shear span ($a'$) may be estimated by comparing the behaviour of the type II beams and the type III beams without web reinforcement but with the same load carrying capacity with respect to Kani’s Valley (Figure 6[18]). The introduction of short stirrups enabled the type II test beams to achieve an average flexural capacity of 94% of their full flexural capacity. The new imaginary shear span ($a'$) of the equivalent type III beams without web reinforcement but with the same relative flexural capacity (94%) is equal
to approximately 1.5d, Figure 3.5.a. It is interesting to note, that this distance is equal to the unconfined length of the beam measured from the support. A study of the shape of the diagonal cracks found in the test beams showed that they were similar to those found in the type III beams without web reinforcement. It can be concluded that the role of the short stirrups was to change the behaviour from that of type II to type III beams without web reinforcement as shown in Figure 3.5.b. It can also be deduced that confining the horizontal leg region will prevent the failure of this region which is characteristic of the failure of type II beams. As a result of confining the horizontal leg, a relatively large load was transmitted to the support through the inclined leg. It is, therefore, anticipated that the unrestrained tensile stresses which cause failure of the inclined leg had initiated failure in the beams.

The difference in the role of confinement between the type II and type III beams in Test Series '1' can be explained as follows:

In the case of a constant level of flexural capacity, the type II beams transmit less load \( P_{II} \) to the support compared to that \( P_{III} \) for the type III beams \( P_{III} \approx 2P_{II} \). As a result, the stress conditions in the inclined leg regions which were not provided with stirrups was not as critical as that for the type III beams. Subsequently, the type II beams were able to achieve a much higher relative flexural capacity \( M_u = 0.94M_f \) compared to that for type III beams \( M_u = 0.75M_f \). The short stirrups succeeded in restraining the most critical stress state which exists in the horizontal leg region of the type II beams. On the other hand, the short stirrups did not have a significant effect on the behaviour of the type III beams because they
were placed in the less critical region of the load path.

3.3.4 Beams Specially Detailed for Shear: Beam Tests Conducted at Birzeit University

An exploratory series of tests[106, 138] to investigate the behaviour of beams was conducted at Birzeit University before the start of this programme of research. The following discussion is based on the results from these tests.

Nine beams having a longitudinal reinforcement ratio ($\rho$) of 1.8% were detailed to resist shear in a similar way to the beams in Test Series ‘1’ except that they were unreinforced for shear along a distance equal to 2d measured from the support. Three type II beams had an a/d ratio equal to 3.2 (three identical beams-type C4[138]) and three type II beams had an a/d ratio of 3.6 (three identical beams-type C[106]). The remaining three type III beams had an a/d ratio of 2.0 (the three beams were identical).

The type II test beams failed in shear and achieved load carrying capacities equal to approximately 80% of their full flexural capacity. The tensile stresses in the region where the compression force changes direction, which influence the critical conditions in the horizontal leg region of type II beams, had not been adequately restrained due to the absence of stirrups in that region. Subsequently, only a slight improvement in the load carrying capacity was obtained. A new imaginary shear span ($a'$) equal to 2d, similar to the beams in Test Series ‘1’, as shown in Figure 3.5.a, may have influenced the behaviour of the beams. It is interesting to note that once
again, this distance was equal to the length which was unreinforced for shear i.e. 2d from the support.

The type II test beams with an a/d ratio equal to 3.6, almost reached their full flexural capacity and showed improved ductility. This was due to the relatively small load which was transmitted to the supports (at a constant moment level) compared with that for the type II beams with an a/d ratio of 3.2. As a result, the stress conditions in the inclined leg (region 3 in Figure 3.1) did not influence the behaviour of the beams although the unconfined length was 2d.

All the type III test beams failed by diagonal cracking in the shear span region in a similar way to that which occurred in the type III beams in Test Series ‘1’.

3.3.5 Beams Specially Detailed for Shear: Beams Tested by Kotsovos

Four type II beams with a/d ratios of 3.3 (two nominally identical beams) and 4.4 (two nominally identical beams) were detailed to resist shear in a similar way to that adopted in the test programme conducted at Birzeit University, except that long stirrups were used instead of the short ones[5]. In all of the test beams, diagonal cracks developed in the shear span where there was no confinement. Further extension of the cracks into the compression zone was prevented by the presence of the long stirrups. Eventually, the beams failed in flexure.

The test programme also included two type III beams with an a/d ratio equal to 1.5 in which the long stirrups were placed under the load points and were also
placed in the mid-span region instead of the shear span region. The beams failed by
diagonal cracking with a load carrying capacity equal to 96% of the corresponding
capacity of traditionally detailed beams. The increase in the flexural capacity of
the test beams was relatively small compared to the corresponding point in Kani's
Valley[18]. However, confining the compression concrete outside the shear span
region, restrained the development of the critical tensile-compressive state of stress
near the load points. This type of detailing resulted in a ductile diagonal failure.
Kotsovos[128] proposed providing the entire length of the shear span with traditional
stirrups in order to prevent diagonal failures. In order to prevent splitting of the
compression concrete outside the shear span, Kotsovos suggested that the stirrups
be placed in the mid-span region near to the loading points for a distance equal to
the depth of the neutral axis. The critical state of stress which develops in regions 2
and 3 (Figure 3.1) of the load path was restrained by this type of detailing. Beams
detailed as described above normally behave in a flexural ductile manner.

Comparison with Type II Beams in Test Series '1' and the Birzeit Uni-
versity Test Programme

The results from the beams in Test Series '1' and the Birzeit University test pro-
gramme appear to contradict the findings from the investigation conducted by
Kotsovos. It was noted from these test programmes that a brittle diagonal fail-
ure may occur in type II beams which may result in the beams not necessarily
reaching their full flexural capacity if the inclined leg region is not confined. How-
ever, the type II beams tested by Kotsovos in which the inclined leg regions were
not provided with stirrups reached their full flexural capacity. The conflicting experimental evidence which has arisen from the beam tests conducted by Kotsovos can be explained as follows:

1. The beams had larger a/d ratios (3.3 and 4.4) compared to those in Test Series ‘1’ (a/d = 3.2). This resulted in the transmission of less load at full flexural capacity of the beams. The smaller loads were not sufficient to result in the development of a critical stress condition in the inclined leg regions. Consequently, the absence of the stirrups in that region did not reduce the overall capacity of the beams.

2. The change in stiffness throughout the load path was less severe because of the use of long stirrups. As a result, the stress concentration in the inclined leg region was less critical.

3. The use of long stirrups restrained the growth of the diagonal cracks compared with the cracks in the other beams. This maintained the integrity of the flexural stiffness of the beams.

4. The long stirrups seem to be more effective in restraining the tensile stresses which usually develop at the point at which the load path changes direction (region 2 in Figure 3.1).

5. The reduced size of the test beams (d = 90mm) compared with the beams included in this series (d = 260mm) may have affected their capacity. It is known that small size beams exhibit a larger ultimate flexural capacity at diagonal cracking[139]. This may be attributed to the confinement effect of
the compression concrete due to the large strain gradient present in small beams.

3.3.6 Structural Behaviour of Beams under Transverse Loading

The effect of variations in the values of a/d and $\rho$ on the ultimate flexural capacity of beams ($M_u$) reflects their influence on the critical state of stress along the load path. A relatively higher load is transmitted to the support (under constant bending moment) in the case of beams with small a/d ratios (type III beams). This initiates the critical state of stress in the inclined leg region particularly when the longitudinal reinforcement ratios are high. On the other hand, a relatively small load is transmitted to the supports when the a/d ratios are large (type II beams). In this case, the state of stress in the inclined leg region is not critical and failure occurs in the horizontal leg region.

(a) Behaviour of Type II Beams

In the type II beams, the most critical stress conditions exist in the horizontal leg region. However, restraining these stresses by confining the horizontal leg with closed short stirrups forces the critical state of stress in the inclined leg region to be the dominating parameter. This means that the capacity of the inclined leg is less than that which corresponds to the full flexural capacity ($M_f$) of the beam. This also shows that, although, the shear span is subjected to a constant shear force, the inclined and the horizontal leg regions have different load carrying capacities.

Despite the fact that the causes of failure and the load carrying capacities of the
various regions of the shear span (load path) are different, the design approaches present in Codes of Practice are based on the existence of a critical section for shear. In the four-point loading system, the entire length of the shear span is subjected to a constant shear force and hence designs based on the Codes result in the same requirements for shear reinforcement for the entire shear span. If Codes of Practice were based on actual structural behaviour and realistic causes of diagonal failures, different arrangements of transverse reinforcement would have been required for the various regions of the shear span.

(b) Behaviour of Type III Beams

In the case of type III beams, the critical tensile-compressive state of stress exists in the inclined leg regions. The confinement of the inclined leg regions (shear spans) alone enables the beams to achieve their full flexural capacity. However, some beams may fail in a less ductile manner due to possible spalling of the compression concrete outside the shear span region but close to the loading points. This implies that the transverse reinforcement is also needed for part of the horizontal leg.

3.3.7 Conclusions

The proposed flexural-shear interaction design model has been developed based on the understanding of the actual structural behaviour of beams under transverse loading discussed above. The following remarks are drawn from that discussion:

1. The actual structural behaviour of beams under transverse loading can be explained from the understanding that failure is associated with the actual
state of stress which exists in the concrete compression region as given by the CFP concept.

2. The collapse of a beam occurs as a result of the development of transverse tensile stresses along the CFP which consists of horizontal and inclined leg regions.

3. For each type of beam, the critical state of stress exists in either the horizontal (beam type II) or the inclined (beam type III) leg regions. However, restraining only the most critical stress state does not necessarily prevent diagonal failures in beams.

4. Conclusions on the effect of confinement on the enhancement of the flexural capacity cannot be deduced from available test results. The addition of double the amount of stirrups required to prevent diagonal failure, did not result in significant increases in the flexural capacity of the beams included in Test Series '1'. This is because the beams failed by diagonal cracking.

5. To prevent diagonal failures in beams, the horizontal and the inclined legs should be provided with stirrups.

3.4 NEW DETAILING ARRANGEMENT FOR BEAMS

3.4.1 Introduction

In the case of the unified structural concrete design approach the satisfactory behaviour of a structure is determined by rational design concepts and detailing ap-
proaches which give more attention to overall force paths and resisting elements. The behaviour of structures is usually influenced by the way they are detailed. A new detailing arrangement for stirrups is proposed in the following section.

3.4.2 Proposed Detailing for Preventing Diagonal Failures

The requirements of the unified structural concrete design approach can be achieved by utilising the effect of confinement to enhance the strength and ductility of the compression concrete which is the main element in resisting axial, flexure, and shear forces in structural concrete members.

In the case of beams subjected to transverse loading, the load path is the compressive force path, and the compression concrete is the resisting element. The observed structural behaviour of beams indicates that diagonal failures initiate from critical diagonal cracks in the inclined leg regions and collapse of beams occurs as a result of splitting of the compression concrete in either the horizontal or the inclined leg regions. In this case, a rational detailing procedure is proposed such that the compression concrete in the region of the compressive load path is confined by closed stirrups.

The principal requirement of the proposed detailing arrangement for preventing diagonal failures, Figure 3.6.a, is that the inclined leg regions are to be provided with long (traditional) stirrups starting from the supports and extending over a length equal to either '1.5d' (type II beams) or 'a' (type III beams). On the other hand, the horizontal leg region (except in the mid-span region away from the loading
points which is not subjected to shear) must be provided with short stirrups as a continuation of the long stirrups. The short stirrups only extend down to half of the beam depth. They overlap the neutral axis in order to provide anchorage to the horizontal leg thus preventing its possible instability. This also enables the stirrups to resist the tensile stresses which develop in the region where the force changes direction (region 2 in Figure 3.1). To prevent spalling of the compression concrete inside the mid-span region in type III beams, short stirrups must be placed outside the shear span for a distance equal to approximately half of the beam depth starting from the loading points.

3.4.3 Proposed Detailing Arrangement for the Evaluation of the Effect of Confinement on Flexural Capacity

To evaluate the effect of confinement on the flexural behaviour of beams, test programmes in which the horizontal legs were provided with closely spaced short closed stirrups were carried out at Heriot-Watt University (Test Series ‘1’)[130] and Birzeit University[106, 138]. However, because the test beams failed by diagonal cracking in the inclined leg regions, the effect of confinement on the flexural capacity could not be determined. In the proposed detailing arrangement for preventing diagonal failures, failure of the horizontal and the inclined legs is prevented. To evaluate the effect of confinement on flexural behaviour, the proposed detailing approach is modified slightly such that short stirrups are provided along the entire length of the horizontal leg including the mid-span region which is subjected to the maximum bending moment as shown in Figure 3.6.b. This change in the stirrup arrangement should not affect the resistance of beams with respect to diagonal failures since the
mid-span region is subjected to only bending moment. Consequently, the test beams served two purposes; the investigation of the applicability of the proposed detailing approach for the prevention of diagonal failures and the study of the effect of confinement on the flexural capacity of beams.

The suitability of the proposed detailing approach for the prevention of diagonal failures and for the study of the flexural behaviour was investigated experimentally ('Test Series ‘A’). For comparison purposes, geometric and material properties of the test beams were chosen to be consistent with those used in the beams in Test Series ‘1’[130].

3.5 TEST SERIES ‘A’: Applicability of the proposed detailing approach

3.5.1 Description of Test Beams

The test programme consisted of eight rectangular beams of 200mm x 300mm cross section. All the beams had an overall length of 3500mm and an effective span of 2800mm. The shear span to depth ratio was either 2.0 (4 beams) or 3.2 (4 beams). Two nominally identical beams of each beam type were cast and subsequently tested. The detailing of the test beams (beam types NA2, NA3, NB2, and NB3) are shown in Figure 3.7. Beam types A1 and B1 which were included in Test Series ‘1’ are also shown in Figure 3.7 for comparison purposes. The test beams were identical except for the lateral reinforcement. The only variable investigated was the spacing of the stirrups. Each beam was reinforced longitudinally with three 20mm nominal
diameter deformed steel bars with a yield strength of 532.6 MPa. Two types of stirrups were used; short stirrups (extending down to only half of the beam depth) and normal stirrups (extending down the effective depth of the beams). The stirrups were fabricated from 10mm nominal diameter plain round mild steel bars with a yield strength of 453.6 MPa. The short and the normal size stirrups were used in the horizontal and the inclined leg regions respectively. The determination of the stirrup spacing was based on the provisions of BS 8110[9]. The same stirrup spacing was used along the entire length of some of the beams (beam types NA2 and NB2). In the remaining beams (beam types NA3 and NB3) the spacing of the stirrups in the horizontal leg region was reduced by 50% in order to investigate the effect of the level of confinement on the strength and ductility of the beams.

3.5.2 Experimental Work

Details of the concrete mix used are given in Table 3.1 and the corresponding compressive strengths of the concrete used in the beams are given in Table 3.2. The beams were cast horizontally in steel shutters. One beam was cast from each batch of concrete. The beams and the control cubes were cast, compacted, and then left in the moulds in the laboratory under ambient conditions. The top cast surfaces were covered with damp hessian over which polythene sheeting was placed. The beams were removed from the shutters prior to testing and whitewashed for easier identification of cracking under loading.

The beams were tested under a four point loading system. The beams were
loaded using a servo-controlled universal test machine.

Three Linear Variable Differential Transducers (LVDT's) were used to measure the deflection of the beams at the loading points and at the mid-span. Two dial gauges were used to measure the relative displacements between the beam and the supports.

The loading system was operated under displacement control at a constant rate of 1mm/minute for two minutes. On completion of each increment of displacement the beams were inspected for cracks which were then measured using a crack micrometer. Cracks were marked on the beam surface. The magnitudes of the applied loads, deflections, and crack widths were also recorded at each stage. A more detailed description of the test procedures used is included in Appendix A.

3.5.3 Test Results

The measured loads were corrected for comparison purposes to allow for the actual geometric and material properties. Each load was multiplied by a correction factor \( F = \frac{P_1}{P_{\text{actual}}} \), where \( P_1 \) and \( P_{\text{actual}} \) were the calculated ultimate load capacities of the beam for the assumed and the actual geometric and material properties respectively. The behaviour of the test beams with respect to failure mode, strength, ductility, and stiffness should be compared with the corresponding results obtained from the traditionally detailed beams. In the initial stages of this programme of research (Test Series '1'[130]), beams similar to the beams included in this test series
except that the stirrups were traditionally detailed, were tested (beam types A1 and B1[130]). Therefore, the results obtained from these beams were corrected in the same way as discussed above and were considered together with the corrected test results from Test Series 'A'. The corrected measured loads for all the beams (beam types A1, NA2, NA3, B1, NB2, and NB3) are shown in Table 3.2. A comparison of the load capacities obtained from the beams which were detailed in accordance with the proposed detailing approach with those obtained from the traditionally detailed beams are shown in Table 3.3. Table 3.4 summarises the crack width and deflection measurements at different load levels. The crack patterns after failure for the four beam types NA2, NA3, NB2, and NB3 are shown in Figure 3.8.

The load-deflection curves show the maximum load capacities, the stiffnesses, and the ductilities of the beams. The load-deflection (at mid-span) curves for the beams are shown in Figure 3.9. It should be emphasised that testing of the beams with closely spaced stirrups was stopped, on the ground of safety, when a mid-span deflection of approximately 120mm was reached.

3.5.4 Discussion of Test Results

(a) Modes of Failure

All beams, regardless of the stirrup spacing and the shear span to depth ratio, failed in flexure as shown in Figure 3.8. In all the beams hair-line flexural cracks developed in the lower part of the beams and extended vertically towards the neutral axis before the appearance of diagonal cracks. As loading was continued, the flexu-
ral cracks proliferated and widened and diagonal cracks appeared in the shear spans. The short stirrups succeeded in preventing further extension of the diagonal cracks into the compression zone, thus preventing diagonal failures in all of the beams. The concrete cover in the compression regions started to crack in all of the beams at a load level close to the maximum value. Spalling of the concrete cover occurred at strain levels in excess of the corresponding ultimate load levels. In the beams in which the stirrup spacing was reduced by 50% (beam types NA3 and NB3) the confining stirrups delayed failure after spalling of the concrete cover. In the remaining beams (beam types NA2 and NB2) the compression bars buckled and spalling of the compression concrete occurred at failure.

At load levels equal to half of the maximum load (refer to Table 3.4), the flexural and the diagonal crack widths were almost equal and had an average value of 0.25mm for all beams with a shear span to depth ratio equal to 2.0 (beam types NA2-1, NA2-2, NA3-1, and NA3-2). These beams had larger flexural crack widths (about 2.5mm) compared with their diagonal crack widths (about 1.0mm) under maximum loading. On completion of the tests on these beams but just before unloading the flexural crack widths reached values approaching 12mm while the diagonal crack widths remained almost unchanged. On the other hand, for all of the beams with a shear span to depth ratio equal to 3.2 (beam types NB2-1, NB2-2, NB3-1, and NB3-2) the widths of the flexural and diagonal cracks were similar (0.2mm and 0.3mm respectively) at half the maximum load level. These crack widths were also similar to those found in beam types NA2 and NA3. However, the flexural crack
widths at maximum load levels for beam types NB2 and NB3 (1.6mm) were smaller than the corresponding values for beam types NA2 and NA3 (2.5mm). This could have resulted from the relatively large maximum loading capacities experienced in beam types NA2 and NA3 (exceeding 460 kN) compared with those obtained from beam types NB2 and NB3 (about 260 kN). The widths of the diagonal cracks at the maximum load level for beam types NB2 and NB3 (2.5mm) were larger than those obtained from beam types NA2 and NA3 (1.0mm). It seems that the absence of long stirrups over most of the shear spans in beam types NB2 and NB3 resulted in wider diagonal crack widths. The widths of the cracks in these beams just before unloading support this conclusion. While the diagonal crack widths for beam types NA2 and NA3 remained almost unchanged after reaching the maximum load level, the widths of the corresponding diagonal cracks in beam types NB2 and NB3 reached a value approaching 12.0mm while the maximum widths of the flexural cracks at this level approached 8.0mm. It is concluded that up to the maximum load level additional long stirrups placed along the entire length of the inclined leg region are not necessary for restraining the growth in the widths of the diagonal cracks found in beam types NB2 and NB3. However, to restrain the diagonal crack widths at post ultimate load levels, additional long stirrups may be necessary in these beam types. It should be noted that the proposed detailing arrangements are intended to be applied to beams subjected to static loading conditions for which post ultimate behaviour is not important. Nevertheless, it is suggested that additional long stirrups are introduced over a distance equal to 2d, Figure 3.6, (instead of 1.5d) from the support for all beam types.
(b) Load Carrying Capacity and Ductility

The test beams, based on current Code approaches for shear design, should have attained a load carrying capacity similar to those obtained from beams without web reinforcement since the assumed truss action could not possibly have developed (stirrups did not extend down the full depth of the beams). All the test beams, however, achieved their full flexural behaviour and had an ultimate flexural load capacity similar to the capacity of the traditionally detailed beams (beam types A1 and B1) as shown in Tables 3.3 and Figure 3.9. The load carrying capacities ranged from 95.2% (beam type NA3-2) to 99.1% (beam type NB2-1) of the flexural capacity of the traditionally detailed beams.

Double the amount of stirrups used in beam types NA2 and NB2 were provided in beam types NA3 and NB3 respectively. However, beam types NA3 and NB3 did not exhibit any increase in load carrying capacity compared with that obtained from beam types NA2 and NB2. The flexural capacities of beams resulting from confinement are discussed in Chapter 6.

All beams exhibited flexural ductile behaviour which is judged to be better than that obtained from the traditionally detailed beams (refer to Table 3.4 and Figure 3.9). The ductility (mid-span deflection at failure) of beam type NA2 was double that which was obtained in the traditionally detailed beams. The corresponding increase in the ductility of beam type NB2 was 150%. When the spacing of the stirrups was reduced by 50%, the increase in ductility was approximately
three times greater than that for the two traditionally detailed beam types. The increased ductilities can only be attributed to the confinement of the compression regions of the beam structures with closed stirrups. When the level of confinement was increased (stirrup spacing reduced by 50%) the ductility increased significantly.

Confinement of the compression concrete had also a significant effect on the post ultimate load carrying capacity of the beams (refer to Table 3.3). In beam types A1 and B1 which were traditionally detailed to resist shear the reduction in load carrying capacity on completion of the test (at failure) approached 99% and 63% respectively. When stirrups with the same spacing used in the traditionally detailed beams were extended to confine the mid-span region of beam types NA2 and NB2 the decrease in the load carrying capacity was of the order of 28% and 23% respectively. However, when the spacing of the stirrups was reduced by 50% an increase in load carrying capacity of up to 4.5% was recorded (beam type NA3-1).

(c) Serviceability

The serviceability of concrete structures has recently become a much more important design consideration, mainly because more efficient design procedures have enabled engineers to satisfy the ultimate limit state requirements with lighter but more highly stressed structural concrete members[140, page 156]. Somerville[141] predicted that the serviceability of concrete structures will be one of the important requirements in future as the time factor will be the single most dominant issue in structural concrete design.
The methods for either the prediction or the improvement of the serviceability of structures is not within the scope of this investigation. However, the influence of the new detailing approach on the serviceability of the beams should be investigated in comparison with traditionally detailed beams assuming that these beams do conform with the Code provisions with respect to the serviceability limit state. Ideally, to judge the effect of the proposed detailing approach on the serviceability of the test beams, the crack history and deflections of these beams should be compared with those obtained from the traditionally detailed beams. Unfortunately, crack widths were not measured in the traditionally detailed beams i.e. beam types A1 and B1 which were included in Test Series ‘1’[130]. As an alternative a comparison was based on their respective crack patterns (Figure 3.8), stiffnesses (Figure 3.9) and between the flexural and diagonal crack widths of each test beam (Table 3.4).

The development of cracking under loading was similar in both beam types i.e. the traditionally detailed beams and the beams in this test series. The cracks were closely spaced in the test beams. All beams had similar stiffnesses (inclination of the load-deflection curve), Figure 3.9. In all beams, Table 3.4, flexural cracks occurred before the diagonal cracks and the widths of the flexural cracks were either smaller or almost equal to the widths of diagonal cracks up to the maximum load level. The serviceability limits of the test beams were therefore based on a consideration of the flexural rather than diagonal cracks which in turn should not be affected by the proposed detailing approach since Code provisions[8, 9] do not re-
quire stirrups to be provided for the control of flexural cracking. Crack widths are normally controlled by limiting the maximum distance between bars in tension[8, 9].

Crack width measurements for traditionally detailed beams were recorded and compared with those obtained from the beams designed and detailed using the proposed design approach and included in Test Series 'C' which is discussed in detail in Chapter 4.

3.5.5 Summary and Conclusions

1. Prevention of failure by diagonal cracking was achieved by confining the compression concrete with closed stirrups in the region of the path along which the loads are transmitted to the supports. The definition of the path was based on the compressive force path concept.

2. The test beams were provided with either the same or double the amount of stirrups, based on the provisions of BS 8110, which were required to prevent diagonal failure.

3. The level of serviceability of the test beams was not adversely affected by the proposed detailing approach. The diagonal cracks occurred after the development of the flexural cracks. As a result, they cannot be considered in the evaluation of the serviceability limits of the beams. The increased widths of the diagonal cracks, after the maximum loading had been reached, could be considered to be an advantage because they give an additional warning of impending failure.
4. The absence of long stirrups in the horizontal leg regions in the beams reduces the bond strength of the longitudinal bars which may result in a local bond failure. Nevertheless, this did not reduce the overall carrying capacities of the test beams. It is acknowledged\[142, pages: 280–284\] that the occurrence of bond failure along a length of up to 75% of the total beam length does not reduce the average bond strength. Also, test results\[143\] have indicated in the case of type II beams the load carrying capacity is increased with poor bond. The local bond failure did not impair the serviceability of the beams since it occurred at an advanced stage of loading after yielding of the longitudinal reinforcement.

5. Confinement of the mid-span region with short stirrups did not enhance the flexural capacity of the test beams. However, the ductility of the beams was improved significantly by increasing the amount of stirrups in this region.

3.6 THEORETICAL BASIS OF FLEXURE-SHEAR INTERACTION DESIGN MODEL

3.6.1 Introduction

The philosophy in most current design approaches to shear imposes an artificial separation between the shear and the flexural resistances of beams. The design procedures adopted by Codes of Practice do not relate a given level of moment capacity to a given amount of shear reinforcement. This artificial separation could result in a design that prevents the development of the full moment capacity of a
beam (e.g. the diagonal failure of members during earthquakes). It would therefore be better to determine the optimum amount of shear reinforcement which will ensure attainment of the full moment capacity of a member.

3.6.2 Main Features of the Proposed Model

1. At the ultimate limit state, loads are assumed to be transmitted to the supports in accordance with the Compressive Force Path (CFP) concept. In the case of a simple beam the shape of the path at ultimate load is considered to be bi-linear thus forming a concrete compression frame as shown in Figure 3.10. The frame consists of a horizontal and two inclined members or legs. The depth of these members is equal to the depth of the concrete compression zone calculated using normal flexural theory.

2. The causes of diagonal failure are associated with the actual state of stress which exists in the concrete compression regions rather than the tension zone below the neutral axis. Failure occurs as a result of the development of secondary tensile stresses which lead to degradation of the concrete compressive strength thus leading to a reduction in the load carrying capacity of beams.

3. In diagonal failures, collapse of beams occurs as a result of spalling of the compression concrete in either the inclined leg region (type III beams) or in the horizontal leg region (type II beams). In order to prevent the ultimate collapse of beams (ultimate limit state), the compression concrete in the various leg regions of the concrete compression frame must be provided with different amounts and configurations of closed stirrups. The inclined leg regions
irrespective of the a/d ratio must be provided with conventional full length stirrups and the horizontal leg regions in the shear spans with short stirrups (the stirrups must be extended into the mid-span region for a distance equal to approximately a half of the beam depth for type III beams). In order to ensure confinement of the entire concrete compression region, the short stirrups must extend down beyond the neutral axis to half of the beam depth.

4. In the proposed model, the role of stirrups is to restrain the development of the secondary transverse tensile stresses which cause failure. This can be regarded as enhancing the tensile strength of the concrete.

5. The proposed design approach for the type II and the type III beams is the same. The required amount of stirrups for each leg region is to be determined from the reduction in the concrete compressive strength due to shear and the relative flexural capacity of each leg ($\frac{M_i}{M_f}$).

The ultimate flexural capacity of each leg region without web reinforcement ($M_i$) is less than the full flexural capacity of the beams ($M_f$). The reduction in the flexural capacity occurs as a result of the development of the tensile stresses in the concrete compression region. Ideally, design methods to prevent diagonal failure must be based on the evaluation of the magnitude of the tensile stresses. Unfortunately, the determination of the magnitude of the tensile stresses is difficult[144]. Also, there is disagreement between researchers on the significance of transverse tensile stresses on the concrete compressive
strength[145, 146, 147]. Therefore, a simpler approach for the prevention of diagonal failures is proposed. It is considered that the development of the tensile stresses reduces the concrete compressive strength by $\Delta f_c$. It will subsequently reduce the flexural capacity of the beams by $\Delta M$. To regain the full flexural capacity of the beam ($M_f$), the reduction in the concrete compressive strength ($\Delta f_c$) must be offset. It is proposed to achieve this by utilising the effect of confinement to enhance the concrete compressive strength by the same amount i.e. $\Delta f_c$. After determining $\Delta f_c$, the amount of stirrups required to achieve this added strength can be determined.

3.7 ADVANTAGES OF THE PROPOSED DESIGN APPROACH

The advantages of the proposed design concept over traditional design approaches such as the truss analogy are summarised below:

1. The development of the new design approach has been based on a better understanding of the actual structural behaviour of beams under static loading and also of concrete at the material level. It is realised that concrete compression failure occurs as a result of the development of transverse tensile stresses resulting from the dilation of concrete under the action of compression stresses. Thus all types of beam failures including diagonal failures are associated with the multiaxial stress state in the concrete compression region[16].

2. The proposed design approach relies on the concrete compression regions (the stronger part of the beam structure) to resist the applied loads rather than
the traditional approaches for shear design which rely mainly on the strength of cracked concrete below the neutral axis (the weaker part of the beam structure).

3. The conventional design methods for structural concrete beams treat shear and flexural actions separately although in practice they occur simultaneously. The traditional design approach for shear relies on the transverse reinforcement at the ultimate limit state to resist the shear stress in excess of that assumed to be carried by the concrete section ($v_c$). Such an approach assumes that two types of mechanisms act simultaneously in the beam structure, namely the beam/arch action for $v_c$ (concrete contribution) and the truss action for $v_s$ (steel contribution). The acceptance of the presence of two mutually exclusive mechanisms in beams thus leads to the confusion in the design procedures for shear. On the other hand, in the proposed new design approach only one mechanism is assumed to be required to resist the applied loads (flexure and shear). The stirrups are intended to enhance the strength of the concrete in the compression zone, which is the main element in the beam structure to resist the imposed loadings (axial, shear, and bending moment) when the load level exceeds the capacity of the compression concrete. The adoption of an approach based on the use of a single mechanism to resist the applied loadings could form the basis of a unified design approach for structural concrete members.

4. The proposed design approach considers the interactive relationship between flexure and shear not only in the evaluation of the relative flexural capac-
ity of beams without web reinforcement \( (\frac{M_u}{M_f}) \), but also in the evaluation of the amount of confinement (stirrups) required to prevent diagonal failures in beams. This is directly related to the reduction in flexural capacity due to the presence of shear i.e. \( \Delta M = M_f - M_t \).

The consideration of the ultimate flexural capacity at diagonal failure \( (M_u) \) rather than the nominal shear strength \( (v_u) \) of beams is regarded to be a better basis for the development of design methods for preventing diagonal failures in beams. This has been the aim of many researchers[18, 44, 148] for the following reasons:

(a) The upper limit for \( M_u \) is the actual flexural strength \( (M_f) \) which is dependent on a limited number of parameters and therefore can be obtained using a simple calculation.

(b) The lower limit for \( M_u \) is usually in the vicinity of 0.50\( M_f \). Thus, all the values of \( M_u \) range between 50% to 100% of \( M_f \), instead of the large variation in the ultimate shear strength \( (v_u) \) which may reach 1500%[18].

(c) The prevention of premature failure because of the formation of a diagonal crack is commonly associated with the term “shear failure”. As an example, if we obtain a diagonal failure at 70% of the flexural failure load, this means, that we are just 30% short of our goal, i.e. the full flexural capacity of the cross section.
(d) The purpose of the web reinforcement is, of course, to increase the strength of the beam so that it reaches 100% of $M_f$. Thus, a result where $M_u$ is equal to $0.7M_f$ for a beam without web reinforcement shows the requirement for the provision of “web reinforcement which increases the capacity of the beam by 30% of $M_f$”. Thus the problem of “shear strength” would become an investigation of, and a search for, the type and quantity of web reinforcement required to increase the ratio $\left( \frac{M_u}{M_f} \right)$ to 1.0.

5. Failure of structural concrete members by diagonal cracking has frequently been experienced particularly during recent earthquakes[149]. The failure was related mainly to the evaluation of stirrup requirements based on the conservative flexural capacity given by Codes of Practice. However, in the proposed design approach, evaluation of the confinement (stirrups) requirements is related directly to the full flexural capacity of beams ($M_f$) and expressed in the form of $\Delta M = M_f - M_t$. So any enhancement in the flexural capacity can be accounted for directly in the design. Such a design approach would minimise the risk of diagonal failures during earthquakes.

6. The confinement requirements of the various leg regions of the different beam types (beam types II and III) are determined using the same approach i.e. confinement requirements are determined from the required enhancement of the concrete compressive strength ($\Delta f_c$) for the various leg regions. This is unlike the design method based on the compressive force path concept[5] which proposes different design approaches for the different regions and types
of beams.

7. It is anticipated that the proposed design approach would result in a more economical design compared with the traditional approach to design. This is attributed to the fact that this approach is derived from the actual structural behaviour of beams under load. It will result in less uncertainties in the evaluation of the confinement requirements to prevent diagonal failures. Also, the additional strength from the other contributors to shear resistance, although they are believed to be secondary, is not considered. The reduced significance of the uncertainties and overlooking the possible additional strength would consequently allow a reduction in the risk factors (safety factors) which are usually introduced in the design. In addition, the use of short stirrups instead of conventional full length (normal) stirrups in the horizontal leg regions also leads to a more economical design.

8. The proposed new detailing arrangement is more efficient in the case of the upgrading and the maintenance of existing structural concrete elements such as beams or slabs in existing bridges in order to allow them to be able to sustain more demanding loading levels by increasing their load carrying capacity and ductility. In this case, the enhancement of the major parts of the elements (the horizontal leg regions) can easily be achieved since the short stirrups do not need to be extended down the entire depth of the members. The overall depth of the members may only require treatment in the inclined leg regions.
3.8 SUMMARY

In this chapter, a new approach to the understanding of the actual structural behaviour of beams has been developed from a theoretical basis and from available test results of beams either without stirrups or with stirrups of different configurations. It was concluded that the diagonal failure of beams occurred primary as a result of the development of transverse secondary tensile stresses in the concrete compression region of the beam structure. Therefore, a new detailing arrangement for the prevention of diagonal failures was proposed such that the stirrups are positioned in such a way as to confine the compression concrete in the region of the load path (the compressive force path). The applicability of the proposed detailing arrangement was verified experimentally. The theoretical basis of the flexure-shear interaction design model, based on the new understanding of the behaviour of beams under transverse loading, has been outlined. In the proposed model, the reduction in the flexural capacity of each leg region along the compressive force path is regained by utilising the effects of confinement from the stirrups to enhance the concrete compression strength and hence the flexural capacity of the beams.

In order to implement the proposed flexure-shear interaction model for the design of beams, it is necessary to determine the influence of shear on reducing the flexural capacity of beams and concrete compressive strength. Also, it is necessary to evaluate the flexural capacity of the various leg regions of the load path \( M_l \) for the whole of Kani's Valley (leg capacity valley). The implementation as well as
the experimental verification of the proposed model are addressed in the following Chapter.
Table 3.1: Concrete Mix Constituents used in Test Series ‘A’.

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>500</td>
</tr>
<tr>
<td>Water</td>
<td>225</td>
</tr>
<tr>
<td>Fine Aggregate</td>
<td>537</td>
</tr>
<tr>
<td>Coarse Aggregate (20mm)</td>
<td>1065</td>
</tr>
<tr>
<td>Water/Cement ratio</td>
<td>0.45</td>
</tr>
<tr>
<td>Slump</td>
<td>30-60 (mm)</td>
</tr>
</tbody>
</table>

Table 3.2: Summarised Results from Test Series ‘1’ and Test Series ‘A’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Stirrup spacing (mm)</th>
<th>Beam size (mm x mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>Correction factor (F)</th>
<th>Measured load (P) (kN)</th>
<th>Corrected load ($P_r$) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-1</td>
<td>2.0</td>
<td>70</td>
<td>204.7 x 308.3</td>
<td>46.6</td>
<td>0.9669</td>
<td>480</td>
<td>464.1</td>
</tr>
<tr>
<td>A1-2</td>
<td></td>
<td></td>
<td>204.9 x 309.3</td>
<td>45.9</td>
<td>0.9694</td>
<td>455</td>
<td>441.1</td>
</tr>
<tr>
<td>NA2-1</td>
<td>2.0</td>
<td>70</td>
<td>202.5 x 311</td>
<td>55.8</td>
<td>0.9416</td>
<td>467</td>
<td>439.7</td>
</tr>
<tr>
<td>NA2-2</td>
<td></td>
<td></td>
<td>203 x 303.5</td>
<td>44.9</td>
<td>0.9747</td>
<td>446</td>
<td>434.7</td>
</tr>
<tr>
<td>NA3-1</td>
<td>2.0</td>
<td>35</td>
<td>204.5 x 307</td>
<td>49.3</td>
<td>0.9579</td>
<td>464</td>
<td>444.5</td>
</tr>
<tr>
<td>NA3-2</td>
<td></td>
<td></td>
<td>202 x 306.5</td>
<td>46.9</td>
<td>0.9680</td>
<td>445</td>
<td>430.8</td>
</tr>
<tr>
<td>B1-1</td>
<td>3.2</td>
<td>120</td>
<td>204.2 x 313.4</td>
<td>56.8</td>
<td>0.9382</td>
<td>281</td>
<td>263.6</td>
</tr>
<tr>
<td>B1-2</td>
<td></td>
<td></td>
<td>207.4 x 308.2</td>
<td>62.1</td>
<td>0.9255</td>
<td>282</td>
<td>261.0</td>
</tr>
<tr>
<td>NB2-1</td>
<td>3.2</td>
<td>120</td>
<td>202.5 x 309</td>
<td>49.1</td>
<td>0.9603</td>
<td>271</td>
<td>260.2</td>
</tr>
<tr>
<td>NB2-2</td>
<td></td>
<td></td>
<td>202 x 309</td>
<td>48.8</td>
<td>0.9614</td>
<td>266</td>
<td>255.7</td>
</tr>
<tr>
<td>NB3-1</td>
<td>3.2</td>
<td>60</td>
<td>203.5 x 308</td>
<td>49.9</td>
<td>0.9570</td>
<td>270</td>
<td>258.4</td>
</tr>
<tr>
<td>NB3-2</td>
<td></td>
<td></td>
<td>203.5 x 310</td>
<td>49.2</td>
<td>0.9594</td>
<td>268</td>
<td>257.1</td>
</tr>
</tbody>
</table>

Table 3.2: Summarised Results from Test Series ‘1’ and Test Series ‘A’.
<table>
<thead>
<tr>
<th>Beam</th>
<th>Corrected measured load ($P_r$) (kN)</th>
<th>Corrected measured flexural capacity $M_i$ (kN.m)</th>
<th>$\frac{M_i}{M_{\text{ave.}}} \times 100$</th>
<th>$\frac{M_{\text{max.}}}{M_{\text{ave.}}} \times 100$</th>
<th>% reduction in loading on completion of testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-1</td>
<td>464.1</td>
<td>120.7</td>
<td>102.5</td>
<td>100</td>
<td>84.6</td>
</tr>
<tr>
<td>A1-2</td>
<td>441.1</td>
<td>114.7</td>
<td>97.5</td>
<td></td>
<td>98.9</td>
</tr>
<tr>
<td>NA2-1</td>
<td>439.7</td>
<td>114.3</td>
<td>97.1</td>
<td>96.6</td>
<td>37.5</td>
</tr>
<tr>
<td>NA2-2</td>
<td>434.7</td>
<td>113.0</td>
<td>96.0</td>
<td></td>
<td>28.3</td>
</tr>
<tr>
<td>NA3-1</td>
<td>444.5</td>
<td>115.6</td>
<td>98.2</td>
<td>96.7</td>
<td>-4.5 (rise)</td>
</tr>
<tr>
<td>NA3-2</td>
<td>430.5</td>
<td>112.0</td>
<td>95.2</td>
<td></td>
<td>-1.6 (rise)</td>
</tr>
<tr>
<td>B1-1</td>
<td>263.6</td>
<td>109.7</td>
<td>100.5</td>
<td>100</td>
<td>62.3</td>
</tr>
<tr>
<td>B1-2</td>
<td>261.0</td>
<td>108.6</td>
<td>99.5</td>
<td></td>
<td>53.6</td>
</tr>
<tr>
<td>NB2-1</td>
<td>260.2</td>
<td>108.2</td>
<td>99.1</td>
<td>98.3</td>
<td>11.4</td>
</tr>
<tr>
<td>NB2-2</td>
<td>255.7</td>
<td>106.4</td>
<td>97.5</td>
<td></td>
<td>22.9</td>
</tr>
<tr>
<td>NB3-1</td>
<td>258.4</td>
<td>107.5</td>
<td>98.5</td>
<td>98.3</td>
<td>3.1</td>
</tr>
<tr>
<td>NB3-2</td>
<td>257.1</td>
<td>107.0</td>
<td>98.0</td>
<td></td>
<td>15.7</td>
</tr>
</tbody>
</table>

where:

$M_{\text{ave.}}$ is the average capacity of two identical beams of types A1 or B1.

**Table 3.3: Comparison between Test Results.**
<table>
<thead>
<tr>
<th>Beam type</th>
<th>At half ultimate load</th>
<th>At ultimate load</th>
<th>Before unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>load (kN)</td>
<td>Defl. (mm)</td>
<td>crack width (mm)</td>
</tr>
<tr>
<td>A1-1</td>
<td>232.1</td>
<td>16.0</td>
<td>484.1</td>
</tr>
<tr>
<td>A1-2</td>
<td>220.5</td>
<td>12.0</td>
<td>441.1</td>
</tr>
<tr>
<td>NA2-1</td>
<td>219.9</td>
<td>6.6</td>
<td>0.3</td>
</tr>
<tr>
<td>NA2-2</td>
<td>217.4</td>
<td>9.7</td>
<td>0.2</td>
</tr>
<tr>
<td>NA3-1</td>
<td>208.6</td>
<td>9.2</td>
<td>0.2</td>
</tr>
<tr>
<td>NA3-2</td>
<td>213.5</td>
<td>8.2</td>
<td>0.25</td>
</tr>
<tr>
<td>B1-1</td>
<td>131.8</td>
<td>9.3</td>
<td>263.6</td>
</tr>
<tr>
<td>B1-2</td>
<td>130.5</td>
<td>10.0</td>
<td>261</td>
</tr>
<tr>
<td>NB2-1</td>
<td>130.1</td>
<td>9.0</td>
<td>0.1</td>
</tr>
<tr>
<td>NB2-2</td>
<td>127.9</td>
<td>9.4</td>
<td>0.1</td>
</tr>
<tr>
<td>NB3-1</td>
<td>128.9</td>
<td>9.2</td>
<td>0.2</td>
</tr>
<tr>
<td>NB3-2</td>
<td>128.9</td>
<td>8.5</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 3.4: Summary of Crack Width and Deflection Measurements.
Figure 3.1: Causes of Diagonal Failures at Different Regions along the CFP.

Figure 3.2: Failure of Beams without Web Reinforcement. (a) Type II Beams. (b) Type III Beams.
Figure 3.3: Failure of Beams Traditionally Detailed for Shear. (a) Type II Beams. (b) Type III Beams.

Figure 3.4: Test Series '1'. (a) Type II Beams. (b) Type III Beams.
Figure 3.5: Behaviour of the Beams in Test Series ‘1’ based on Kani’s Valley.

(a) Kani’s Valley. (b) Transformation from Type II to Type III Beam Behaviour.
Figure 3.6: Proposed Detailing Arrangement for Beams. (a) To Prevent Diagonal Failure. (b) Study of the Effect of Confinement on the Flexural Capacity of Beams.
Figure 3.7: Detailing of the Beams in Test Series ‘A’.
Figure 3.8: Crack Patterns and Failure Modes for the Beams in Test Series ‘A’.
Figure 3.9: Load-Midspan Deflection Curves for the Beams in Test Series 'A'.
For Type II Beams

For Type III Beams

Figure 3.10: Concrete Compression Frame.
Chapter 4

IMPLEMENTATION OF THE FLEXURE-SHEAR INTERACTION MODEL IN THE DESIGN OF BEAMS

4.1 INTRODUCTION

In order to implement the proposed concept in the analysis and design of structural concrete beams, it is necessary to determine the magnitude of the reduction in the concrete compressive strength ($\Delta f_c$) and in the flexural capacity of the leg regions ($\Delta M = M_f - M_l$) in the beam due to the presence of shear. Also, it is important to adopt and if necessary modify a suitable confinement model in order to determine the amount of stirrups needed to achieve the required added strength ($\Delta f_c$). These requirements as well as the experimental verification of the proposed design model are discussed in this Chapter. The notation used in the derivation of the proposed equations is detailed in Table 4.1.

4.2 EVALUATION OF THE SHEAR EFFECT ON REDUCING THE CONCRETE COMPRESSIVE STRENGTH (CONFINEMENT REQUIREMENTS)

4.2.1 Bending Moments

By definition,
\[ \Delta f_c = f_{cc} - f_c = (K_s - 1)f_c \]
\[ \frac{\Delta f_c}{f_c} = K_s - 1 \]  \hspace{1cm} (4.1)

where:

\[ f_{cc} = K_s f_c. \]

\( K_s \) is the confinement enhancement factor (strength gain factor).

Based on flexural theory:

\[ \frac{\Delta f_c}{f_c} = \frac{\text{Constant}(M_{f_d} - M_l)}{M_f} = \frac{M_{f_d} - M_l}{M_f} = \frac{M_r}{M_f} \]  \hspace{1cm} (4.2)

In the above relationship the flexural capacity of the leg \( (M_l) \) is a function of the ultimate flexural capacity of a beam \( (M_u) \). \( M_f \) is the full flexural capacity of a beam and \( M_{f_d} \) is the design bending moment which may differ from \( M_f \).

Equations (4.1) and (4.2) can be expressed as follows:

\[ K_s = \frac{M_{f_d} - M_l}{M_f} + 1 = \frac{M_r}{M_f} + 1 \]  \hspace{1cm} (4.3)
4.2.2 Shear Forces

\[ M_f = V_f \cdot a \]
\[ M_{fa} = V_{fa} \cdot a \]
\[ M_l = V_l \cdot a \]
\[ M_r = (V_{fa} - V_l) \cdot a \]
\[ \Delta f_c \frac{f_c}{f_c} = \frac{M_r}{M_f} = \frac{V_{fa} - V_l}{V_f} \quad (4.4) \]

From equation (4.3):

\[ K_s = \frac{V_{fa} - V_l}{V_f} + 1 \quad (4.5) \]

Where \( V_f \) and \( V_l \) are the shear forces corresponding to the full and to the leg flexural capacities respectively and \( V_{fa} \) is the design shear strength corresponding to the design flexural strength \( (M_{fa}) \).

4.2.3 Variable Shear Forces

When the applied shear force is not constant, such as in the case of a beam subjected to distributed loading, and the design shear force \( (V_{fa}) \) is larger than the leg capacity \( (V_l) \), then the confinement requirements are determined using equation (4.5). On the other hand, if \( V_{fa} \) is less than \( V_l \) then theoretically confinement is not required. However, a minimum amount of stirrups may be provided in order to satisfy Code requirements.
4.2.4 Enhanced Flexural Capacity (Flexure-Shear Interaction)

In practice members may have an actual flexural capacity which is higher than the theoretical value which is determined using flexural theory e.g. enhanced flexural capacity resulting from confinement. To prevent diagonal failures in such members under increasing load, which may be imposed on them during earthquakes, the determination of the required amount of stirrups should be based on the enhanced flexural capacity. In the proposed design method this can be achieved by equating the design flexural moment \( M_{fd} \) in equation (4.3) to the enhanced flexural strength of members. Alternatively, this can be achieved by considering the design shear loading \( V_{fd} \) in equation (4.5) to correspond to the enhanced flexural capacity of the members. This flexure-shear interaction design approach would minimise the risk of the occurrence of brittle diagonal failures of structural concrete members during earthquakes.

4.3 LEG CAPACITY \( (M_l) \)

4.3.1 Introduction

The load carrying capacity of a beam is influenced by the state of stress in the various leg (the horizontal and the inclined) regions along the load path. To determine the amount of stirrups needed to prevent a diagonal failure in each leg, the reduction in the flexural capacity of that leg \( (M_f - M_l) \) due to shear must be determined.

For this purpose, the flexural capacities of the horizontal leg \( (M_{h}) \) and the inclined leg \( (M_{i}) \) are determined from available test results including those obtained from
Test Series '1'[130] and the Birzeit University[106, 138] test programme. However, additional tests are still required in order to evaluate the leg capacities of beams having a/d ratios covering the whole of Kani’s Valley.

4.3.2 Beam Tests Required for the Evaluation of the Horizontal Leg Capacity ($M_{h}$)

The horizontal leg capacity is determined on the basis that failure of the inclined legs must be prevented. Failures of the inclined legs (region 3 in Figure 3.1) are prevented by considering the shape of the CFP. This is done by providing a length of the test beams equal to 2d from the supports with conventional full length stirrups. At this stage, as was the case with Test Series ‘1’, the amount of stirrups is determined from the Code of Practice[9] provisions for shear. It is acknowledged that the traditional design of type III beams with an a/d ratio equal to or less than 2.0 prevents failure of the inclined leg region (the shear span). Testing of beams in which the a/d ratio only varied between 2.5 and 5.0 was, therefore, required.

4.3.3 Beam Tests Required for the Evaluation of the Inclined Leg Capacity ($M_{i}$)

In order to determine the load carrying capacity of the inclined legs, consideration must be given to the prevention of failure of the horizontal leg (regions 1 and 2 in Figure 3.1) by considering the shape of the CFP and all possible failure mechanisms for the horizontal leg region. The horizontal leg starting from a distance equal to 1.5d away from the supports must be provided with short stirrups (extending down to only half the depth of the beam). At this stage, the determination of the amount of stirrups is also based on the Code provisions for shear.
It was only necessary to test beams with an a/d ratio equal to 2.5 because of the availability of results from beams included in Test Series ‘1’[130] and in the Birzeit University[106, 138] test programme.

The additional beam tests required for the evaluation of the flexural capacity of both the horizontal and the inclined leg regions (Test Series ‘B’) are described in the following section.

4.4 TEST SERIES ‘B’: Evaluation of the leg capacities

4.4.1 Description of Test Beams

For the purposes of comparison and in order to utilise the available related beam test results, dimensional details and material properties ($\rho$, $d$, $\frac{h}{d}$, $f_{cu}$, etc.) of the beams included in this test series were selected to be consistent with those used in Test Series ‘1’[130] and the Birzeit University[138] and Toronto[18] test programmes.

The test series included twelve beams with a 200mm x 300mm cross section, Figure 4.1. All the beams had an overall length of 3500mm and an effective span of 2800mm except for one beam (beam type D5) in which the effective span was 3000mm. Each beam was reinforced with three longitudinal 20mm nominal diameter deformed bars with a yield strength of 532.6 MPa. The stirrups used were fabricated from 10mm nominal diameter plain round mild steel bars with a yield strength of
364.4 MPa. The determination of the amount of stirrups required for each leg was based on the provisions of BS 8110. Two identical beams (beam type C2.5) with a shear span to depth ratio (a/d) equal to 2.5 were used for the evaluation of the capacity of the inclined leg. Ten beams were used for the evaluation of the horizontal leg capacity. The beams had a/d ratios of 2.5 (beam type D2.5), 3.0 (beam type D3), 3.5 (beam type D3.5), 4.0 (beam type D4), 4.5 (beam type D4.5), and 5.0 (beam type D5). The details of all the test beams are shown in Figure 4.1.

4.4.2 Experimental Work

The casting, curing, and test procedures as well as the approach used to correct the load measurements were identical to those used in Test Series 'A' described in Chapter 3.

4.4.3 Test Results

The resulting concrete compressive strengths, beam dimensions, ultimate load, and the corrections to the actual ultimate loads obtained from the various test beams in Test Series '1' and Test Series 'B' are given in Table 4.2. The corrected relative flexural strengths of each leg \( \frac{M_i}{M_f} \) for all beams included in this test series as well as those beams in the Test Series '1'[130] are given in Table 4.3. The full flexural capacity \( M_f \) which is required to permit the calculation of the ratio \( \frac{M}{M_f} \) was obtained from the beams in Test Series '1' (beam types A1 and B1). For the purpose of discussion the load-mid span deflection curves for the test beams are shown in Figure 4.2. The relative flexural capacities of the inclined and the horizontal legs
(leg capacity valleys) are shown in Figures 4.3 and 4.4 respectively. The relative flexural capacities of the legs are discussed with reference to Kani's Valley. For this purpose, Kani's Valley shown in Figures 4.3 and 4.4 was based on the results from the tests[18, 138] on beams which were similar to those in the present test series. Finally, the mode of failure of beam type D4 is shown in Figure 4.5.

4.4.4 Discussion of Test Results

The beams used for the evaluation of the inclined leg capacity (beams type C2.5) failed by diagonal cracking in the inclined leg region where no stirrups had been provided. Flexural cracks developed in the lower part of the beams at a load level approximately equal to 0.2 times the ultimate load. Diagonal cracks appeared in the inclined leg regions at a load level approximately equal to 30% of the ultimate load. Additional flexural and diagonal cracks appeared and widened under increasing loads. The width of the flexural crack reached 0.3mm while the corresponding width of the diagonal cracks at ultimate load was of the order of 10.0mm. The presence of short stirrups near the point where the load path changes direction prevented propagation of the diagonal cracks into the compression zone in that region thus resulting in an increase in the load carrying capacity of the beams. The beams achieved 93% of their full flexural capacity as shown in Tables 4.2 and 4.3. An increase in the flexural capacity in excess of 30% over beams without lateral reinforcement based on Kani's Valley, Figure 4.3, was achieved by preventing failure of the horizontal leg region through the introduction of short stirrups. As a result of the increased load carrying capacity the biaxial state of stress in the inclined leg regions
became critical and the beams failed in a brittle manner by diagonal cracking in that region. The maximum mid-span deflection at failure was less than 30mm, Figure 4.2.

In the test beams used for the evaluation of the inclined leg capacity (beam types D2.5, D3, D3.5, D4, D4.5, and D5) the conventional full length stirrups in the inclined leg regions succeeded in preventing a diagonal failure in those regions. Most of the beams with relatively small a/d ratios (beam types D2.5 and D3) failed in a flexural mode and achieved almost their full flexural capacity, Tables 4.2 and 4.3.

For beam type D2.5 flexural cracks appeared at load levels approaching 20% of the maximum loads. The diagonal cracks appeared at load levels approaching 30% of the maximum loads. The number of flexural and diagonal cracks and their widths increased with increasing load. The widths of the two types of cracks were approximately similar up to failure. They reached a value approaching 2.6mm under the peak loading and had reached 4.0mm at failure. The absence of stirrups in part of the shear span near the loading points resulted in the development of excessive diagonal cracking although the beam failed in a flexural mode by spalling of the compression concrete in the mid-span region. The average capacity of the beams was 99% of their full flexural capacity, Tables 4.2 and 4.3. The prevention of a diagonal failure allowed the beams to undergo relatively large deflections (up to 47mm) compared to that obtained, for example, from beam type C2.5 (25mm) which had the same a/d ratio but failed by diagonal cracking. It is concluded that for this type of beam the most critical state of stress exists in the inclined leg regions and
that a diagonal failure can be prevented by the presence of conventional full length stirrups in these regions.

The structural behaviour of beam type D3 was similar to that described above for beam type D2.5. The beams achieved their full flexural capacity, Tables 4.2 and 4.3. In one beam (beam type D3-1) the critical diagonal cracks propagated into the compression zone inside the mid-span region because of the absence of stirrups in the horizontal leg region. At failure these cracks propagated into the compression concrete which subsequently spalled in the mid-span region. The diagonal crack width approached 5.0mm at failure while the flexural crack width was only of the order of 2.5mm. Nevertheless, the cracking mechanism did not impair either the strength or the ductility of the test beams, Figure 4.2.

Beam types D3.5, D4, D4.5, and D5 all failed by diagonal cracking without reaching their full flexural capacity. Flexural cracks appeared at load levels varying from 20% to 60% of the maximum load for beams with a/d ratios equal to 3.5 and 5.0 respectively. Similarly the diagonal cracks appeared at load levels varying from 40% to 85% of the maximum loads. It should be noted that for a constant flexural capacity the maximum loads carried decrease as the a/d ratio increases. All beams failed suddenly when they reached their ultimate capacity. The mid-span deflections at diagonal failure was as low as 15mm (beam type 4.5), Figure 4.2. Beam types D4, D4.5 and D5 all failed in a typical diagonal-tension mode, Figure 4.5, which is characteristic of the behaviour of type II beams. On the other hand, in beam type D3.5
the critical diagonal cracks at failure propagated into the compression zone inside
the mid-span region. The type of failure obtained was characteristic of that for
type III beams. For beam type D3.5 the presence of the conventional full length
stirrups in the inclined leg regions allowed the beams to achieve an increase in their
load carrying capacity. The average capacity obtained was equal to 95% of their full
flexural capacity, Tables 4.2 and 4.3. As the a/d ratio increased the biaxial state
of stress in the inclined leg regions became less critical and failure results from the
state of stress in the horizontal leg regions. Confining the inclined leg regions did
not result in any enhancement in the load carrying capacity of the beams with an
a/d ratio greater than 4 (beams type D4.5 and D5), Figure 4.4. It is concluded that
for beams with large a/d ratios (larger than 4) confinement of the inclined leg region
has no effect on the load carrying capacity of beams.

The aim of this test series was to determine the flexural capacity of the various leg
regions in the beams for the implementation of the proposed flexure-shear interaction
design model. For this purpose the test results obtained and the results from other
test programmes[106, 130], Tables 4.2 and 4.3, were used to draw the leg capacity
curves shown in Figures 4.3 and 4.4.

4.4.5 Relative Flexural Capacity of the Inclined Leg ($\frac{M_i}{M_f}$)

Figure 4.3 shows the relative flexural capacity curve for the inclined leg ($\frac{M_i}{M_f}$).
The curve shows that beams with an a/d ratio greater than 4.0 can reach their
full flexural capacity ($\frac{M_i}{M_f} = 1.0$) without having to confine the inclined leg regions.

On the other hand, based on Kani's Valley, for type III beams with an a/d ratio

151
less than 2.0 confinement of the horizontal leg had no effect on the load carrying capacity of beams. Figure 4.3 also shows that in the case of beams with an a/d ratio less than 4.0 the confinement requirements increase ($M^h_i / M_f$ decreases) as the a/d ratio decreases. The reason for this is that more load is carried to the supports as the a/d ratio decreases which results in a more severe biaxial state of stress in the inclined leg region. This behaviour continues until an a/d ratio equal to 2 where more load is carried directly to the supports thus reducing the shear stresses.

### 4.4.6 Relative Flexural Capacity of the Horizontal Leg ($\frac{M_i^h}{M_f}$)

Figure 4.4 shows the relative flexural capacity curve for the horizontal leg ($\frac{M_i^h}{M_f}$). The curve shows that for type III beams confinement of the inclined leg region (the shear span) which is subjected to the critical biaxial state of stress, is sufficient for the beams to achieve their full flexural capacity ($\frac{M_i^h}{M_f} = 1.0$). For such beams the state of stress in the horizontal leg region is not critical, thus confinement of that region is not necessary as was shown in Figure 4.3. On the other hand, for type II beams with increasing a/d ratios less load is transmitted to the supports thus the state of stress in the inclined leg region becomes less critical. Thus, confinement of the inclined leg regions becomes less effective as the a/d ratio increases. The ratio $\frac{M_i^h}{M_f}$ decreased from about 1.0 (full flexural capacity) for beams with an a/d ratio equal to 3 to approximately 0.9 for beams with an a/d ratio equal to approximately 4. For beams with an a/d ratio greater than 4.0 the state of stress in the inclined leg regions did not influence their behaviour under loading. The state of stress in the horizontal leg is the most critical for these beams, thus providing the inclined leg regions with stirrups had no effect on the load carrying capacity as shown in
4.4.7 Conclusions

The observations described above obtained from the leg capacity curves confirm the conclusions reached in Chapter 3 regarding the proposed understanding of the structural behaviour of beams. For beams with small a/d ratios (type III beams) the biaxial state of stress in the inclined leg regions is the most critical and thus it is sufficient to confine the inclined leg region in order to achieve the full flexural capacity. For beams with a large a/d ratio (type II beams) the state of stress in the horizontal leg region is the most critical. However, confining the horizontal leg regions increases the load carrying capacity of beams which in turn may initiate the critical biaxial state of stress in the inclined leg regions (for type II beams with an a/d ratio less than 4.0). In the case of these types of beams both the horizontal and the inclined leg regions require confinement.

4.5 GENERAL METHOD FOR THE EVALUATION OF THE LEG CAPACITIES

Figures 4.3 and 4.4 were based on the results obtained from Test Series ‘B’, Test Series ‘1’, and the Birzeit University test programme. In practice, it is obviously necessary to design beams with different material and geometric properties to those used in the test beams. It is therefore necessary to evaluate the leg capacity for such beams. This can be accomplished by modifying the resulting horizontal and inclined leg capacity ($M_i$) curves, Figures 4.3 and 4.4. It is proposed that the leg
capacity curves are modified in accordance with the ultimate flexural capacity of the design beams ($M_u$). The justification for this is that $M_u$ is influenced by the stress conditions in both legs. The relative flexural capacity of a beam ($\frac{M_u}{M_f}$) is increased in situations where favourable stress conditions exist in both legs, which is associated with for example a smaller value of $\rho$. This implies that the relative leg capacities ($\frac{M_L}{M_f}$) are also increased due to the less severe stress conditions which are present in these regions. On the other hand, if the stress conditions become more severe, for example as a result of increasing the value of $\rho$, $\frac{M_u}{M_f}$ and $\frac{M_L}{M_f}$ will both be reduced.

In both cases (increasing or decreasing the value of $\rho$) the relative leg capacity $\frac{M_L}{M_f}$ is a function of the change in the relative flexural capacity of the beam ($\frac{M_u}{M_f}$). It is proposed for the generalisation of the flexure-shear interaction design model, therefore, that the new (general) relative leg capacity is determined using the following relationship:

Let $\tau_u = \frac{M_u}{M_f}$, and $\tau_l = \frac{M_L}{M_f}$.

Then,

$$\tau_{\text{new}} = \tau_{\text{old}} + \frac{1 - \tau_{\text{old}}}{1 - \tau_{\text{old}}} (\tau_{\text{new}} - \tau_{\text{old}})$$

(4.6)

where:

$\tau_{\text{new}}$ is the required relative leg capacity value for the new geometric and material properties of the beams.
is the measured relative leg capacity value obtained from the test beams, Figures 4.3 and 4.4.

Let \( r_{l_{\text{old}}} = r_{l_{(\text{old,measured})}} \), and \( \Delta r_{l_{(\text{old,measured})}} = 1 - r_{l_{(\text{old,measured})}} \).

\( r_{u_{\text{old}}} \) is the predicted relative ultimate flexural capacity value of the test beams based on the assumed flexure-shear interaction model (see next section for the model selected).

Let \( r_{u_{\text{old}}} = r_{u_{(\text{old,model})}} \).

\( r_{u_{\text{new}}} \) is the predicted relative ultimate flexural capacity value for the new geometric and material properties of the beams designed using the flexure-shear interaction model which has been adopted.

Let \( r_{u_{\text{new}}} = r_{u_{(\text{new,model})}} \).

Equation (4.6) can be represented in the following form:

\[
\frac{r_{l_{\text{new}}} - r_{l_{(\text{old,measured})}}}{\Delta r_{l_{(\text{old,measured})}}} = \frac{(r_{u_{(\text{new,model})}} - r_{u_{(\text{old,model})}})}{(1 - r_{u_{(\text{old,model})}})}
\]

(4.7)

The applicability of equation (4.7) can be shown by considering the following two cases:

1. For the geometric and material properties used in the test programmes:

\[
\frac{r_{u_{(\text{new,model})}} - r_{u_{(\text{old,model})}}}{(1 - r_{u_{(\text{old,model})}})} = r_{u_{(\text{old,model})}}
\]

(4.8)

Then equation (4.7) becomes:
This implies that the relative leg capacity of the test beams is the measured one which is the case in practice.

2. For the case of beams which are predicted to attain their full flexural capacity:

\[ r_{u(\text{new, model})} = 1.0 \]  

(4.10)

Then equation (4.7) becomes:

\[ r_{l_{\text{new}}} = r_{l(\text{old, measured})} + \Delta r_{l(\text{old, measured})} \left( \frac{1 - r_{u(\text{old, model})}}{1 - r_{u(\text{old, model})}} \right) = 1.0 \]  

(4.11)

This implies that the relative leg capacity is equal to the full flexural capacity of the beams i.e. such beams do not need stirrups to attain their full flexural capacity which is the case in practice.

4.6 Ultimate Flexural Capacity of Beams (\(M_u\))

The determination of the magnitude of the required enhancement in the concrete compressive strength (\(\Delta f_c\)) is based on the evaluation of the leg capacity (\(M_l\)), equations (4.1) - (4.5). The evaluation of the leg capacity is a function of the ultimate flexural capacity of the beams (\(M_u\)), equation (4.7). For this purpose, it is necessary
to adopt a model which is able to predict the relative ultimate flexural capacity of beams (flexure-shear interaction model).

Various analytical models have been proposed to determine the reduction in the flexural capacity of beams because of the influence of shear\cite{17, 18, 44, 148, 150, 151, 152, 153}. The majority of these models are based either on simplified models derived directly from test results\cite{151, 153} or on a separation between beam and arch actions in the computation of the resisting moments\cite{17, 18, 148}. There is currently strong disagreements on the mechanism of diagonal failure and hence on the theoretical basis of any of the proposed analytical models. Nevertheless, in this investigation an analytical model is not intended to be used to determine the actual relative flexural capacity of beams, but only as a measure of the change in the relative leg capacity of beams, equation (4.7). For this purpose, a model recently developed by Russo, et. al.\cite{44}, equations (4.12) and (4.13), which has proved to give the most accurate prediction of the results of tests conducted by many researchers\cite{44} was adopted. However, other models could have been considered for comparison purposes.

\[
\frac{M_u}{M_f} = \zeta \frac{0.83\rho^{\frac{1}{2}}(f'_c)^{\frac{1}{2}}g + 206.9\rho^{\frac{1}{2}}\left(\frac{g}{d}\right)^{\frac{3}{2}}}{\rho f_u \left(1 - \frac{\rho f_u}{1.7f'_c}\right)} \tag{4.12}
\]

where:

\[
\zeta = \frac{1}{\sqrt{1 + \frac{d}{(25d_a)}}} \tag{4.13}
\]

where \(d_a\) is the maximum aggregate size.
4.7 PROVISIONS OF THE CONFINEMENT REQUIREMENTS

4.7.1 Introduction

Transverse steel in structural concrete members serves a three-fold function; confinement of the compression concrete, prevention of lateral buckling of the longitudinal reinforcement steel, and the provision of shear resistance. The properties of confined concrete with respect to strength and ductility differ significantly from that of plain concrete. Carefully detailed transverse reinforcement improves the properties of the concrete through confinement.

It is generally accepted that spiral reinforcement results in increased strength and ductility of confined concrete. It is also accepted that concrete confined by rectilinear ties leads to increased ductility but there is a division of opinion on both the magnitude of this ductility and on the contribution from the ties to the enhancement of the strength of confined concrete[154].

An acceptable stress-strain curve for plain concrete exists. On the other hand, numerous laboratory based studies have been carried out to investigate the behaviour of concrete confined by transverse reinforcement[154, 155]. Several models have been proposed with various degrees of sophistication to describe the stress-strain relationship of confined concrete. A comparative study by Sakai et. al.[155] showed that the majority of these analytical models were only applicable to the lim-
ited data on which they were developed. This was attributed to the difference in the
details of the test specimens used and the variables considered in the development
of the empirically based analytical models.

While in this section no attempt has been made to review all the confinement
models, related basic concepts and important models have been reviewed in order
to assist in the adoption of the most reliable model. The model adopted will be
modified later in accordance with the proposed design methods for beams under
transverse static loadings.

4.7.2 Types of Confinement

Confinement can be divided into three types: (a) active, (b) passive, and (c) active-
 passive confinement.

(a) Active Confinement

Active confinement is when concrete is subjected to externally applied transverse
stresses such as in the case of axially loaded cylinders subjected to transverse hy-
draulic pressure. Richart et. al.[156] (from reference [13]) suggested that relation-
ships derived from active confinement tests can be applied, with some reservations,
to concrete confined using spirals. The active confinement tests showed that both the
strength and the ductility of concrete increase with confinement. When the effects of
instrumentation, size, and shape of specimens on confinement were investigated[157],
it was found that the boundary constraints prevent transverse deformations of the
specimens and will therefore result in the introduction of incorrect material parameters.

(b) Passive Confinement

In practice, concrete may be confined by transverse reinforcement, commonly in the form of spirals or rectilinear ties. The concrete becomes effectively confined under axial stresses approaching its uniaxial strength (60-70% of the cylinder strength[158]). When the transverse strains become very high because of progressive internal cracking, the concrete bears against the transverse reinforcement which then applies a confining reaction to the concrete. In this case confinement is referred to as passive.

It was suggested[145] that, ideally, concrete should never fail as long as there is an appropriate level of lateral pressure acting on it thus preventing the internal tensile cracks from propagating.

Tests on passively confined specimens have shown that confinement enhances both the ductility and the strength of concrete[156, 159, 160, 161, 162, 163]. However, circular stirrups (spirals) confine concrete more effectively than rectangular stirrups. This is because the shape of spirals is such that they are in axial hoop tension and thus provide continuous confining pressure around the circumference. In this case, confinement depends on the pitch of the spirals and if the spirals are spaced closely enough together confinement can approach that provided by uniform lateral pressure. On the other hand, rectangular ties are subjected to both bending and tensile stresses. In this case, confinement is provided by the arch action between
adjacent transverse bars and to a certain extent by arch action between longitudinal bars. The dependency of rectilinear ties on the spacing of vertical bars and the corresponding bending stresses in these ties make rectilinear ties less efficient when compared to spirals in confining concrete.

In studying the effect of confinement on the stress-strain relationships for passively confined concrete, the following factors are usually considered to be relevant[155].

1. Types and strength of concrete.

2. Amount and distribution of longitudinal reinforcement.

3. Amount, spacing, and configuration of transverse reinforcement.

4. Size and shape of confined concrete.

5. Ratio of confined area to gross area.


7. Strain gradient.

8. Supplementary cross ties.

9. Cyclic loading.


11. Level of axial load in the case of flexural behaviour.
When failure of confined concrete is due to yielding of the transverse reinforcement, the use of high tensile steel stirrups provides a higher degree of confinement[164]. However, to develop high stresses in the transverse reinforcement, large lateral strains in the concrete must develop and as a result excessive internal damage occurs. This may adversely affect the durability and serviceability of the concrete. To overcome this problem, prestressing of the spiral confinement has been suggested[165] in order to create a triaxial state of stress which is independent of the transverse deformation of the concrete during the initial stages of the application of the loads (active confinement). Eventually, when the precompression in the concrete is neutralised by the concrete dilation, the concrete becomes passively confined.

This research programme includes beams with rectilinear and circular stirrups and therefore only passive confinement models will be considered in the following sections.

4.7.3 Confinement Models for Circular Sections

In general, for axially loaded members, a combination of lateral confinement pressure and axial compression results in a triaxial state of stress. Lateral pressure counteracts the tendency of concrete to dilate laterally and results in increased strength. An expression to describe the triaxial strength of concrete in terms of uniaxial stress and lateral confinement pressure is given below:
Where \( f'_{cc} \), and \( f'_{co} \) are the confined and unconfined strengths respectively of concrete in a member. The coefficient \( K1 \) is a function of Poisson’s ratio. The variation in \( K1 \) with respect to lateral pressure \( f_l \) is obtained from the results of tests on cylinders subjected to different levels of hydraulic pressure. Richart et. al.[156] (from reference [13]) suggests that \( K1 \) should be equal to 4.1. Other researchers[166] have proposed different relationships, nevertheless, a value of 4.1 has been adopted in Codes of Practice[8].

For concentrically loaded columns, assuming spirals are spaced sufficiently close together to apply a near-uniform pressure[32, 166], the confining pressure \( f_l \) is calculated from the hoop tension developed in the spiral steel. When the lateral reinforcement yields, the axial confined concrete compressive strength \( f'_{cc} \) is calculated as follows[32, page:25]:

\[
f'_{cc} = f'_{co} + K1f_l
\]

(4.14)

where

\[
f'_{cc} = f'_{co} + 8.2 \frac{f_y A_{sp}}{d_s s}
\]

(4.15)

where

- \( d_s \) is the diameter of the spiral.
- \( A_{sp} \) is the area of the spiral bar.
- \( s \) is the pitch of the spiral.
Different confinement models with various degrees of sophistication have been proposed by researchers to describe the stress-strain relationship of spirally reinforced concrete\[159, 166, 167, 168, 169, 170, 171, 172\]. However, it is generally considered to be sufficient to adopt the simple relationship given by equation (4.15) to calculate the concrete confined compressive strength\[32\].

4.7.4 Confinement Models for Rectangular Sections

Numerous laboratory based studies have been reported on the behaviour of concrete confined with ties. Previous research was conducted on small-size specimens with simple tie configurations. The confined core compared to the gross area was small, so the confinement strength of the specimens did not exceed the unconfined strength. The extra strength due to confinement was assumed to be offset only by the reduction in strength resulting from the spalling of the concrete cover.

A number of stress-strain curves for confined concrete have been proposed. Several variables have been considered in these models such as: the amount of lateral reinforcement, the strength of the concrete and the steel, and the distribution of longitudinal steel and the resulting tie configurations.

The beneficial effect of ties on the strength of concrete has generated a lot of disagreement among researchers. Sheikh\[154\] and Sakai and Sheikh\[155\] conducted comparative studies on several analytical models used to describe the behaviour of
confined concrete. They reported that in all the models the amount of longitudinal steel had no effect on the properties of the confined concrete. Also, in specimens with only four corner bars the effectively confined concrete at the critical section between the ties, could be very small compared with the core area bounded by the centre lines of the tie thus resulting in poor confinement. Small size specimens, simple steel arrangements, low volumetric ratios of lateral steel to concrete core, and low ratios of the core area to the gross area of specimens would result in small increases in the strength and ductility of the confined concrete.

It was concluded from the comparative studies that the most relevant models for passive confinement are those which are based on testing full-size specimens with various arrangements and numbers of longitudinal bars as well as overlapping ties.

Models by Park et. al.[161, 172, 173] and by Sheikh et. al.[163, 174, 175, 176, 177, 178] are considered, based on the above criteria, to be realistic models. When predictions from various models were checked against experimental results[154], it was concluded that the model which was developed by Sheikh[174] gave the best prediction.

4.7.5 Influence of Strain Gradient

Studies on the influence of strain gradient on confinement have shown conflicting results. While most researchers[154, 158, 173, 179, 180, 181] reported an increase in strain due to strain gradient, there have been disagreement on its effect on the
concrete compressive strength. Several researchers[158, 179, 180] reported no increase in strength due to strain gradient while others[181] reported an increase of up to 20%. Scott et. al.[173] stated that the presence of a strain gradient results in a smaller decrease in the load and the moment carried as the strain increases beyond that associated with the peak load when predicted using the stress-strain curves from concentric load tests. They concluded that when the neutral axis lies within the section, estimating the strength using stress-strain curves obtained from concentric load tests was undoubtedly conservative.

The influence of strain gradient on the strength of confined concrete was related to its influence on the area of the concrete which was effectively confined[175]. When the ratio of the depth of the neutral axis to the depth of the confined concrete \( \frac{d}{B} \) was less than 0.5, the ratio of the concrete effectively confined area to the core area \( (\lambda) \) based on concentric compression overestimates the efficiency of confinement. However, the increase in concrete compressive strength due to steep strain gradients, which produces additional confinement, compensates for the low efficiency. On the other hand, when the \( \frac{d}{B} \) is larger than 0.5, the concrete strength has significant influence on behaviour. In this case, the efficiency of confinement is at least equal to that experienced under concentric compression. It should be noted that, due to the presence of shear, a high confined concrete strength may be developed. This was attributed to the early mobilisation of the lateral steel[177].

When applying the confinement model originally developed by Sheikh et. al.[174]
to specimens under the combined action of axial load and flexure, the results were consistently conservative under large curvatures[154]. However, the flexural strength predictions changed from conservative to unsafe as the magnitude of the axial load was increased[178]. It was explained that under high axial loads, the strain gradient reduces (i.e. less confinement) which results in lower concrete strengths.

It can be concluded from the above discussion that the presence of a strain gradient has a beneficial effect on the ductility of the confined concrete. The flexural capacity of beams is not influenced significantly by a moderate variation in concrete strength[176] even although the compressive strength of confined concrete may increase as a result of strain gradients.

4.7.6 Summary and Conclusions

Equation (4.15) has been adopted for beams with circular cross sections in this research programme. However, other relationships[172] will also be considered for estimating the compressive strength of the confined concrete.

For rectangular beams, the beneficial effects of the presence of a strain gradient on ductility[175] and the influence of high axial load levels on flexural capacity[178] are included in the confinement model developed by Sheikh et. al.[175, 178]. The accuracy of the model was tested against the results from different experimental programmes and gave very good results[154, 155, 175, 176, 177, 178]. Therefore, this model has been adopted in the case of this research programme into the behaviour
of beams with rectangular cross sections. However, modifications with respect to
the calculation of the effectively confined area of the concrete will be made to make
the model applicable to beams under only transverse loading.

In the following sections, the confinement model adopted for rectangular beams
and the proposed modifications to it have been outlined.

4.7.7 Confinement Model for Rectangular Beams

In the model[175, 178], the increase in concrete strength is calculated based on an
effectively confined concrete area, which is less than the concrete core area enclosed
by the centre line of the perimeter tie. The distribution of core concrete between the
contained and the effectively confined categories is a function of the distribution of
the longitudinal and the lateral reinforcement. Closer spacing of both the longitudi-
nal and the lateral reinforcement results in a higher proportion of the confined area
becoming effective as shown in Figure 4.6, and also in higher strength and ductility
of the concrete.

The stress-strain curve OABCDE shown in Figure 4.6 represents the behaviour
of concrete in the core (centre to centre of the perimeter tie) for concentric compres-
sion loading[174]. The curve consists of three parts. Part ‘OA’ is a second degree
parabola with point ‘A’ at \( f_{cc1}, \varepsilon_{s1} \). The term \( f_{cc} \) represents the compressive strength
of confined concrete which is equal to \( K_s f_{cp} \), in which \( f_{cp} \) is the compressive strength
of the plain concrete and \( K_s \) is the confinement enhancement factor (strength gain
factor). The terms \( \varepsilon_{s1} \) and \( \varepsilon_{s2} \) are the minimum and the maximum strain values re-
spectively corresponding to the maximum stress. The term \( \varepsilon_{85} \) is the value of strain corresponding to 85% of the maximum stress in the unloading branch of the curve. Parts 'AB', and 'BC' of the curve are straight lines. The curve is assumed to continue in the same straight line beyond the point 'C' until the stress drops to about 30% of its maximum value (point 'D'). A horizontal line is assumed beyond the point 'D'.

The effect of the presence of a strain gradient on the ductility of confined concrete, was included in the model by extending part 'AB' of the stress-strain curve to 'AB'' as shown in Figure 4.6. In this case, the stress-strain curve \( OABB'C'D'E \) represents the behaviour of confined concrete subjected to a strain gradient.

To include the influence of a high axial load level on the concrete strength, the confined concrete strength \( f_{cc} \) was multiplied by a reduction factor \( (\eta)[178] \).

In reference to Figure 4.6, the governing equations of the confinement model which have been adopted in this investigation are given below (SI units):

\[
f_{cc} = \eta K_s f_{cp} \tag{4.16}
\]

\[
\eta = 1 - 0.575 \frac{P - P_b}{f_{cp} A_g} \leq 1.0 \tag{4.17}
\]

\[
K_s = 1 + \frac{B H}{P_{occ}} \left[ \left( 1 - \frac{\sum_{i=1}^{n} c_i^2}{\alpha A_{os}} \right) \left( 1 - 0.5 \frac{s \tan \theta}{B} \right) \left( 1 - 0.5 \frac{s \tan \theta}{H} \right) \right] \beta(\rho f_{sv})^\gamma \tag{4.18}
\]

\[
\varepsilon_{s1} = 0.0022 K_s \tag{4.19}
\]
\[ \varepsilon_{s1} = 80K \varepsilon_c (10^{-6}) \]  
\[ \varepsilon_{s2} = 1 + \left( \frac{248}{c} \left[ 1 - 5 \left( \frac{s}{B} \right)^2 \right] + 3\sqrt{\frac{B}{c}} \right) \frac{\rho_s f_{s'}}{f_{c'}} \]  
\[ \varepsilon_{s5} = 0.225 \rho_t \sqrt{\frac{B}{s}} + \varepsilon_{s2} \]

and
\[ Z = \frac{1.0}{1.5 \rho_t \sqrt{\frac{B}{s}}} \]  

In equation (4.18), \( \alpha = 5.5, \beta = \frac{1}{140}, \theta = 45^\circ, f_{sv} = f_{yy} \) (test results support this assumption[177] and in addition confinement is not sensitive to the stress in the tie steel), and \( \gamma = 0.5 \).

where:

- \( P_b \) is the balanced-failure axial load.

- \( A_{co} \) is the area of the core measured from centre to centre of the perimeter tie.

- \( A_s \) is the area of the longitudinal steel.

- \( B \) is the core size (width) measured from the centre to centre of the perimeter tie.

- \( c \) is the distance between laterally supported longitudinal bars.

- \( f_{cp} \) is the strength of unconfined concrete in the structural concrete member and is equal to \( K_p f_c' \).

- \( K_p \) is the ratio of unconfined concrete strength in the structural concrete member to \( f_c' \). It is evaluated assuming:
\[ K_p = 1.0 \text{ for } P < P_b. \]
\[ K_p = \left(1 - 0.15 \frac{P - P_b}{P_o - P_b}\right) \] for \( P_b < P < P_o. \)

\( P_o \) is the axial load strength.

\( f_{sv} \) is the stress in the lateral steel.

\( n \) is the number of arcs containing concrete which is not effectively confined and is also equal to the number of laterally supported longitudinal bars.

\[ P_{occ} = K_p f_c'(A_{cc} - A_s), \] unconfined strength of the concrete core.

\( s \) is the tie spacing \((s_v)\).

\( \rho_t \) is the ratio of the volume of the tie steel to the volume of the core.

\( \varepsilon_o \) is the strain corresponding to the maximum stress in the unconfined concrete.

\( Z \) is the slope of the descending part of the stress-strain curve (part 'B'D').

4.7.8 Equivalent Concrete Compressive Block

To simplify the application of the stress-strain model in flexure, an equivalent stress block, Figure 4.6, has been proposed [175]. The proposed equivalent stress block has a width equal to \( \beta f_{cc} \), and a length equal to \( \alpha x \), where \( x \) is the depth to the neutral axis. The different parameters relating to the stress block are calculated as follows:

Zone 1: \((\varepsilon_c \leq \varepsilon_{st})\)

\[ \alpha = \frac{4 - \Omega}{2(3 - \Omega)} \] (4.24)
and

\[ \beta = \frac{2\Omega(3 - \Omega)^2}{3(4 - \Omega)} \]  
(4.25)

**Zone 2:** \((\varepsilon_{s1} < \varepsilon_c \leq \varepsilon_{s2})\)

\[ \alpha = \frac{6\Omega^2 - 4\Omega + 1}{2\Omega(3\Omega - 1)} \]  
(4.26)

and

\[ \beta = \frac{2(3\Omega - 1)^2}{3(6\Omega^2 - 4\Omega + 1)} \]  
(4.27)

**Zone 3:** \((\varepsilon_{s2} < \varepsilon_c \leq \varepsilon_{s30})\)

\[
\begin{align*}
\alpha\beta &= 1 - \frac{1}{3\Omega} - 0.075 \left(\frac{GD}{D - G}\right) \left(1 - \frac{1}{D}\right)^2 \\
\alpha &= 2 - \frac{1}{\alpha\beta} \left[1 - \frac{1}{6\Omega^2} - \frac{G(2D^3 - 3D^2 + 1)}{20D^2(D - G)}\right]
\end{align*}
\]  
(4.28, 4.29)

where:

\[ \varepsilon_c \] is the extreme fibre concrete strain.

\[ \Omega = \frac{\varepsilon_c}{\varepsilon_{s1}}. \]

\[ D = \frac{\varepsilon_c}{\varepsilon_{s2}}. \]

\[ G = \frac{\varepsilon_c}{\varepsilon_{s30}}. \]

**4.7.9 Proposed Modifications to the Confinement Model**

The confinement model which has been adopted was originally derived for square columns under concentric loading. It was then modified to account for the presence of a strain gradient and for the magnitude of the axial load. To make the model applicable to beams detailed in accordance with the proposed flexure-shear interaction
design model, it was envisaged that the following modifications would be necessary
to account for the effectively confined area in such beams:

(a) Confinement Requirement for the Horizontal Leg

In reference to Figure 4.7.a, it is proposed to modify equation (4.18) as follows:

\[
K_s = 1 + \frac{BH1}{P_{occl}} \left[ \left( 1 - \frac{\sum_{i=1}^{n} c_i^2}{\alpha A_{col}} \right) \left( 1 - \frac{0.5 s \tan \theta}{B} \right) \left( 1 - \frac{0.25 s \tan \theta}{H1} \right) \right] \beta(\rho_t f_{yd})^7
\]

(4.30)

where:

x shown in Figure 4.7, is the depth to the neutral axis calculated using normal
flexural theory.

\[A_{col} = BH1.\]

\[P_{occl} = BH1 \ast K_p f'c = BH1 \ast f'c \quad \text{(since } P=0.0).\]

\[\sum_{i=1}^{n} c_i^2 \text{ in this case } = 2c1^2 + c2^2.\]

\[\rho_t = \frac{2(H+B)A_{ts}}{HB s_v}.\]

(b) Confinement Requirement for the Inclined Legs

(i) Near the Point of Force Changing Direction

The same modifications required for the horizontal leg were necessary except that
the calculation of \(\rho_t\) is based on the conventional full length stirrup configuration as
shown in Figure 4.7.b.
Away From the Point of Force Changing Direction

In reference to Figure 4.7.b, the confinement enhancement factor \( K_s \) is calculated as follows:

\[
K_s = 1 + \frac{Bx}{P_{occ2}} \left[ \left( 1 - \frac{\sum_{i=1}^{n} c_i^2}{\alpha A_{co2}} \right) \left( 1 - \frac{0.5 s \tan \theta}{B} \right) \right] \beta(\rho_t f'_{yu})^n \quad (4.31)
\]

where:

- \( x \) is the depth to the neutral axis calculated in accordance with flexural theory.
- \( A_{co2} = Bx \).
- \( P_{occ2} = Bx \cdot f'_{c'} \).
- \( \sum_{i=1}^{n} c_i^2 \) in this case equal to \( 2x^2 \).
- \( \rho_t = \frac{2(H+B)A_{xx}}{HBx_s} \).

4.7.10 General Remarks

1. The confinement enhancement factor \( K_s \) exists only for the compression concrete inside the confined area \( (A_{co} = BH) \), or \( Bx \). The concrete compressive strength is less than \( f'_{c'} \) due to shear. The presence of stirrups offsets the reduction in the strength and enables the concrete to reach its unconfined compressive stress \( (f'_{c'}) \).

2. At full flexural strength, the concrete compressive strength inside and outside the confined area will not increase beyond the ultimate strength \( (f'_{c'}) \). In this
case, flexural theory is applicable. Therefore it is justifiable to calculate the position of the neutral axis based on flexural theory.

3. To prevent a diagonal failure, the role of stirrups in the proposed design approach is to maintain the unconfined concrete compressive strength. As a factor of safety, the direct shear resistance of the stirrups, if any, is not considered.

4.8 IMPLEMENTATION OF THE FLEXURE-SHEAR INTERACTION DESIGN MODEL

4.8.1 Analysis Steps

The steps of the analysis are summarised as follows:

1. Calculate the required confinement enhancement factor \( K_{\text{required}} \) based on either equation (4.3) or equation (4.5).

2. Calculate the confinement enhancement factor \( K_{\text{provided}} \) based on either equation (4.30) or equation (4.31).

3. For safe design \( K_{\text{provided}} \) should not be less than \( K_{\text{required}} \).

The evaluation of the confinement requirements of the beams which were included in Test Series 'A' has been carried out in Appendix B (a detailed discussion of that test programme was included in Chapter 3). The final results of the analysis are summarised in Table 4.4. The analysis shows that the confinement (stirrups) provided and given by the factor \( K_s \) (provided) is equal to or greater than the required confinement given by the factor \( K_s \) (required). These results could be
considered as part of the experimental verification of the proposed flexure-shear interaction design model. An experimental verification of the model for various types of beams has been included in a following section.

4.8.2 Design Steps

The proposed design steps can be summarised as follows:

1. The required confinement enhancement factor \( K_s \) is calculated based on the reduction in the compressive strength in the different legs due to shear (equations (4.3) or (4.5)).

2. In accordance with flexural theory and the proposed design method the geometry of the confined section is determined \((x, c_i, B, H, \text{etc})\).

3. The diameter of the stirrups is specified based on practical considerations.

4. The spacing of the stirrups \( (s_r) \) is determined using either equation (4.30) or equation (4.31). For a distance equal to approximately ‘d’ from the support it is suggested that equation (4.31) should be used. This equation results in lower confinement requirements because of the relatively large effective confined area.

5. Finally, the calculated spacing is checked against the limiting values specified in Codes of Practice.
4.9 TEST SERIES ‘C’: Verification of the proposed flexure-shear interaction design model

4.9.1 Description of Test Beams

Test Series ‘C’ included ten normal-size beams made with normal-strength concrete, Figures 4.8, 4.9, and 4.10. All beams had an overall length of 3500mm and an effective span of 2800mm. The test beams were selected to cover the whole of Kani’s Valley for the different beam types. The variations included the shear span to depth ratio ($\frac{a}{d} = 1.75, 3.2, 3.9, \text{ and } 4.0$), the longitudinal reinforcement ratio ($\rho = 1.5\%, 1.8\%, \text{ and } 2.0\%$), the beam cross section ($200\text{mm} \times 300\text{mm} \text{ and } 200\text{mm} \times 400\text{mm}$), the loading arrangement (four and three point loading systems), and the detailing of the lateral reinforcement (proposed and traditional approaches). The design concrete compressive strength ($f_{cu}$) was equal to 40 MPa. High-strength deformed steel bars with a nominal diameter of either 20mm ($A_s = 310.35\text{mm}^2$ and $f_y = 500 \text{ MPa}$) or 25mm ($A_s = 483.85\text{mm}^2$ and $f_y = 526 \text{ MPa}$) were used for the longitudinal reinforcement. The stirrups were fabricated from plain round mild steel bars of either 8mm nominal diameter ($A_s = 49.9\text{mm}^2$ and $f_y = 420.8 \text{ MPa}$) or 10mm nominal diameter ($A_s = 78\text{mm}^2$ and $f_y = 441 \text{ MPa}$).

Beam type A, Figure 4.8, had an a/d ratio equal to 1.75, a cross section of $200\text{mm} \times 400\text{mm}$, and a value of $\rho$ equal to either 1.5 (beam type A1.5) or 2 (beam type A2). These beams were used to validate the proposed model for type III beams. It should be noted that beams with an a/d ratio equal to 2 were included in Test Series ‘A’ which was described in Chapter 3. The analysis of the beams based on
the proposed model was discussed in an earlier section in this Chapter.

It should be noted that the resulting detailing of the type III beams based on the proposed and the traditional design approaches is similar i.e. all of the shear span is provided with conventional full size stirrups. However, short stirrups were provided along a distance equal to approximately half the beam depth inside the mid-span region of the type A beams. The similarities in the two design methods are not just apparent in the resulting detailing arrangement but also in the calculated stirrup spacing, Table 4.5. Thus, the inclusion of traditionally designed beams for the purpose of comparison between the two design methods was believed not to be necessary for this beam type.

Four type B beams having an a/d ratio of 3.2, Figure 4.9, and four type C beams having an a/d ratio of either 3.9 or 4.0, Figure 4.10, were used to validate the proposed model for the design of type II beams. The test beams had values for $\rho$ of 1.5 (beam types B1.5 and C1.5), 1.8 (beam types B1.8, B1.8(T), C1.8, and C1.8(T)), and 2.0 (beam types B2 and C2). The beams with an a/d ratio equal to 1.8 had cross sections of 200mm x 300mm. The remaining beams had cross sections of 200mm x 400mm. Beam types C1.5 and C2 were tested under a three point loading system.

Test Series 'A' described in Chapter 3 included beams with an a/d ratio of 3.2 and a value for $\rho$ ratio of 1.8. The discussion of the results obtained from these
beams highlighted the need for the testing of similar beams in which the cracking process had to be monitored. One beam was therefore designed using an approach based on the proposed model (beam type B1.8) and one beam traditionally designed (beam type B1.8(T)) were included in this test series. Electrical resistance strain gauges were used to measure the strains at different loading levels in the stirrups in the two beams. The locations of the strain gauges are shown in Figure 4.11. The type C beams also included one traditionally designed beam (beam type C1.8(T)) for comparison purposes. Beam type C1.8 was identical to beam type C1.8(T) except that it was designed and detailed using the proposed model.

4.9.2 Design of Test Beams

For comparison purposes all the beams were designed in accordance with BS 8110 and the proposed flexure-shear interaction design model, Table 4.5. Only two traditionally designed beams (beam types B1.8(T) and C1.8(T)) were, however, included in the test series. In the design of all of the beams the load and resistance factors were not included. The actual structural steel properties, the design concrete strengths, and the assumed dimensions of the beams were used in the design calculations. To prevent anchorage failure all beams were extended beyond the supports for a distance equal to 350mm and the longitudinal bars were also bent up at the ends. The anchorage lengths provided and the radii of the bends in the reinforcement were found to satisfy the appropriate Code requirements.

In the design of the beams based on the proposed model, the design steps de-
scribed in the previous section, the design equations, the leg capacity, the generalised leg capacity curves, the modified confinement model, and the modified confinement area described in this Chapter were used. The determination of the location of the neutral axis was based on the approach put forward in BS 8110.

Table 4.5 summarises the input data and design requirements for all of the beams included in this test series. The traditional design solution for these beams is also included in the Table 4.5 for comparison purposes. Appendix B includes a prototype design example (design of beam type B1.5). Table B.2 in Appendix B gives a summary of the design data and the results obtained from the design calculations for all of the beams including the equivalent traditional design. The design results indicate that, in general, the proposed model results in a more economical design. The spacing of the stirrups in the inclined and horizontal leg regions is smaller in the traditionally designed beams than in the beams which were designed using the proposed model except for the horizontal leg regions of beam type C. Nevertheless, the stirrups in the horizontal leg regions of the beams designed using this model do not extend down the full depth of the beam, thus they produce a more economical solution even if the stirrup spacing is relatively small. It is worthwhile noting that test results[46, 47, 48, 49, 182, 183] have shown that the shear strength can be up to 80% higher than that predicted by the truss analogy. The design results based on the proposed approach support this conclusion since the spacing of the stirrups in some of the beams, Table 4.5, was significantly larger than that required using the truss analogy and the beams did not fail due to diagonal cracking. A detailed cal-
calculation of the savings in the amount of stirrups provided using the proposed model cannot be generalised for all of the beams since it depends on the dimensions of the beams and the properties of the material. In addition, it is believed that since the proposed model is based on concepts derived from a more realistic understanding of the actual structural behaviour of beams, the model will reduce the uncertainties present in current design approaches. This will increase the level of confidence in the design process and will thus allow a reduction in the risk factors (or safety factors) which are usually included in design. The possible reduction in the risk factors will also result in more economical designs.

Since the design was based on the assumed concrete strength and the cross section geometry, re-working of the design procedures was considered on completion of the test series. It was found that the variation in the concrete strength ($f_{cu,ass.} = 34.11 \, MPa$) and in the beam dimensions (Table 4.6) from the values specified had a relatively insignificant influence on the design results. In the prototype design example in Appendix B, for example, if the actual properties ($f_{cu} = 31.1 \, MPa$, $f'_c = 25.4 \, MPa$, and cross section= 209mm x 412mm) were used, the required stirrup spacing was found to be 165mm instead of 155mm for the traditional design. In the case of the proposed model, the stirrup spacings would be 190mm, 175mm, and 225mm instead of 175mm, 160mm, and 220mm for the horizontal leg region, near the point at which the load path changes direction, and near the supports in the inclined leg region respectively.
In the evaluation of the confinement requirements which are necessary to prevent diagonal failures in the test beams, the stirrups were assumed to yield at diagonal failure. The strain measurement results at ultimate load given in Table 4.9 show that the maximum strain in the stirrups in beam type B1.8 was 0.00146 (the stirrup inside the shear span). The stirrup therefore reached 70% of its ultimate strength based on the steel stress-strain curve given in Appendix A. The reason for this is that the beam was not subjected to severe shear stresses \((a/d = 3.2)\) and the beam did not fail by diagonal cracking which usually results in yielding of the stirrups. The implication of this is that there was still a reserve capacity in the stirrups against diagonal failure when the beam reached its full flexural capacity.

4.9.3 Experimental Work

The casting, curing, and test procedures (apart from the loading arrangement described previously for beam types C1.5 and C2) as well as the approach used to correct the load measurements were identical to those used in Test Series ‘A’ described in Chapter 3. In addition, strains in the stirrups in beam types B1.8 and B1.8(T) were measured using strain gauges having a gauge length of 5mm (for more detailed information on the experimental work, refer to Appendix A).

4.9.4 Test Results

The concrete compressive strength \((f_{cu})\), actual beam dimensions, actual ultimate load, correction factors, and corrected ultimate load for all of the test beams are given in Table 4.6. Table 4.7 summarises the theoretical \((M1)\) and the corrected
actual flexural capacities \( (M_f) \) for all of the test beams. The evaluation of the actual flexural capacity \( \frac{M_f}{M_{f1}} \) is also given in Table 4.7. In order to evaluate the serviceability requirements the diagonal and flexural crack widths, and the mid-span deflections at working load, at ultimate capacity, and just before unloading are given in Table 4.8. Table 4.9 summarises the strains present in the stirrups in beam types B1.8 and B1.8(T) under maximum loading and after failure. The measured strains in the stirrups at various load levels are given in Table B.3 in Appendix B. Figures 4.12, 4.13, and 4.14 show the load-mid span deflection curves for beam types A, B, and C respectively. The crack patterns at failure for all of the beams are shown in Figure 4.15. Finally, the deflected shapes of typical test beams after failure are shown in Figure 4.16 (beam types A2, B1.8, and C1.5).

4.9.5 Discussion of Test Results

(a) Modes of Failure

All the beams reached their full flexure capacity and eventually failed in flexure by spalling of the concrete compression zone in the regions of the beams subjected to maximum bending moments as shown in Figure 4.15.

In the type A beams diagonal cracks developed in the shear spans as an extension of existing flexure cracks. The diagonal cracks extended towards the loading points and were arrested by the confining influence of the stirrups in the concrete compression region. The flexure and the diagonal cracks proliferated and widened with increasing load. The short stirrups adjacent to the loading points inside the
mid-span region, Figure 4.8, succeeded in preventing splitting of the compression concrete in that region. Also, they succeeded in preventing the extension of the diagonal cracks inside the mid-span region which is usually characteristic of the failures of short beams. After reaching ultimate load, spalling of the concrete cover was noted in the beams. The compression concrete started to crack and the load carrying capacity was reduced because of the absence of the stirrups in the mid-span region. The beams eventually failed by spalling of the compression concrete and widening of the flexural cracks in the mid-span region. At failure, due to the relatively large diameter of the longitudinal steel bars ($\Phi = 25\text{mm}$), bond cracks widened and joined up with the flexural cracks to extend through to the cracked compression concrete as shown in Figure 4.16.

In beam type B, Figure 4.15, the traditionally designed beam type B1.8(T) failed in a typical flexural failure mode. The flexural and diagonal cracks developed and widened under increasing loads. The diagonal cracks developed after the flexural cracks and were generally wider than the flexural cracks up to the ultimate load. Beam type B1.8 had similar material and dimensional properties except that it was designed and detailed using the proposed model. This beam also failed in a typical flexural failure mode as shown in Figure 4.16. The flexure and shear cracks developed and widened under increasing loads. Spalling in the concrete cover was noted as the ultimate capacity of the beams was approached. Spalling of the compression concrete in the mid-span region occurred at failure. The diagonal cracks did not extend into the mid-span region although the short stirrups were not provided near
to the loading points inside the mid-span region. The diagonal cracks which developed after the flexural cracks in beam type B1.8 were always smaller in width than the flexural cracks and the diagonal cracks which developed in beam type B1.8(T) at the different load levels.

Beam types B1.5 and B2, Figure 4.15, also failed in a typical flexural failure mode similar to that described for beam type B1.8. However, in beam type B1.5 the diagonal cracks were wider at a load level approximately equal to 75% of the ultimate load. At failure, the most critical diagonal crack bypassed the loading point and entered the concrete compression region adjacent to the loading point where spalling of the compression concrete had occurred. This type of behaviour resulted from not including stirrups in the mid-span region near to the loading points. This type of cracking was not observed in the beams included in Test Series ‘A’ in which short stirrups were provided along the entire length of the horizontal leg. This type of behaviour did not prevent the beam from achieving its full flexural capacity. Nevertheless, the extension of the short stirrups into the mid-span for a distance equal to approximately half of the beam depth is recommended for all types of beams regardless of the a/d ratio.

In type C beams, Figure 4.15, the traditionally designed beam (beam type C1.8(T)) failed in a typical flexural failure mode similar to that described previously for the traditional beam type B1.8(T). Beam type C1.8 had similar material and dimensional properties except that it was designed and detailed using the proposed model.
This beam also failed in a typical flexural failure mode. However, the diagonal crack widths were larger than the flexural crack widths at load levels from 68% of the ultimate load up to failure.

Beam types C1.5, Figure 4.16, and C2, Figure 4.15, also failed in flexure which is characteristic of the failure of beams subjected to a centrally positioned load. These two beams were subjected to a three point loading system which forced the failure to take place in the shear spans. Diagonal cracks developed as an extension of the flexural cracks. The flexure and diagonal cracks proliferated and widened under increasing loads. The short stirrups succeeded in preventing the diagonal cracks from propagating into the concrete compression zone. The compression concrete adjacent to the loading point started to crack just before failure. Spalling of the cracked compression concrete occurred at failure.

(b) Load Carrying Capacity and Ductility

The validation of the model was examined by considering the strength and ductility of the test beams. For this purpose the corrected measured flexural capacities ($M_f$) were compared with the predicted theoretical values ($M_1$), Table 4.7. All of the test beams reached their full flexural capacity as can be noted from Table 4.7. The ratio ($\frac{M_f}{M_1}$) ranged from 1.03 (beam type C2) to 1.19 (beam type B1.8) with an average value of 1.11. In structural design, the ductility of a beam can be related to its ability to undergo a significant deflection in the post-elastic range without a substantial reduction in its strength. In the following discussion the mid-span de-
flection obtained before a significant reduction in strength occurred was considered as a measure of ductility. The measured mid-span deflections for the test beams are shown in Figures 4.12, 4.13, and 4.14 where it can be noted that all beams behaved in a typical ductile manner. The ductility obtained from the test beams ranged from 30mm (beam type B1.5) to 45mm (beam type C1.5).

Beam type A in which the stirrup detailing was similar to that used in the traditionally designed beams, achieved an average ratio of 113%, Table 4.7. The ductilities of beam types A1.5 and A2 were 40mm and 35mm respectively. The additional increase in the ductility of beam type A1.5 is due to its low longitudinal reinforcement ratio (ρ = 1.5) compared to that for beam type A2 (ρ = 2). It is known that ductility increases as the value of ρ decreases[32, page 207]. In both beams the extension of the short stirrups into the mid-span region close to the loading points prevented spalling of the compression concrete in that region and thus resulted in a relative improvement in ductility compared to that obtained from beams in which the stirrups were not provided in the mid-span region. On the other hand, the ductility of these beams decreased significantly (up to three times) when compared to the type III beams included in Test Series ‘A’. The reason for this is the absence of confinement in the mid-span regions in the test beams. Short stirrups should therefore also be provided in the mid-span region in situations where more ductile behaviour is required e.g. to survive severe earthquakes or to allow redistribution of bending moments.
Beam type B achieved an average ratio of 113%, Table 4.7. The ductility obtained from these beams was as low as 30mm (type B1.5). The reason for the decrease in the ductility was due to the absence of short stirrups in the mid-span region including the region near the loading points. It is interesting to note that all of the beams failed by spalling of the compression concrete near the loading points, Figure 4.15, where no stirrups were provided. Beam types B1.5 and B2 achieved similar ductilities (about 30mm) although beam type B2 had a larger value of \( \rho \) (\( \rho = 2 \)) compared to that for beam type B1.5 (\( \rho = 1.5 \)). It appears that the increase in concrete strength for beam type B2 (\( f_{cu} = 35.5 \, MPa \)) compared to that for beam type B1.5 (\( f_{cu} = 31.1 \, MPa \)) resulted in its increased ductility. It is known that ductility increases with an increase in concrete strength[32, page 207].

The test results, Table 4.7 and Figure 4.13, confirm the similarity in the behaviour of beam type B1.8 (designed and detailed based on the proposed model) and beam type B1.8(T) (traditionally designed and detailed). The stiffness (inclination of the load-deflection curve), the ductility (40mm), and the load carrying capacity (\( \frac{N}{M_1} = 1.19 \) and 1.15 for beam types B1.8 and B1.8(T) respectively) obtained from the two beams were almost identical. It can be concluded from these results that although the traditional design approach resulted in a less economical solution, it did not result in improved better behaviour in terms of strength and ductility. The strain measurements in the stirrups in these two beams, Figures 4.11 and A.1-Appendix A and Tables 4.9 and B.3-Appendix B indicate the following:

- The strains in the stirrups at the ultimate load in beam type B1.8 (maximum
strain = 0.0015) did not reach the elastic limit of the steel (∼ 0.002). On the other hand, the strain measurements in the stirrups in the traditionally detailed beam type B1.8(T) (maximum strain = 0.0024) showed that they had yielded. The reason for this is that in the traditional beam the widths of the diagonal cracks were higher at all load levels compared with the values obtained from beam type B1.8. The wider diagonal cracks corresponded to the presence of higher strains in the stirrups. The implication of this is that at the ultimate limit state beam type B1.8, which was designed using the proposed model, had a higher reserve strength against diagonal failure than that obtained in the traditionally designed beam.

- The actual tensile strains in the top transverse leg of all of the stirrups were higher than those in the lower and vertical legs. The reason for this is that the top concrete compression region is subjected to the highest compression stresses which results in an increased dilation of the concrete thus leading to higher strains.

- The tensile strains in the stirrups inside the shear span region were higher than those in the stirrups near the loading points in all of the beams. The reason for this is that the stirrups inside the shear span region were crossed by diagonal cracks which were wider than those which crossed the stirrups adjacent to the loading points. In addition, the confinement resulting from the presence of the loading plates might have counteracted the dilation of the concrete and thus resulted in lower strains in the stirrups at that location.
Type C beams achieved an average $\frac{M_f}{M_1}$ ratio of 108%, Table 4.7. Beam types C1.5 and C2 did, however, fail through spalling of the compression concrete inside the shear span (the beams did not have a mid-span region). They achieved 118% and 103% of their theoretical full flexural capacity with approximate ductilities of 45mm and 35mm respectively.

The test results, Table 4.7 and Figure 4.14, confirm the similarity in the behaviour of beam types C1.8 (designed and detailed using the proposed model) and C1.8(T) (traditionally designed and detailed). The stiffness (inclination of the load-deflection curve), the ductility (about 40mm), and the load carrying capacity ($\frac{M_f}{M_1} = 1.04$ and 1.08 for beam types C1.8 and C1.8(T) respectively) obtained from the two beams were almost identical.

4.9.6 Serviceability

In day-to-day practical design, the serviceability limit state requirements (deflection and flexural crack widths at service load levels) are satisfied by following straightforward procedures in which the maximum deflection and flexural crack width are assigned limiting values. The flexural crack width should not exceed 0.3mm and the final deflection should not exceed either the span/250 based on the requirements of BS 8110. The corresponding values in the ACI Code of Practice are 0.41mm and span/360 (live load deflection) respectively. In the case of the test beams the allowable deflection under working load conditions, assuming that the long term
deflection is included, would be approximately 11mm. In the case of the test beams the working load level was estimated, for comparison purposes, to be equal to 60% of the ultimate load (the average capacity reached was 111% of the theoretical ultimate capacity and the average load factor was assumed to be 1.5). A more sophisticated analysis for the serviceability limit state requirements is usually undertaken when the serviceability of members is of major significance. The serviceability requirements are mainly related to the distribution of longitudinal reinforcement (related to the limitation on the 'flexure' crack width) and the depth of members (related to the limitation on the deflection). Therefore, detailing based on the proposed model should not influence significantly the serviceability of the beams. However, the actual widths of the diagonal cracks can influence the serviceability of beams. Little work has been done on how to control diagonal crack width, but it is generally accepted that traditional design approaches result in diagonal crack widths which satisfy the serviceability limit state requirements with respect to crack width. A detailed study of the diagonal cracking mechanism and the resulting crack widths was not within the scope of this investigation. However, to validate the proposed flexure-shear model for the design of beams it is important to compare the serviceability (deflection and crack width) obtained from beams designed using the proposed model with those obtained from traditionally detailed beams. Also, the serviceability obtained from the beams was checked against the requirements of the serviceability limit state. For this purpose, deflection and crack width measurements were taken at the estimated working load levels, the ultimate load level, and before unloading for all of the test beams. The respective values are given in Table 4.8.
The deflection at working load levels for all the beam types included in this test series was approximately 11mm. The measured values just satisfy the serviceability limit state requirements. Table 4.8 shows that the deflections at working load levels obtained from beam types B1.8 (11mm) and C1.8 (12mm) were similar to those obtained from the traditionally detailed beam types B1.8(T) (12mm) and C1.8(T) (11mm). This shows that the proposed detailing approach does not significantly influence the deflection of beams at the working load level. This conclusion can be applied in general terms based on the deflections obtained at different load levels, Table 4.8, for all of the beams.

Table 4.8 shows that the flexural crack widths at working load levels obtained from all of the beams were less than or equal to 0.3mm, except for beam type A which had a crack width approaching 0.4mm. The measured crack widths satisfy the serviceability limit state requirements (0.3mm or 0.41mm based on the requirements of either the BS8110 or ACI Codes of Practice respectively). It should be noted that although the design of beam type A was based on the proposed model, the detailing was similar to that for traditionally designed beams. The diagonal crack widths at working load levels obtained from all of the test beams did not exceed 0.4mm except for beam type A2 (0.45mm) which was traditionally detailed to resist shear. The diagonal crack widths at working load levels obtained from beam types B1.8 (0.15mm) and C1.8 (0.3mm) were less than or similar to that obtained from the traditionally designed and detailed beam types B1.8(T) (0.4mm)
and C1.8(T) (0.2mm). The cracking process summarised in Table 4.8 shows that the widths of the diagonal cracks were always smaller than the flexural cracks at each load level for the majority of the beams. Therefore, it can be concluded that the detailing approach based on the proposed model does not adversely affect the serviceability limit state requirements with respect to crack width. It should also be noted that for practical design purposes, factors of safety are usually included which would require the introduction of additional stirrups. This could result in an improvement in the serviceability and ductility of the beams compared with that obtained from this test series.

4.9.7 Conclusions

The following conclusions can be drawn from the results obtained from the beams in Test Series 'C'.

1. All the beams which were designed and detailed using an approach based on the proposed flexure-shear interaction design model failed in flexure after reaching their full flexural capacity. Therefore, with respect to the ultimate limit state, the proposed model has been validated experimentally for normal-size beams made from normal-strength concrete.

2. The serviceability (deflection and crack widths at service load levels) of the beams designed using the proposed model was found, in general, to satisfy the serviceability limit state requirements. It was also found that the serviceability of the test beams was at least comparable to the corresponding values obtained from the traditionally designed beams. However, it cannot be concluded that
the proposed detailing approach leads to a significant enhancement in the serviceability of beams.

3. To prevent the extension of the diagonal cracks into the concrete compression zone in the mid-span region near the loading point, short stirrups should be placed in the mid-span region over a distance from the load point approximately equal to half the beam depth for all beams regardless of the a/d ratio.

4. The ductility of the beams designed using the proposed model was similar to that found in the traditional beams.

5. The measured ductilities of all of the test beams, including the traditionally designed beams, were less than those obtained from the beams which were included in Test Series ‘A’. If more ductile behaviour is required, the short stirrups should be placed along the entire length of the mid-span region of the beam.

6. The proposed model normally results in a more economical design compared with that which is based on traditional design approaches. This is due to the use of a reduced amount of stirrups (spacing and length) and the anticipated rise in the level of confidence in the design procedure resulting from the use of an approach based on a more realistic understanding of the actual structural behaviour of beams. It is accepted[46, 47, 48, 49, 182, 183] that Code provisions for shear based on truss analogy are overconservative for beams with web reinforcement.
4.10 SUMMARY

In this Chapter the flexural-shear interaction design model which was outlined in Chapter 3 has been developed and implemented for the design of beams. It was necessary for the implementation of the model to determine the leg capacity curves (Test Series ‘B’). The determination of the leg capacity for all types of beams was derived theoretically based on the measured leg capacity curves and the theoretical relative ultimate flexural capacity of beams. To evaluate the amount of stirrups (confinement) required to prevent a diagonal failure, the confinement model developed recently by Sheikh et. al.[178] was modified to comply with the proposed design approach. Finally, the proposed model to prevent diagonal failure was validated experimentally for normal-size beams made from normal-strength concrete (Test Series ‘C’).

In order to generalise the proposed flexure-shear interaction design model, the sensitivity of the approach to the factors which are traditionally believed to influence diagonal failures in beams is required to be investigated.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a/d</td>
<td>shear span to depth ratio</td>
</tr>
<tr>
<td>$f_c$</td>
<td>concrete compressive strength from standard tests</td>
</tr>
<tr>
<td>$f_{cc}$</td>
<td>compressive strength of confined concrete</td>
</tr>
<tr>
<td>$f_{cp}$</td>
<td>compressive strength of unconfined concrete in beam</td>
</tr>
<tr>
<td>$K_s$</td>
<td>confinement enhancement factor</td>
</tr>
<tr>
<td>$M_f$</td>
<td>full flexural capacity of beam</td>
</tr>
<tr>
<td>$M_{fd}$</td>
<td>design bending moment</td>
</tr>
<tr>
<td>$M_l$</td>
<td>ultimate flexural capacity of leg</td>
</tr>
<tr>
<td>$M_{lh}$</td>
<td>ultimate flexural capacity of horizontal leg</td>
</tr>
<tr>
<td>$M_{li}$</td>
<td>ultimate flexural capacity of inclined leg</td>
</tr>
<tr>
<td>$M_r$</td>
<td>decrease in flexural capacity of leg due to shear</td>
</tr>
<tr>
<td>$M_u$</td>
<td>ultimate flexural capacity of beam</td>
</tr>
<tr>
<td>$r_l$</td>
<td>relative ultimate flexural capacity of leg</td>
</tr>
<tr>
<td>$r_{l,new}$</td>
<td>relative ultimate flexural capacity of leg for design beam</td>
</tr>
<tr>
<td>$r_{l,old}$</td>
<td>relative ultimate flexural capacity of leg for test beam</td>
</tr>
<tr>
<td>$r_u$</td>
<td>relative ultimate flexural capacity for beam</td>
</tr>
<tr>
<td>$r_{u,new}$</td>
<td>predicted relative ultimate flexural capacity of design beam</td>
</tr>
<tr>
<td>$r_{u,old}$</td>
<td>predicted relative ultimate flexural capacity of test beam</td>
</tr>
<tr>
<td>$V_f$</td>
<td>shear force corresponding to full flexural capacity of beam</td>
</tr>
<tr>
<td>$V_{fd}$</td>
<td>shear force corresponding to design bending moment</td>
</tr>
<tr>
<td>$V_l$</td>
<td>shear force corresponding to ultimate flexural capacity of leg</td>
</tr>
<tr>
<td>$\Delta f_c$</td>
<td>reduction in compressive concrete strength due to shear</td>
</tr>
<tr>
<td>$\Delta M$</td>
<td>reduction in flexural capacity due to shear</td>
</tr>
<tr>
<td>$\Delta r_l$</td>
<td>reduction in relative ultimate flexural capacity of leg due to shear</td>
</tr>
</tbody>
</table>

Table 4.1: Notation used in the Proposed Design Approach.
### Full load carrying capacity ($P_f = \frac{2M_f}{a}$)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Beam size (mm x mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>Correction factor ($F$)</th>
<th>Measured ultimate load (kN)</th>
<th>Corrected load (kN)</th>
<th>Average load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1-1</td>
<td>2</td>
<td>204.7 x 308.3</td>
<td>46.6</td>
<td>0.9669</td>
<td>480</td>
<td>464.1</td>
<td>452.6</td>
</tr>
<tr>
<td>A1-2</td>
<td></td>
<td>204.9 x 309.3</td>
<td>45.9</td>
<td>0.9694</td>
<td>455</td>
<td>441.1</td>
<td></td>
</tr>
<tr>
<td>B1-1</td>
<td>3.2</td>
<td>204.2 x 313.4</td>
<td>56.8</td>
<td>0.9382</td>
<td>281</td>
<td>263.6</td>
<td>262.3</td>
</tr>
<tr>
<td>B1-2</td>
<td></td>
<td>207.4 x 308.2</td>
<td>62.1</td>
<td>0.9255</td>
<td>282</td>
<td>261</td>
<td></td>
</tr>
</tbody>
</table>

### Load carrying capacity of the inclined leg ($P_i = \frac{2M_i}{a}$)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Beam size (mm x mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>Correction factor ($F$)</th>
<th>Measured ultimate load (kN)</th>
<th>Corrected load (kN)</th>
<th>Average load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2-1</td>
<td>2</td>
<td>200.9 x 308.6</td>
<td>48.8</td>
<td>0.9623</td>
<td>322</td>
<td>309.9</td>
<td>338.9</td>
</tr>
<tr>
<td>A2-2</td>
<td></td>
<td>204.4 x 306.8</td>
<td>48.9</td>
<td>0.9593</td>
<td>376</td>
<td>360.7</td>
<td></td>
</tr>
<tr>
<td>A3-1</td>
<td></td>
<td>206.3 x 310.7</td>
<td>46.0</td>
<td>0.9679</td>
<td>407</td>
<td>393.9</td>
<td></td>
</tr>
<tr>
<td>A3-2</td>
<td></td>
<td>207.1 x 311.3</td>
<td>47.1</td>
<td>0.9632</td>
<td>302</td>
<td>290.9</td>
<td></td>
</tr>
<tr>
<td>B2-1</td>
<td>3.2</td>
<td>200.8 x 302</td>
<td>46.7</td>
<td>0.9696</td>
<td>251</td>
<td>243.4</td>
<td>247.5</td>
</tr>
<tr>
<td>B2-2</td>
<td></td>
<td>205.6 x 311.5</td>
<td>71.8</td>
<td>0.9113</td>
<td>302</td>
<td>275.2</td>
<td></td>
</tr>
<tr>
<td>B3-1</td>
<td></td>
<td>208.7 x 311</td>
<td>50.0</td>
<td>0.9528</td>
<td>264</td>
<td>251.5</td>
<td></td>
</tr>
<tr>
<td>B3-2</td>
<td></td>
<td>207.9 x 308.9</td>
<td>46.2</td>
<td>0.9650</td>
<td>228</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>C2.5-1</td>
<td>2.5</td>
<td>207 x 308</td>
<td>37.5</td>
<td>1.008</td>
<td>322</td>
<td>324.6</td>
<td>323.7</td>
</tr>
<tr>
<td>C2.5-2</td>
<td></td>
<td>207 x 310</td>
<td>43.6</td>
<td>0.9779</td>
<td>330</td>
<td>322.7</td>
<td></td>
</tr>
</tbody>
</table>

### Load carrying capacity of the horizontal leg ($P_h = \frac{2M_h}{a}$)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Beam size (mm x mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>Correction factor ($F$)</th>
<th>Measured ultimate load (kN)</th>
<th>Corrected load (kN)</th>
<th>Average load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2.5-1</td>
<td>2.5</td>
<td>206 x 310</td>
<td>40.2</td>
<td>0.9943</td>
<td>349</td>
<td>347.0</td>
<td>345</td>
</tr>
<tr>
<td>D2.5-2</td>
<td></td>
<td>206 x 308</td>
<td>45.5</td>
<td>0.9713</td>
<td>353</td>
<td>342.9</td>
<td></td>
</tr>
<tr>
<td>D3-1</td>
<td>3</td>
<td>215 x 312</td>
<td>44.6</td>
<td>1.0071</td>
<td>291</td>
<td>293.1</td>
<td>287.5</td>
</tr>
<tr>
<td>D3-2</td>
<td></td>
<td>210 x 310</td>
<td>38.9</td>
<td>1.0362</td>
<td>272</td>
<td>281.9</td>
<td></td>
</tr>
<tr>
<td>D3.5-1</td>
<td>3.5</td>
<td>207 x 309</td>
<td>49.1</td>
<td>0.9979</td>
<td>230</td>
<td>229.5</td>
<td>227</td>
</tr>
<tr>
<td>D3.5-2</td>
<td></td>
<td>206 x 309</td>
<td>46.6</td>
<td>1.0069</td>
<td>223</td>
<td>224.5</td>
<td></td>
</tr>
<tr>
<td>D4-1</td>
<td>4</td>
<td>203 x 308</td>
<td>27.0</td>
<td>1.1420</td>
<td>166</td>
<td>189.6</td>
<td>187.7</td>
</tr>
<tr>
<td>D4-2</td>
<td></td>
<td>204 x 310</td>
<td>40.6</td>
<td>1.032</td>
<td>180</td>
<td>185.8</td>
<td></td>
</tr>
<tr>
<td>D4.5</td>
<td>4.5</td>
<td>211 x 312</td>
<td>40.2</td>
<td>1.0288</td>
<td>159</td>
<td>163.6</td>
<td>163.6</td>
</tr>
<tr>
<td>D5</td>
<td>5</td>
<td>210 x 310</td>
<td>33</td>
<td>1.0737</td>
<td>150</td>
<td>161.1</td>
<td>161.1</td>
</tr>
</tbody>
</table>

Table 4.2: Results from the Beams in Test Series '1' and Test Series 'B'.

197
### Relative flexural capacity of the inclined leg ($\frac{M_{il}}{M_f}$)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Average load (kN)</th>
<th>Flexural capacity ($M_{il}$) (kN.m)</th>
<th>Full flexural capacity ($M_f$) (kN.m)</th>
<th>Relative flexural capacity ($\frac{M_{il}}{M_f}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2-1 to A3-2</td>
<td>2</td>
<td>338.9</td>
<td>88.11</td>
<td>117.76</td>
<td>0.75</td>
</tr>
<tr>
<td>B2-1 to B3-2</td>
<td>3.2</td>
<td>247.5</td>
<td>102.96</td>
<td>109.12</td>
<td>0.94</td>
</tr>
<tr>
<td>C2.5</td>
<td>2.5</td>
<td>323.7</td>
<td>105.2</td>
<td>113.44</td>
<td>0.93</td>
</tr>
<tr>
<td>C†</td>
<td>3.6</td>
<td></td>
<td></td>
<td></td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Relative flexural capacity of the horizontal leg ($\frac{M_{ih}}{M_f}$)

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Average load (kN)</th>
<th>Flexural capacity ($M_{ih}$) (kN.m)</th>
<th>Full flexural capacity ($M_f$) (kN.m)</th>
<th>Relative flexural capacity ($\frac{M_{ih}}{M_f}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2.5</td>
<td>2.5</td>
<td>345</td>
<td>112.13</td>
<td>113.44</td>
<td>0.99</td>
</tr>
<tr>
<td>D3</td>
<td>3</td>
<td>287.5</td>
<td>112.13</td>
<td>109.12</td>
<td>1.03</td>
</tr>
<tr>
<td>D3.5</td>
<td>3.5</td>
<td>227</td>
<td>103.29</td>
<td>109.12</td>
<td>0.95</td>
</tr>
<tr>
<td>D4</td>
<td>4</td>
<td>187.7</td>
<td>97.6</td>
<td>109.12</td>
<td>0.89</td>
</tr>
<tr>
<td>D4.5</td>
<td>4.5</td>
<td>163.6</td>
<td>95.7</td>
<td>109.12</td>
<td>0.88</td>
</tr>
<tr>
<td>D5</td>
<td>5</td>
<td>161.1</td>
<td>104.7</td>
<td>109.12</td>
<td>0.96</td>
</tr>
</tbody>
</table>

* is the full flexural capacity of either type III (for a/d less than 2.5) or type II (for a/d larger than 2.5) beams obtained from Test Series ‘1'[130]. For a/d = 2.5 the average value was considered.
† After Kuttab[106].

Table 4.3: Relative Flexural Capacities of the beams in Test Series ‘1’ and Test Series ‘B'.
<table>
<thead>
<tr>
<th>Beam Type</th>
<th>$K_s$ (Required)</th>
<th>$K_s$ (Provided)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA2-1</td>
<td>- 1.22</td>
<td>- 1.21</td>
</tr>
<tr>
<td>NA2-2</td>
<td>- 1.23</td>
<td>- 1.26</td>
</tr>
<tr>
<td>NA3-1</td>
<td>- 1.22</td>
<td>- 1.43</td>
</tr>
<tr>
<td>NA3-2</td>
<td>- 1.23</td>
<td>- 1.45</td>
</tr>
<tr>
<td>NB2-1</td>
<td>1.05 1.06</td>
<td>1.11 1.11</td>
</tr>
<tr>
<td>NB2-2</td>
<td>1.05 1.06</td>
<td>1.11 1.11</td>
</tr>
<tr>
<td>NB3-1</td>
<td>1.05 1.06</td>
<td>1.25 1.25</td>
</tr>
<tr>
<td>NB3-2</td>
<td>1.05 1.06</td>
<td>1.27 1.27</td>
</tr>
</tbody>
</table>

Table 4.4: Evaluation of the Confinement Requirements ($K_s$) for the Prevention of Diagonal Failure of the Beams in Test Series ‘A’.
<table>
<thead>
<tr>
<th>Beam type</th>
<th>$\frac{a_d}{d}$</th>
<th>$\rho$ (%)</th>
<th>Cross section (mm x mm)</th>
<th>Spacing of stirrups† (mm)</th>
<th>Traditional horizontal leg</th>
<th>Proposed design model horizontal leg</th>
<th>Inclined leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.5</td>
<td>1.75</td>
<td>1.5</td>
<td>200 x 400</td>
<td>110</td>
<td>-</td>
<td>115</td>
<td>170</td>
</tr>
<tr>
<td>A2</td>
<td>2</td>
<td></td>
<td>200 x 400</td>
<td>85</td>
<td>-</td>
<td>100</td>
<td>115</td>
</tr>
<tr>
<td>B1.5</td>
<td>1.5</td>
<td></td>
<td>200 x 400</td>
<td>155</td>
<td>175</td>
<td>160</td>
<td>220</td>
</tr>
<tr>
<td>B1.8</td>
<td>3.2</td>
<td>1.8</td>
<td>200 x 300</td>
<td>140</td>
<td>160</td>
<td>150</td>
<td>195</td>
</tr>
<tr>
<td>B1.8(T)</td>
<td>1.8</td>
<td></td>
<td>200 x 300</td>
<td>140</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>2</td>
<td></td>
<td>200 x 400</td>
<td>115</td>
<td>175</td>
<td>160</td>
<td>200</td>
</tr>
<tr>
<td>C1.5</td>
<td>3.9</td>
<td>1.5</td>
<td>200 x 400</td>
<td>215</td>
<td>150</td>
<td>270</td>
<td>270</td>
</tr>
<tr>
<td>C1.8</td>
<td>4.0</td>
<td>1.8</td>
<td>200 x 300</td>
<td>195</td>
<td>120</td>
<td>195</td>
<td>195</td>
</tr>
<tr>
<td>C1.8(T)</td>
<td>4.0</td>
<td>1.8</td>
<td>200 x 300</td>
<td>195</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C2</td>
<td>3.9</td>
<td>2</td>
<td>200 x 400</td>
<td>155</td>
<td>125</td>
<td>270</td>
<td>270</td>
</tr>
</tbody>
</table>

† Stirrup diameter was 10 mm for beam types A1.5 and A2, and 8 mm for the remaining beams.

‡ Required spacing near the horizontal leg.

* Required spacing near the supports.

Table 4.5: Predicted Results for the Prevention of Diagonal Failure in the Beams in Test Series ‘C’.
Table 4.6: Results from Test Series ‘C’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Beam size (mm×mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>$f'_{c}$ (MPa)</th>
<th>Correction factor (F)</th>
<th>Measured ultimate load (kN)</th>
<th>Corrected load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.5</td>
<td>1.75</td>
<td>204×412</td>
<td>34.6</td>
<td>24.7</td>
<td>1.0235</td>
<td>591</td>
<td>604.9</td>
</tr>
<tr>
<td>A2</td>
<td></td>
<td>203×414</td>
<td>32.3</td>
<td>23.0</td>
<td>1.0579</td>
<td>762</td>
<td>806.1</td>
</tr>
<tr>
<td>B1.5</td>
<td>3.2</td>
<td>209×412</td>
<td>31.1</td>
<td>25.4</td>
<td>1.0414</td>
<td>306</td>
<td>318.7</td>
</tr>
<tr>
<td>B1.8</td>
<td></td>
<td>208×310</td>
<td>39.2</td>
<td>28.8</td>
<td>0.9962</td>
<td>291</td>
<td>289.9</td>
</tr>
<tr>
<td>B1.8(T)</td>
<td></td>
<td>215×312</td>
<td>34.2</td>
<td>31.7</td>
<td>1.0178</td>
<td>275</td>
<td>279.9</td>
</tr>
<tr>
<td>B2</td>
<td></td>
<td>205×415</td>
<td>35.5</td>
<td>31.5</td>
<td>1.0253</td>
<td>412</td>
<td>422.4</td>
</tr>
<tr>
<td>C1.5</td>
<td>3.9</td>
<td>208×416</td>
<td>31.3</td>
<td>25.4</td>
<td>1.0410</td>
<td>281</td>
<td>292.5</td>
</tr>
<tr>
<td>C1.8</td>
<td>4.0</td>
<td>204×312</td>
<td>33.5</td>
<td>23.9</td>
<td>1.0351</td>
<td>195</td>
<td>201.8</td>
</tr>
<tr>
<td>C1.8(T)</td>
<td>4.0</td>
<td>210×311</td>
<td>33.3</td>
<td>23.2</td>
<td>1.0296</td>
<td>204</td>
<td>210.0</td>
</tr>
<tr>
<td>C2</td>
<td>3.9</td>
<td>205×416</td>
<td>36.1</td>
<td>28.1</td>
<td>1.0206</td>
<td>317</td>
<td>323.5</td>
</tr>
</tbody>
</table>

Table 4.7: Flexural Capacities of the Beams in Test Series ‘C’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>$\alpha$</th>
<th>Corrected load (kN)</th>
<th>$M_f$ (kN.m)</th>
<th>$M_f$ (kN.m)</th>
<th>$M_f$ (kN.m)</th>
<th>$M_f$ (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.5</td>
<td>1.75</td>
<td>604.9</td>
<td>292.5</td>
<td>205.3</td>
<td>173.6</td>
<td>173.6</td>
</tr>
<tr>
<td>A2</td>
<td></td>
<td>806.1</td>
<td>121.5</td>
<td>105.7</td>
<td>101.8</td>
<td>101.8</td>
</tr>
<tr>
<td>B1.5</td>
<td>3.2</td>
<td>318.7</td>
<td>117.3</td>
<td>117.3</td>
<td>101.8</td>
<td>101.8</td>
</tr>
<tr>
<td>B1.8</td>
<td></td>
<td>289.9</td>
<td>243.3</td>
<td>220.5</td>
<td>220.5</td>
<td>220.5</td>
</tr>
<tr>
<td>B1.8(T)</td>
<td></td>
<td>279.9</td>
<td>243.3</td>
<td>220.5</td>
<td>220.5</td>
<td>220.5</td>
</tr>
<tr>
<td>B2</td>
<td></td>
<td>422.4</td>
<td>243.3</td>
<td>220.5</td>
<td>220.5</td>
<td>220.5</td>
</tr>
<tr>
<td>C1.5</td>
<td>3.9</td>
<td>292.5</td>
<td>292.5</td>
<td>205.3</td>
<td>205.3</td>
<td>205.3</td>
</tr>
<tr>
<td>C1.8</td>
<td>4.0</td>
<td>210.8</td>
<td>105.7</td>
<td>101.8</td>
<td>101.8</td>
<td>101.8</td>
</tr>
<tr>
<td>C1.8(T)</td>
<td>4.0</td>
<td>210.0</td>
<td>110.0</td>
<td>101.8</td>
<td>101.8</td>
<td>101.8</td>
</tr>
<tr>
<td>C2</td>
<td>3.9</td>
<td>323.5</td>
<td>227.1</td>
<td>220.5</td>
<td>220.5</td>
<td>220.5</td>
</tr>
</tbody>
</table>

201
<table>
<thead>
<tr>
<th>Beam type</th>
<th>At working load level</th>
<th>At ultimate load</th>
<th>Before unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>load (kN)</td>
<td>Defl.</td>
<td>crack width (mm)</td>
</tr>
<tr>
<td>A1.5</td>
<td>383</td>
<td>11</td>
<td>0.4</td>
</tr>
<tr>
<td>A2</td>
<td>486</td>
<td>17</td>
<td>0.35</td>
</tr>
<tr>
<td>B1.5</td>
<td>191</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td>B1.8</td>
<td>176</td>
<td>12</td>
<td>0.3</td>
</tr>
<tr>
<td>B1.8(T)</td>
<td>1166</td>
<td>12</td>
<td>0.3</td>
</tr>
<tr>
<td>B2</td>
<td>733</td>
<td>10</td>
<td>0.5</td>
</tr>
<tr>
<td>C1.5</td>
<td>176</td>
<td>10</td>
<td>0.2</td>
</tr>
<tr>
<td>C1.8</td>
<td>121</td>
<td>12</td>
<td>0.3</td>
</tr>
<tr>
<td>C1.8(T)</td>
<td>126</td>
<td>11</td>
<td>0.3</td>
</tr>
<tr>
<td>C2</td>
<td>194</td>
<td>12</td>
<td>0.3</td>
</tr>
</tbody>
</table>

1 the working load level is assumed to be equal to 0.6 times the ultimate measured load.

Table 4.8: Crack Width and Deflection Measurements for the Beams in Test Series ‘C’.

202
<table>
<thead>
<tr>
<th>Load level (kN)</th>
<th>Adjacent to loading point</th>
<th>Inside shear span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical chord</td>
<td>Horizontal chord</td>
</tr>
<tr>
<td></td>
<td>Upper</td>
<td>Lower</td>
</tr>
<tr>
<td>291 (Max. load)</td>
<td>470.5</td>
<td>998.6</td>
</tr>
<tr>
<td>92 (Failure)</td>
<td>350.9</td>
<td>895.3</td>
</tr>
<tr>
<td>275 (Max. Load)</td>
<td>1067.4</td>
<td>1453.5</td>
</tr>
<tr>
<td>91 (Failure)</td>
<td>556.3</td>
<td>788.0</td>
</tr>
</tbody>
</table>

† For the location of stirrups refer to Figure 4.11.

Table 4.9: Strains in the Stirrups in Beam Types B1.8 and B1.8(T).
Figure 4.1: Details of the Beams in Test Series ‘B’.
Figure 4.2: Load-Deflection Curves for the Beams in Test Series 'B'.
Figure 4.3: Relative Flexural Capacity Curve (Valley) for the Inclined Leg.

Figure 4.4: Relative Flexural Capacity Curve (Valley) for the Horizontal Leg.
Figure 4.5: Mode of Failure of Beam Type D4 in Test Series ‘B’.
Determining effectively confined concrete area

Stress-strain curves for confined concrete

Figure 4.6: Confinement Model.

(after Sheikh and Yeh[175, 178])

Figure 4.7: Concrete Confined Area for the Prevention of Diagonal Failure. (a) Horizontal leg and Inclined Leg Regions Away from the Supports. (b) Inclined Leg Near to the Supports.

208
Figure 4.8: Details of Type A Beams in Test Series ‘C’.
Figure 4.9: Details of Type B Beams in Test Series 'C'.

Beam Type B1.5

Beam Type B1.8(T)

Beam Type B1.8

Beam Type B2
Figure 4.10: Details of Type C Beams in Test Series ‘C’.

Beam Type C1.5

Beam Type C1.8(T)

Beam Type C1.8

Beam Type C2
Figure 4.11: Location of the Strain Gauges used in Beam Types B1.8 and B1.8(T).
Figure 4.12: Load-Deflection Curves for Type A Beams in Test Series ‘C’.
Figure 4.13: Load-Deflection Curves for Type B Beams in Test Series 'C'.
Figure 4.14: Load-Deflection Curves for Type C Beams in Test Series 'C'.
Figure 4.15: Crack Patterns after Failure for all of the Beams in Test Series ‘C’.
Figure 4.16: Beam Types A2, B1.8, and C1.5 in Test Series ‘C’ after Failure.
Chapter 5

FACTORS AFFECTING DIAGONAL FAILURES IN BEAMS

5.1 INTRODUCTION

Statistical and laboratory based studies have highlighted several factors which are believed to influence the load carrying capacity of structural concrete beams which experience diagonal failures. In this Chapter the influence of these factors on diagonal failures is briefly discussed. The discussion is based on the understanding of the behaviour of structural concrete beams which was developed in Chapter 3. Special emphasis has been placed on examining the influence of beam size (size effect) and high-strength concrete.

5.2 FACTORS AFFECTING THE LOAD CARRYING CAPACITY OF BEAMS AT DIAGONAL FAILURE

5.2.1 Shear Span to Depth Ratio ($\frac{a}{d}$)

It is generally recognised that the a/d ratio has a significant influence on the load carrying capacity and the mode of failure of beams[17, 23, 184, 185, 186, 187, 188]. The influence of the a/d ratio on the relative ultimate flexural capacity ($\frac{M_u}{M_f}$) of beams was illustrated with respect to Kani's Valley as shown in Figure 2.1-Chapter 2. The a/d ratio influences the critical states of stress which exist in the horizontal and
diagonal leg regions of the compressive load path. It was concluded that, for type II beams, the relative ultimate flexural capacity of beams ($\frac{M_u}{M_{lf}}$) increased as the a/d ratio increased. On the other hand, for type III beams, $\frac{M_u}{M_{lf}}$ increased as the a/d ratio decreased.

5.2.2 Longitudinal Reinforcement Ratio ($\rho$)

The results from laboratory based investigations[18, 61, 81, 189, 190], as well as those from statistically and theoretically based studies[63, 64, 91, 93, 191] have shown the effect that the value of $\rho$ has on the nominal shear strength ($\nu_c = \frac{V_c}{V_{ld}}$). It was concluded from these studies that the shear strength increases as the value of $\rho$ increases. Codes of Practice[8, 9, 10] have recognised this and have included $\rho$ in the shear equations. Test results[18, 61, 81, 189] have, nevertheless, shown that the ACI[8] Code provisions for shear are unsafe in the case of low longitudinal reinforcement ratios ($\rho$).

It is believed that the nominal shear strength should not be considered to be a realistic indicator of the load carrying capacity of beams at diagonal failure. The relative flexural capacity of beams ($\frac{M_u}{M_{lf}}$), which has been adopted in the proposed flexure-shear interaction model, offers a more realistic representation of the capacity of beams at diagonal failure. Test results[17, 18] have shown that the ($\frac{M_u}{M_{lf}}$) ratio, unlike the nominal shear strength, decreases as the value of $\rho$ increases.
5.2.3 Yield Strength of Longitudinal Reinforcement

Test results[189, 192] have shown the dependency of shear strength on the yield strength of the longitudinal steel bars ($f_y$). The shear strength was found to decrease when the value of $\rho$ was reduced to compensate for the increase in the yield strength in order to maintain the same flexural capacity. This was believed[13] to be due to the decrease in dowel action resulting from the decrease in the longitudinal reinforcement content. This effect may also be attributed to the increase in the width and length of the cracks in beams in which the longitudinal reinforcement content was low. The propagation of these cracks deep into the compression zone will also adversely affect the critical state of stress in the compressive force path region and will result in a relatively low ultimate flexural capacity at diagonal failure.

5.2.4 Bond Characteristics

Tests by Leonhardt and Walther[143] have shown that shear strength decreases when the bond between the longitudinal bars and the concrete was enhanced. This conclusion was corroborated by Kani[17] who argued that the capacity of the concrete teeth decreases as the bond between the steel and the concrete increases (good bond results in smaller concrete teeth forming between cracks). Cairns[193] found that the load carrying capacity of a beam with longitudinal reinforcement exposed over 63% of the length of the span was 6% higher than that obtained from a similar beam with bonded reinforcement. He also noted that cracks did not develop in the shear spans in the beams in which the bars were exposed. The increase in the load carrying capacity of beams in which the bond was either poor or where the bars
were exposed can be attributed to the insignificance or even the absence of cracking in the shear spans which would normally adversely affect the controlling state of stress in the load path regions.

### 5.2.5 Shape of the Cross Section

(a) **Width to Depth Ratio of Rectangular Cross Sections**

The results from beam tests[139, 187] with different width to depth ratios have shown that the beam width had an insignificant influence on the strengths of the beams. Leonhardt and Walter[143] tested slab strips and found slightly higher nominal ultimate shear strengths compared with that for beams. It was also reported by de Cossio[194] that shear strength increases with increasing values of the b/d ratio. The reported slight increase in beam strength with increasing b/d ratio may be attributed to the more effective confinement of the core of the concrete compression zone when the b/d ratio was increased.

In conclusion, although there are doubts regarding the adoption of the nominal shear strength as an indicator of the load carrying capacity of beams at diagonal failure[16], the omission of the width parameter (b) from any of the relationships for shear strength of rectangular beams is justified.

(b) **I & T Cross Sections**

The results of tests from beams with I and T cross sections[13, 195] have shown that these types of beams possesses an increased shear strength which is usually of
the order of 13% to 60% compared with that for rectangular beams with the same values of $b_w$, $d$, $\rho$, and $f'_c$. It was found, based on available test results, that Codes of Practice in general underestimate the load carrying capacity of I beams by as much as 80%[196]. This is because Codes of Practice while overestimating the contribution of web shear resistance, tend to underestimate the contribution of the flange. The increase in the load carrying capacity can be attributed to the contribution of the uncracked compression concrete (flanges of I or T sections) which is believed to be the main element in resisting shear. Also, in this case the flanges provide the concrete compression cores in the beams with a considerable amount of lateral confinement. It was suggested[197] that recommendations which require closed hoops as shear reinforcement may be somewhat conservative for flanged sections or beams cast monolithically with slabs because of the confinement available.

(c) Circular Section

Only a few researchers have attempted to investigate the shear behaviour of beams with circular cross sections. The circular section has been treated separately in Chapter 7.

5.2.6 Axial and Prestressing Loads

For beams subjected to axial tensile forces, flexural cracks occur early and extend almost vertically thus reducing the shear strength[13]. In the ACI Code of Practice[8], it is suggested that the concrete shear strength ($\nu_c$) is neglected when beams are subjected to significant tensile forces. On the other hand, shear strength is increased
when beams are subjected to either axial compression or prestressing forces[13]. The effect of these actions has been incorporated into different Codes of Practice[8, 9, 10]. The effects of axial forces on shear strength can be related to the effect these have on the controlling state of stress which exists in the compressive force path region. While applied axial compression forces enhance the state of stress, tensile stresses on the other hand adversely affect the critical state of stress.

5.2.7 Web Reinforcement

The lateral load capacity of beams can be enhanced by the use of stirrups which also change a brittle diagonal failure mode into a ductile one. The following factors are related to the influence of web reinforcement on shear strength[198, 199].

1. Spacing of stirrups: The same stirrup content but using stirrups spaced more closely together will increase shear capacity. This is explained in terms of the possibility of the diagonal cracks penetrating more deeply into the beam in the case of widely spaced stirrups. This will adversely affect the critical state of stress in the region of the compressive force path. Also, the effectiveness of the passive confinement of stirrups is a function of stirrup spacing[163].

2. Anchorage of web reinforcement: Stirrups should be adequately anchored by means of standard hook ends. This is to support the diagonal compression of the web and the bond forces in the longitudinal bars. This can also be explained in terms of the stirrup anchorage which is essential to enable stirrups to restrain the development of the secondary tensile stresses which result in failure.
3. Distribution of stirrup legs: Multiple stirrup legs distributed transversely across the cross section of a beam are more effective than single rectangular shaped stirrups. This can be explained by recognising that the effectively confined area is larger for a more closely knit cage\[163\]. This results in an increase in the load carrying capacity of beams.

In studying the influence of web reinforcement, researchers\[13, 200\] found that the existence of small amounts of stirrups had a greater effect on shear strength than that predicted by the truss analogy\[8\]. This can be explained by considering the enhancing confinement effect of stirrups upon the compression concrete, the main element in resisting shear.

5.2.8 Implications on the Proposed Flexure-Shear Interaction Design Model

The proposed flexure-shear interaction design model directly incorporates the influence of basic factors which affect diagonal failures in beams such as the a/d ratio, the value of $\rho$, and the characteristics of the concrete and the longitudinal reinforcement. These factors influence the relative ultimate flexural capacity ($M_u/M_f$) of a beam which determines the amount of confinement required for the prevention of diagonal failures.

The main aim of this programme of research was to investigate the basic behaviour of beams under lateral loading. It is believed, however, that the theoretical basis of the proposed model is applicable to all types of beams regardless of the shape of their cross-section and static loading systems. Possible modifications to
the model and subsequent validation by experiment for such cases is still required.

In the present investigation emphasis has been placed on studying the influence of beam size and concrete strength on the diagonal failure in beams. The implications of these two factors on the proposed flexure-shear interaction design model are investigated first of all theoretically and then experimentally in the following sections.

5.3 SIZE EFFECTS

5.3.1 Introduction

The results from studies into the influence of beam size (size effects) on the load carrying capacity of structural concrete beams with and without web reinforcement are contradictory. The proposed flexure-shear interaction design model is concerned with the determination of the amount and form of lateral reinforcement. However, the model required the determination of the reduction in the ultimate flexural capacity of beams at diagonal failure ($M_u$ and $M_f$). Therefore, to reach a definitive conclusion with respect to the implications of size effects on the proposed analytical model, its effect on beams with and without web reinforcement has been considered in the following section. A detailed study of size effects has been reported elsewhere[201].
5.3.2 Beams Without Web Reinforcement

Laboratory based studies on beams subjected to four-point loading systems have indicated that the nominal shear strength \( v_n = \frac{V}{bd} \) decreases with increasing beam size \( (d \text{ or } t) \). The magnitude of the decrease reported by several researchers has varied considerably\[18, 139, 143, 202, 203, 204, 205, 206, 207, 208]\.

The conflicting experimental evidence regarding size effects was thought to have been related to several factors; the geometrical differences in the test beams, the method used to interpret the test results, and the influence of other interrelated factors which might affect the shear strength e.g. percentage and diameter of longitudinal reinforcement, aggregate type and size, water/cement ratio, brittleness of concrete, etc.

A new approach to size effects based on fracture mechanics\[15, 60, 75, 78, 79, 209, 210, 211, 212, 213\] has been developed in an attempt to reduce the extent of the scatter in the test results. The new approach did not treat fracture as a point phenomenon, but recognised that in brittle heterogeneous materials such as concrete, the fracture propagates over a relatively large fracture process zone in which progressive microcracking gradually reduces the tensile stress to zero\[84, 85\]. Size effects based on fracture mechanics approach result from the release of strain energy from the beam into the cracking zone as the cracking zone extends. The larger the structure, the greater is the energy release and the smaller is the nominal shear strength.
In an attempt to solve the shear problem the influence of the size of beams having an $\frac{a}{d}$ ratio greater than 2.5 was explained[68] in terms of frictional resistance (aggregate interlock) which in turn was based on a tooth model developed by Reineck[67]. In the model size effect was attributed to the critical crack width condition which was assumed to be a function of the tooth friction. It is interesting to note that tests have indicated that size effects do exist in lightweight concrete beams although aggregate interlock action in lightweight concrete is considered to be negligible[80].

Another explanation for size effects was given by Swamy[207] who attributed the increase in the nominal shear strength to the increased strength produced by the extreme strain gradient which is characteristic of small beams. This explanation was corroborated by Walraven[206].

It is believed that size effects can be related more realistically to the strain gradient concept. Its effect can be explained in terms of the additional confinement present in the compression concrete due to the existence of extreme strain gradients[154]. The acceptance that the failure of beams is the result of failure of the compression concrete leads to the conclusion that failure would be delayed if confinement of the compression concrete was provided. In the proposed detailing approach for the prevention of diagonal failures in beams confinement is provided by the stirrups. For small beams, without web reinforcement, the confinement is partially provided by the extreme strain gradient. Therefore, the load carrying ca-
pacity at diagonal failure increases with decreasing beam size i.e. by increasing the
confinement of the compression concrete.

5.3.3 Beams With Web Reinforcement

In the design approaches adopted by the majority of Codes of Practice, it is assumed
that the ultimate capacity of a beam is the sum of the beam capacity without shear
reinforcement \( V_c \) plus the additional capacity due to shear reinforcement \( V_s \). This
means that the size effect should also influence web reinforced beams, but, it seems
that this is not the case. The presence of shear reinforcement mitigates the influence
of size effects. Theoretically, the reduction in the safety margin with increasing size
is less significant when shear reinforcement is present[75, 82].

Bázant and Sun[15] claimed, that based on a comparison of available test results,
size-effects do exist when shear reinforcement is present. However, it is much less
significant than without stirrups but the scatter in the test results is extremely large.
There are obvious doubts on how they measured the concrete shearing capacity \( v_c \)
for web reinforced beams. No meaningful experimental results appear to be available
to permit the measurements of the shear capacity of such beams.

5.3.4 Discussion and Conclusions

(a) Beams Without Web Reinforcement

(i) At Diagonal Failure (Ultimate limit state)

1. Test results have shown a declining trend in the nominal ultimate shear strength
with increasing beam size.

2. For the same a/d ratio, two geometrically similar beams but with different sizes may achieve different ultimate flexural strength levels. While, the smaller might reach its full flexural capacity, the larger one may not.

3. The proposed design approach is in agreement with the size effect being based on the strain gradient concept.

4. The reported influence of size effects on the shear strength varied significantly. It was found to vary from reductions of 5% (Alami and Ferguson[204]) to 100% for very large beams i.e. beams cannot carry loads in excess of their own weight (Kani[139], Băzant and Kim[75], and Gustafsson and Hillerborg[211]).

5. Several equations, based either on nonlinear fracture mechanics or on the statistical analysis of published test data were proposed in order to include the effect of beam size on shear strength. These equations not only give different predictions, but are also contradictory e.g. the size-effect law put forward by Băzant predicts a complete loss in the shear strength for very large beams, while Kim and Eo[79] predicted that size effects tend to become insignificant for such beams.

6. Several equations have been put forward to account for size effects in the prediction of the shear strength of beams. However, empirical equations e.g. the Zsutty formula[63] which did not even consider size effects gave shear strength predictions almost as good as the most sophisticated equations e.g. Băzant
size effect law[75], containing six empirical parameters which were obtained
from extensive data bases using statistical and optimisation approaches.

7. Size effect has been incorporated in some of the Codes of Practice (the Swedish
and the CEB-FIP Model Codes)[211]. However, the scatter in the shear pre-
dictions based on these Codes of Practice when compared to test results is
enormous.

(ii) At Diagonal Cracking (Serviceability Limit State)

1. Experimentally, it is very difficult to measure the load carrying capacity at
the onset of diagonal cracking.

2. It can be argued based on the above explanations, that size effects should not
exist before the onset of cracking. However, test results which indicate the
possible presence of size effects have been related to the difficulties in defining
the cracking load.

(b) Beams with Web Reinforcement

1. There is no significant experimental data which shows the influence of size
effects on the nominal concrete shear strength.

2. The presence of web reinforcement (confinement) mitigates size effects.

3. For all practical purposes, it can be concluded that size effects do not exist in
beams with web reinforcement.
5.3.5 Proposed Design Method and Size Effects

The flexural-shear interaction design model is concerned only with detailing and with the determination of the amount of confinement stirrups required for the prevention of diagonal failures in beams. However, it is necessary for the implementation of the proposed design approach to determine the relative ultimate flexural capacity of beams at diagonal failure \( \frac{M_u}{M_{lf}} \) in order to calculate the relative ultimate flexural capacity \( \frac{M_u}{M_{lf}} \) of the legs required for design. In this context the equations developed by Russo et. al.[44] which accounted for the size of the beams and agreed closely with test data was adopted for the determination of \( \frac{M_u}{M_{lf}} \) for large beams.

An experimental verification of the proposed flexure-shear interaction design model for large beams is detailed in the following section.

5.4 TEST SERIES ‘D’: Large beam

5.4.1 Test Beam

One large beam was designed and detailed using the proposed flexure-shear interaction design model, Figure 5.1. The beam was designed to have a cross section of 230mm×700mm (the actual cross section was 236mm×715mm). The overall length of the beam was 6300mm and the effective span was 5500mm. The a/d ratio was 3.2. Three 32mm nominal diameter high-strength deformed steel bars were used for the longitudinal reinforcement \( (A_s = 751.4mm^2 \) and \( f_y = 527\ MPa) \). The stirrups were fabricated from 10mm nominal diameter plain round mild steel bars \( (A_s = 78mm^2 \)
and $f_u = 356 \text{ MPa}$. The design concrete compressive strength was 40 MPa, Table 5.1.

The required stirrup spacings based on the proposed model were 205mm, 180mm, and 210mm for the horizontal leg, for the inclined leg near the point at which the load path changes direction, and near to the supports respectively. The stirrup spacing which was based on the approach in BS 8110 was found to be 175mm for the whole shear span. It should be emphasised that no partial safety factors were included in the design calculations. The short stirrups in the test beam were placed inside the mid-span region near to the loading points in order to prevent failure of the compression concrete in that location. Two 20mm diameter high-strength steel bars were used for the compression reinforcement in order to prevent accidental failure of the beam during the handling operations. In addition, two lifting points were placed in the beam to produce equal positive and negative bending moments resulting from the self weight of the beam. Two electrically operated overhead cranes were used to move the beam in the laboratory. The beam was free from all types of cracks when it was placed in the test machine prior to testing. The end plate arrangements shown in Figure 5.1 were used to prevent anchorage failure in the main longitudinal reinforcement bars. All of the steel bars were welded to end plates.

Four batches of concrete were used for the casting of the beam. An effective quality control procedure was used in the casting: the same mix proportions were used in all of the mixes with particular care being taken during the batching operation,
and the workability of the fresh concrete was monitored. This procedure resulted in very little variation in the concrete compressive strengths ($f_{cu}$) for all four batches of concrete, Table 5.2. The beam was cast in four layers with the top cast surface being the compression face of the beam. Two standard control cubes and two 150mm $\times$ 300mm cylinders were also cast from each of the batches of concrete. The beam and the control specimens were kept in the moulds inside the laboratory until the time of testing. The loading arrangement, instrumentation and test procedures used in this test were similar to those used in Test Series ‘C’.

5.4.2 Discussion of Test Results

The beam failed in a typical ductile flexure mode after reaching its full flexural capacity. The failure occurred as a result of spalling of the compression concrete in the mid-span region where no short stirrups had been provided. The compression longitudinal reinforcement bars buckled at failure. The diagonal cracks developed under increasing loads as an extension of the existing flexural cracks. The stirrup detailing which was used prevented the widening and propagation of the diagonal cracks into the compression zone. While the maximum flexural crack width was 4mm at failure, the diagonal crack width was only 0.6mm, Table 5.3. The presence of the short stirrups inside the mid-span region near the loading points succeeded in preventing failure of the compression concrete in that region. As a result, the diagonal cracks were not able to propagate into the failed compression concrete in the mid-span region as was experienced in some of the beams in Test Series ‘C’.
The width of the flexural and diagonal cracks (0.25mm and 0.18mm respectively), and midspan deflection (21mm) obtained from the beam at working load (423 kN) showed that the serviceability limit state requirements given by Codes of Practice[8, 9] were satisfied, Table 5.3. Table 5.3 and Figure 5.2 show that the ductility was acceptable. The mid-span deflection exceeded 87mm at failure. The shape of the beam after failure is shown in Figure 5.3.

The beam was able to reach its full flexural capacity. The maximum load and flexural capacities obtained from the beam were 705 kN and 723 kN.m respectively. The theoretical flexural capacity of the beam using the actual material and geometric properties was 690 kN.m. The beam thus achieved 105% of its full theoretical flexural capacity.

It is concluded that the design approach which has been developed enabled the large beam to not only reach its full ductile flexural behaviour but also enabled it to satisfy the serviceability limit state requirements.

5.5 HIGH STRENGTH CONCRETE (HSC)

5.5.1 Introduction

The majority of Codes of Practice divide the shear resistance of a beam into two parts: \( V_c \) provided by concrete and \( V_s \) provided by shear reinforcement. The values of \( V_c \) given in Codes of Practice were based on experimental results obtained from
large numbers of beams made from relatively low strength concrete ($f_{cu}$ less than 42 MPa[214]). Currently, HSC with values of $f'_{c}$ approaching 140 MPa is used in structural concrete construction[141, 215, 216].

The increased use of HSC has focused on the evaluation of the shear strength ($V_c$) of HSC beams. Several investigators have studied the corresponding shear-flexure interaction behaviour of HSC Beams[148]. The shear strength of beams with web reinforcement has also been investigated. The results from the investigations have shown that the concrete strength may have an important influence on the load carrying capacity of beams at diagonal failure. The research also indicated that Code provisions for shear for HSC beams may be conservative in some cases, but may equally be unsafe in others.

Results from previous work into the behaviour of HSC beams are briefly discussed below in order to assess their implications on the proposed design model for the prevention of diagonal failures in HSC beams. The implementation of the model for beams made from HSC has been investigated experimentally and is described in the following section.

5.5.2 Behaviour of HSC Beams

Moody[217], and Van der Berg[218] have concluded that the shear strength ($V_c$) increased significantly as the concrete strength ($f'_{c}$) was increased from 1,000 to 4,500 psi (7 to 31 MPa). They found no significant increase in the shear capacity of
the beams in which $f'_c$ was greater than 4,500 psi (31 MPa).

Taub and Neville[198] deduced from test results reported by Morrow and Viest[188] that the influence of the a/d ratio and the concrete strength are interdependent. The influence of the concrete strength is greater on the ultimate shear strength of type III beams (a/d < 2.5) compared with that for type II beams (a/d > 2.5). In order to offer an explanation of this behaviour, they assumed that as the a/d ratio is decreased the diagonal failure mechanism changes from a beam type to a tied arch type, in which the strength of the compression struts is more dependent on the concrete compressive strength. The design approach developed in this investigation may be used to explain this type of behaviour.

For type III beams, a biaxial state of stress exists in the inclined leg region of the compressive force path as shown in Figure 3.1. A concrete element in this region is subjected to a combination of shear stress ($v_c$) and normal stress ($f_c$). The ratio $\frac{v_c}{f_c}$ which may be considered to be an indication of the critical state of stress which causes a diagonal failure[201] would be less critical for high strength concrete, since the concrete compressive strength ($f_c$) is relatively high. On the other hand, for type II beams, the critical state of stress exists in the horizontal leg region of the compressive force path. In this case, the possible causes of failure e.g. bond failure and load path changing direction, are influenced by other factors in addition to concrete strength. Therefore, the concrete strength would have less of an effect on the shear strength of type II beams.
The findings of Taub and Neville were corroborated by Mphonde and Frants\cite{219} who investigated the effect of low and high strength concrete on the cracking and the ultimate shear capacity of beams without shear reinforcement (the value of $\rho$ was equal to 3.36%). They concluded that the effect of concrete strength on shear strength is more pronounced for small a/d ratios. For an a/d ratio equal to 3.6, they found that the shear equations given in the ACI 318-77 Code of Practice were conservative. However, the factor of safety decreased from 1.64 to 1.2 as $f_c'$ increased from 20 MPa to 103 MPa. For an a/d ratio equal to 2.5, they stated that the ACI Code of Practice gives a reasonable lower bound estimate of the measured shear capacity. For an a/d ratio equal to 1.5, they found that both Zsutty\cite{64} and the ACI relationships underestimated even the lowest measured strength by 40% and 70% respectively. They proposed the following equation, based on their test results, for the prediction of the ultimate shear strength of beams.

\[ v_u = 10.16 \sqrt{f_c'} + 71 \text{ psi} \quad (5.1) \]

Elzanati et. al.\cite{182} tested eighteen beams made from concrete with compressive strengths ($f_{cu}$) ranging from 21 MPa to 83 MPa. Three beams were provided with stirrups. The variables included the value of $\rho$ (0.6\% to 3.3\%) and the a/d ratio (2.0, 4.0, and 6.0). They concluded from their results that the ACI 318-83 Code provisions for shear may overestimate the shear strength by 10 to 30\% for high strength concrete, combined with a low value of $\rho$ and normal to high a/d ratios. A similar conclusion was reported by Roller and Russell\cite{215}. The results of tests carried out by Mphonde and Frants\cite{48, 65} have indicated that as the concrete strength
increases, the shear strength increases but the ratio of measured to predicted shear strengths decreases.

Elzanati et. al.[182] reported that the ACI Code provisions for shear are conservative for all beams with stirrups regardless of concrete strength. A similar conclusion was reported by Kaufman and Ramirez[183]. The type of behaviour obtained can be explained in terms of the confinement provided by stirrups to the concrete compression regions. The truss analogy adopted by Codes of Practice does not take into account the enhancement effect of stirrups with respect to the resistance of the compression zone. For this reason, predictions based on the truss analogy underestimate the capacity of beams.

The results reported by Johnson and Ramirez[220] and Roller and Russell[215] from tests on beams provided with minimum amounts of shear reinforcement in accordance with the ACI 318-83 Code of Practice, showed that the overall reserve shear strength after diagonal cracking may be less for HSC beams. In such cases, where a minimum amount of stirrups is used, confinement would be less significant due to the large spacings which could be used ($s_o$ was 270mm[220]). Also, the efficiency of the passive confinement decreases with increasing concrete strength[170, 221].

Nielsen and Braestrup[74] recommended a significant reduction in the effective concrete strength factor ($\nu$), which was introduced to limit the usable concrete compressive strength, with increasing concrete strength. This was attributed to the
brittle nature of high strength concrete, since $\nu$ is considered to be a measure of the ductility of concrete.

Ahmad et. al. [214] tested thirty six beams with the value of $f'_c$ ranging from 63 MPa to 70 MPa. The primary variables were the value of $\rho$ (1.77% to 6.64%) and the a/d ratio (1 to 4). They found that the relationships given in the ACI 318-83 Code of Practice are conservative for short beams. However, the predicted strength was equal to the measured value in the case of slender beams made from high compressive strength concrete and low values of $\rho$. They concluded that the Code equations underestimate the effect of the value of $\rho$ and the a/d ratio. The authors criticised equation (5.1) for not including both parameters ($\rho$ and a/d).

An analytical flexural-shear interaction model was proposed by Ahmad and Lue [148, 152] based on the results obtained from tests on fifty four high strength concrete beams ($f'_c$ ranged from 66 MPa to 70 MPa). This model was then compared with the experimental results produced by Kani [18], Leonhardt and Walther [143] as well as their own [148]. The comparison showed a reasonably fair correlation, however, errors in the derivation of the equations were subsequently found in the model [44, 152]. There was a significant lack of correlation between the experimental data and the prediction based on the corrected analytical model.
5.5.3 Implication of the Concrete Strength on the Proposed Design Model

(a) Evaluation of the Leg Capacities ($M_t$)

It can be concluded from the previous discussion on the effect of concrete strength, that the load carrying capacity of beams made with HSC is higher at diagonal failure compared with that for normal strength concrete NSC beams. In the proposed design model, the load carrying capacity of the leg ($M_t$) shown in Figures 4.3 and 4.4 was based on test results obtained from NSC beams ($f_{cu} \approx 40$ MPa). In order to make the proposed design model applicable to HSC beams, the value of $M_t$ can be determined using one of the following two approaches.

Approach I

To undertake additional beam tests similar to those already completed in Test Series 'B' using a range of high strength concretes ($f_{cu} > 40$ MPa). The influence of the concrete strength on $M_t$ can then be determined from the test results. The corresponding leg capacity curves can thus be generated.

Approach II

To use the general method for the evaluation of the leg capacity described in Chapter 4.

Obviously, the first approach (Approach I) requires a large number of beam tests to be conducted in order to investigate a wide range of a/d ratios and concrete...
strengths. On the other hand, the second approach (Approach II) is more general and does not require additional testing.

Conclusion

Approach II was adopted in this programme of research. The same flexure-shear model developed by Russo et. al.[44] was used to determine the relative flexural capacity of the beams \( \frac{M_u}{M_f} \). The model accounted directly for the influence of concrete strength on the \( \frac{M_u}{M_f} \) ratio. The correlation between the model and the test results obtained from beams made with a wide range of concrete strengths[44] was good.

(b) Evaluation of the Required Confinement \( (K_s) \)

It can be concluded, from the previous discussion on the effect of HSC on the relative flexural capacity \( \frac{M_u}{M_f} \) of beams with web reinforcement, that the ratio \( \frac{M_u}{M_f} \) increases with increasing concrete strength. However, the efficiency of web reinforcement decreases with increasing concrete strength. This is attributed to the brittleness of HSC and, therefore, the confinement provided by stirrups becomes less effective. This is believed to be the main reason for the decrease in the efficiency of stirrups with increasing concrete strength which has been reported by several researchers[170, 221, 222].

The influence of concrete strength has been included in the confinement model which was adopted and described in Chapter 4. There was good correlation between
the results from the model and the results from tests conducted on HSC specimens ($f_{cu}$ up to 80 MPa[178]).

Conclusion

The implementation of the proposed flexure-shear interaction design model for beams made with HSC, in which values of $f_{cu}$ up to 80 MPa, has been included in this research programme. The confinement model which has been adopted has been validated for concrete strengths up to this value. No further modifications to the confinement model are believed to be required. However, for concrete strengths outside this range, validation of both models (the confinement and the proposed flexure-shear interaction design models) needs further investigation.

5.6 TEST Series ‘E’: Validation of the flexure-shear interaction design model for high-strength concrete beams

5.6.1 Description of Test Beams

This test series included the seven HSC beams shown in Figure 5.4. All of the beams had an overall length of 3500mm and width to depth dimensions of 200mm x 300mm. The effective spans were either 2600mm (beam types A1.86, B2.78(T) and C2.78(T)) or 2800mm (beam types B1.86, B2.78, C1.86 and C2.78). For the validation of the proposed model the test beams were selected to cover most of Kani's Valley. The variations included the a/d ratio (1.75, 3.2, and 4.0), the value of $\rho$ (1.86% and 2.78%), and the detailing arrangement of the lateral reinforcement (proposed and traditional detailing approaches). The traditionally designed and detailed beam
types B2.78(T) and C2.78(T) were included for comparison purposes. The design cube concrete compressive strength was 80 MPa. 25mm nominal diameter high-strength deformed steel bars were used for the longitudinal reinforcement ($A_s = 483.8 \text{mm}^2$ and $f_y = 526 \text{MPa}$). The stirrups were fabricated from 10mm nominal diameter plain round mild steel bars ($A_s = 78.13 \text{mm}^2$ and $f_y = 346 \text{MPa}$).

5.6.2 Design of Test Beams

All of the test beams were designed using either the proposed model or the provisions of BS 8110, Table 5.5. However, only beam types B2.78(T) and C2.78(T) from the traditionally designed beams were included in this test series. The partial safety factors were not used in the design of the beams. The actual mechanical properties of the reinforcement, the assumed concrete strength, and the assumed dimensions of the beams were used in the design. To prevent anchorage failure, the beams were extended beyond the supports for a distance of either 300mm or 400mm, Figure 5.4. Also, the longitudinal bars were bent up a distance of 250mm at their ends. The anchorage lengths provided and the diameters of the bends were found to satisfy Code requirements.

The design procedures used in this test series were similar to those used in the design of normal-strength concrete beams ('C'). However, the dimensions of the confined areas were determined using the provisions from the ACI Code of Practice because it takes a more realistic account of concrete strengths in the determination of the location of the neutral axis. Typical concrete strains were measured.
on the two faces of beam type A1.86, Table 5.10 and Figure 5.6. The depth of the neutral axis was found to be 105mm. The corresponding position of the neutral axis would have been 52.8mm, 56.3mm, and 76.8mm using the simplified and the idealized stress blocks in BS 8110, and the ACI Code of Practice respectively. It is, therefore, concluded that the determination of the neutral axis for the design of the HSC beams based on the simplified concrete compression block given in BS 8110 would be over conservative. This is because of the resulting smaller confined areas compared with those obtained using the ACI Code of Practice.

The strains measured in the stirrups, Figure 5.5 and Table 5.9 indicated that the stirrups did not yield when the maximum capacity of the beams was reached. The implication of this is that there was still a reserve of strength against diagonal failure when the beams reached their full flexural capacity. Thus all the beams failed in a flexural mode.

5.6.3 Experimental Work

The mix proportions of the high strength concrete \( f_{cu} = 80 \ M Pa \) used in this test series are detailed in Table 5.4. The casting, curing, and test procedures, as well as the method used to correct the load measurements obtained in this test series were similar to those used for the normal strength beams included in Test Series ‘C’. Additional information on the experimental work including the design of the high-strength concrete is included in Appendix A.
5.6.4 Test Results

The concrete compressive strengths, actual beam dimensions, correction factors, and the actual and corrected maximum loads for all of the test beams are given in Table 5.6. The corresponding flexural capacities of the beams are given in Table 5.7. The crack width and deflection measurements obtained at several loading levels are given in Table 5.8. These measurements allow the behaviour of the beams at working load levels (serviceability limit state) and up to the failure to be studied. Table 5.9 summarises the actual strains found in the stirrups in beam types B2.78 and B2.78(T). The concrete strains measured in beam type A1.86 are given in Table 5.10. The location of the neutral axis in beam type A1.86 is shown in Figure 5.6. The load-mid span deflection curve obtained from beam type A1.86 is shown in Figure 5.7. The load-mid span deflection curves obtained from the traditionally designed beam types B2.78(T) and C2.78(T) are shown for comparison purposes in Figures 5.8 and 5.9 together with the corresponding curves obtained from beams types B2.78 and C2.78 respectively. The crack patterns after failure for all of the beams are shown in Figure 5.10. Finally, typical failure modes found in beam types B2.78 and C1.86 are shown in Figure 5.11.

5.6.5 Discussion of Test Results

(a) Modes of failure

All of the beams failed in flexure, Figure 5.10. Diagonal cracks developed as an extension of the flexural cracks. The conventional full length and short stirrups succeeded in preventing the increase in widths and lengths of the diagonal cracks.
into the compression zone. The maximum diagonal crack widths were small in all of
the beam types up to the failure, Table 5.8. It was recommended in Test Series ‘C’
that the short stirrups should be placed into the mid-span region near to the loading
points in order to prevent the diagonal cracks extending into the failed compression
concrete. The short stirrups in the beams included in this test series were extended
into the mid-span region as required, Figure 5.4. All of the beams detailed as such,
failed in a typical ductile flexural mode, Figures 5.10 and 5.11.

(b) Load Carrying Capacity and Ductility

For the validation of the flexure-shear interaction design model the ultimate strength
as well as the ductility obtained from the test beams had to be investigated. For
this purpose Table 5.7 includes a comparison between the actual flexural strengths
obtained from the test beams and the theoretical values of \( \frac{M_f}{M_{fi}} \). The ductility of
the beams can be judged from the mid-span deflections given in Table 5.8 as well as
from the load-deflection curves shown in Figures 5.7, 5.8, and 5.9.

Table 5.7 shows that values of \( \frac{M_f}{M_{fi}} \) ranged from 0.93 (beam type B2.78(T)) to
1.00 (beam type B1.86) with an average value of 0.97. It can be concluded that
since all of the test beams failed in flexure their respective full flexural capacities
were achieved in each case. The average value of the ratio \( \frac{M_f}{M_{fi}} \) obtained from the
normal-strength concrete beams included in Test Series ‘C’ was 1.11. It can be con-
cluded from a comparison of the values of \( \frac{M_f}{M_{fi}} \) obtained from this test series (0.97)
that flexural designs are less conservative for HSC than for NSC beams. This con-
clusion has been corroborated by other researchers[152] who stated that the validity of Whitney’s uniform stress distribution for HSC should be re-evaluated.

The values of the $\frac{M_f}{M_i}$ ratios obtained from the beams designed using the proposed model (beam types B2.78 and C2.78) compared to that obtained from the traditionally detailed beams (beam types B2.78(T) and C2.78(T)) were 1.06 and 0.99 respectively. Therefore, it can be concluded that the proposed detailing approach does not have a significant influence on the maximum flexural capacity of the beams.

The load-deflection curves given in Figures 5.7, 5.8, and 5.9 indicate the following:

1. The ductilities obtained (the mid-span deflection exceeded 60mm) were higher than those obtained from the NSC beams included in Test Series ‘C’ (the maximum mid-span deflection was approximately 50mm). The increase in the ductility is due to the increase in the concrete compressive strength. It is recognised that the ductility of beams increases with increasing concrete compressive strength[32, page:207].

2. The ductilities of beam types B2.78 (45mm) and C2.78 (40mm) which were detailed in accordance with the proposed model were larger than those obtained from the traditional beam types B2.78(T) (36mm) and C2.78(T) (33). This is due to the presence of the short stirrups inside the mid-span region close to the loading points. The presence of these stirrups prevented splitting...
of the compression concrete in that region. As a result failure was suppressed which in turn allowed more deflection to take place before failure.

(c) Serviceability

It can be concluded, based on the crack width measurements at working load levels shown in Table 5.8, that the serviceability limit state requirements with respect to crack width (0.3mm and 0.41mm based on the provisions of the BS 8110 and the ACI Codes of Practice respectively) are satisfied for all of the beams (the maximum measured value was 0.36mm). The maximum crack widths at working load levels obtained from the traditional beam types B2.78(T) and C2.78(T) were 0.34mm and 0.26mm respectively. The corresponding widths obtained from beam types B2.78 and C2.78 were 0.36mm and 0.2mm respectively. Therefore, it can be concluded that the proposed detailing approach does not influence the serviceability limit state requirements with respect to crack width. It is interesting to note that diagonal crack widths remained small until failure (maximum crack width was equal to 0.6mm at failure, Table 5.8).

Generally, the deflections at working load levels obtained from all of the beams, including those which were traditionally detailed, were high in comparison with the corresponding values normally obtained from NSC beams. The large deflection obtained can be explained in terms of two factors. Firstly, the use of high strength concrete normally results in an increase in ductility. Secondly, the large load carrying capacity of the beams resulting from the use of high strength concrete combined
with high longitudinal reinforcement ratios (the maximum value of \( \rho \) used was equal to 2.78\%).

The deflection of the beams was believed to be mainly influenced by the depth of the beams. Therefore, it is believed that the proposed approach to the detailing of stirrups had an insignificant influence on the deflections obtained under working load conditions (refer to the corresponding deflections for beam types B2.78, B2.78(T), C2.78 and C2.78(t) in Table 5.8). To satisfy the serviceability limit state requirements on deflection the depth of the beams should be increased. The establishment of new limitations on the depth of high-strength concrete beams in order to control deflection is beyond the scope of this thesis.

5.6.6 Conclusions

The following conclusions can be made from the results obtained from this test series.

1. All of the beams which were designed using the proposed flexure-shear interaction model failed in flexure after reaching their full flexural capacity. The model has therefore been verified for HSC beams with concrete strengths \( (f_{cu}) \) up to 80 MPa. The applicability of the proposed model for higher concrete strengths needs further investigation.

2. The serviceability of beams (crack widths and deflection) was not adversely affected by the proposed detailing approach. It seems that the detailing approach used does not have a significant influence on the performance of beams
at the serviceability limit state. However, the deflections obtained at various load levels including working load for all of the beams were higher than those normally obtained from NSC beams.

3. The ductility of all of the beams were higher than those normally obtained from normal-strength concrete beams. This is due to the influence of concrete strength on ductility.

4. The placement of short stirrups in the mid-span regions near to the loading points enhanced the ductility of the beams. Also, this type of detailing prevented diagonal cracks from extending into the failed compression concrete which was found in some of the beams in Test Series ‘C’. It is therefore recommended that short stirrups should always be provided near to the loading points inside the mid-span region.

5.7 SUMMARY

The influence of factors traditionally believed to affect diagonal failures in beams was investigated in this Chapter. The factors discussed included the shear span to depth ratio, the longitudinal reinforcement ratio, the characteristics of the longitudinal reinforcement, the shape of the beam cross section, the type of loading, the web reinforcement, size effects, and the concrete compressive strength. It has been concluded that the proposed approach to the understanding of the behaviour of beams developed in Chapter 3 offers a better explanation of the various factors which influence the behaviour of beams. Particular emphasis was put on the study
of the behaviour of large beams and beams made from high-strength concrete. The implications of these two factors on the proposed flexure-shear interaction design model were considered both theoretically and experimentally. Two test series: Test Series 'D' and Test Series 'E' were undertaken in order to verify the proposed design model with respect to large and high strength concrete beams respectively. The test results obtained from the beams included in these test series confirmed that the proposed flexure-shear interaction design model was applicable to large and high strength concrete beams.
<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>490</td>
</tr>
<tr>
<td>Water</td>
<td>225</td>
</tr>
<tr>
<td>Sand</td>
<td>750</td>
</tr>
<tr>
<td>Aggregate (10mm)</td>
<td>460</td>
</tr>
<tr>
<td>Aggregate (20mm)</td>
<td>460</td>
</tr>
<tr>
<td>Water/Cement ratio</td>
<td>0.46</td>
</tr>
<tr>
<td>Slump</td>
<td>30-60 (mm)</td>
</tr>
</tbody>
</table>

Table 5.1: Concrete Mix Constituents used in Test Series ‘D’.
### Table 5.2: Concrete Compressive Strength obtained from the beam in Test Series ‘D’.

<table>
<thead>
<tr>
<th>Batch no.</th>
<th>$f_{cu}$ (MPa)</th>
<th>$f'_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>46.1</td>
<td>30.5</td>
</tr>
<tr>
<td>2</td>
<td>50.4</td>
<td>30.1</td>
</tr>
<tr>
<td>3</td>
<td>48.7</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>47.6</td>
<td>32</td>
</tr>
</tbody>
</table>

### Table 5.3: Summary of the Crack Width and Deflection Measurements at Different Load Levels found in the Beam in Test Series ‘D’.

<table>
<thead>
<tr>
<th>At working load level†</th>
<th>At ultimate load</th>
<th>Before unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td>load (kN)</td>
<td>Defl. (mm)</td>
<td>crack width (mm)</td>
</tr>
<tr>
<td>423</td>
<td>21</td>
<td>0.25</td>
</tr>
</tbody>
</table>

† the working load level is assumed to be equal to 0.6 times the ultimate measured load.

Table 5.3: Summary of the Crack Width and Deflection Measurements at Different Load Levels found in the Beam in Test Series ‘D’.
### Material Weight (kg/m³)

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>643</td>
</tr>
<tr>
<td>Water</td>
<td>180</td>
</tr>
<tr>
<td>Sand</td>
<td>700</td>
</tr>
<tr>
<td>Aggregate (10mm)</td>
<td>830</td>
</tr>
<tr>
<td>Aggregate (20mm)</td>
<td>100</td>
</tr>
<tr>
<td>Superplasticiser</td>
<td>2 lit/100 kg cement</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water/Cement ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Slump</td>
<td>30-60 (mm)</td>
</tr>
</tbody>
</table>

Table 5.4: Concrete Mix Constituents used in Test Series ‘E’ (High strength concrete).
<table>
<thead>
<tr>
<th>Beam type</th>
<th>$\frac{a}{d}$</th>
<th>$\rho$</th>
<th>Spacing of stirrups$^{HV}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Traditional design</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>horizontal leg</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$s_1^\dagger$</td>
</tr>
<tr>
<td>A1.86</td>
<td>1.75</td>
<td>1.86</td>
<td>65</td>
</tr>
<tr>
<td>B1.86</td>
<td>3.2</td>
<td>1.86</td>
<td>130</td>
</tr>
<tr>
<td>B2.78</td>
<td>3.2</td>
<td>2.78</td>
<td>90</td>
</tr>
<tr>
<td>B2.78(T)</td>
<td></td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>C1.86</td>
<td>4.0</td>
<td>1.86</td>
<td>130</td>
</tr>
<tr>
<td>C2.78</td>
<td>4.0</td>
<td>2.78</td>
<td>115</td>
</tr>
<tr>
<td>C2.78(T)</td>
<td></td>
<td></td>
<td>115</td>
</tr>
</tbody>
</table>

† 10mm diameter stirrups used in all of the beams.

∇ Maximum spacing between stirrups was 130mm[8].

‡ Spacing required near the horizontal leg.

* Spacing required near the supports.

Table 5.5: Details of Stirrup Configurations required to prevent Diagonal Failure in the HSC Beams in Test Series ‘E’.
<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Beam size (mm x mm)</th>
<th>$f_{cu}$ (MPa)</th>
<th>$f'_{c}$ (MPa)</th>
<th>Correction factor ($F'$)</th>
<th>Measured ultimate load (kN)</th>
<th>Corrected load (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.86</td>
<td>1.75</td>
<td>203 x 320</td>
<td>79.6</td>
<td>58.1</td>
<td>0.9990</td>
<td>518</td>
<td>517.5</td>
</tr>
<tr>
<td>B1.86</td>
<td></td>
<td>202 x 310</td>
<td>70.1</td>
<td>57.2</td>
<td>1.0127</td>
<td>285</td>
<td>288.6</td>
</tr>
<tr>
<td>B2.78</td>
<td>3.2</td>
<td>210 x 330</td>
<td>79.4</td>
<td>62.1</td>
<td>0.9937</td>
<td>410</td>
<td>407.4</td>
</tr>
<tr>
<td>B2.78(T)</td>
<td></td>
<td>209 x 330</td>
<td>83.9</td>
<td>67.3</td>
<td>0.9863</td>
<td>389</td>
<td>383.7</td>
</tr>
<tr>
<td>C1.86</td>
<td></td>
<td>205 x 315</td>
<td>77.6</td>
<td>59.8</td>
<td>1.0006</td>
<td>227</td>
<td>227.1</td>
</tr>
<tr>
<td>C2.78</td>
<td>4.0</td>
<td>204 x 325</td>
<td>80.7</td>
<td>63.4</td>
<td>0.9955</td>
<td>318</td>
<td>316.6</td>
</tr>
<tr>
<td>C2.78(T)</td>
<td></td>
<td>210 x 315</td>
<td>83.3</td>
<td>66.9</td>
<td>0.9937</td>
<td>321</td>
<td>319.0</td>
</tr>
</tbody>
</table>

Table 5.6: Summarised Results from the Beams in Test Series 'E'.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>Corrected load (kN)</th>
<th>Flexural capacity ($M_f$) (kN.m)</th>
<th>Theoretical flexural capacity ($M_1$) (kN.m)</th>
<th>$\frac{M_f}{M_1}$</th>
<th>$\left(\frac{M_f}{M_1}\right)_{ave}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.86</td>
<td>1.75</td>
<td>517.5</td>
<td>119.0</td>
<td>121.52</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>B1.86</td>
<td></td>
<td>288.6</td>
<td>121.2</td>
<td>121.52</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>B2.78</td>
<td>3.2</td>
<td>407.4</td>
<td>171.1</td>
<td>173.2</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>B2.78(T)</td>
<td></td>
<td>383.7</td>
<td>161.2</td>
<td>173.2</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>C1.86</td>
<td></td>
<td>227.1</td>
<td>119.2</td>
<td>121.52</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>C2.78</td>
<td>4.0</td>
<td>316.6</td>
<td>166.2</td>
<td>173.2</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>C2.78(T)</td>
<td></td>
<td>319.0</td>
<td>167.5</td>
<td>173.2</td>
<td>0.97</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7: Actual Flexural Capacities of the Beams in Test Series 'E'.

256
<table>
<thead>
<tr>
<th>Beam type</th>
<th>At working load level</th>
<th></th>
<th></th>
<th>At ultimate load</th>
<th></th>
<th></th>
<th>Before unloading</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>load (kN)</td>
<td>Defl. (mm)</td>
<td>crack width (mm)</td>
<td>flex.</td>
<td>diag.</td>
<td>load (kN)</td>
<td>Defl. (mm)</td>
<td>crack width (mm)</td>
<td>flex.</td>
</tr>
<tr>
<td>A1.86</td>
<td>335</td>
<td>17</td>
<td>0.3</td>
<td>0.3</td>
<td>517.5</td>
<td>63.2</td>
<td>3</td>
<td>63.2</td>
<td>6</td>
</tr>
<tr>
<td>B1.86</td>
<td>188</td>
<td>13</td>
<td>0.32</td>
<td>0.16</td>
<td>288.6</td>
<td>61.7</td>
<td>3</td>
<td>61.7</td>
<td>6</td>
</tr>
<tr>
<td>B2.78</td>
<td>285</td>
<td>17</td>
<td>0.36</td>
<td>0.36</td>
<td>407.4</td>
<td>43.4</td>
<td>3.6</td>
<td>43.4</td>
<td>3</td>
</tr>
<tr>
<td>B2.78(T)</td>
<td>249</td>
<td>14</td>
<td>0.34</td>
<td>0.3</td>
<td>383.7</td>
<td>32</td>
<td>2</td>
<td>32</td>
<td>3</td>
</tr>
<tr>
<td>C1.86</td>
<td>146</td>
<td>13</td>
<td>0.26</td>
<td>0.04</td>
<td>227.1</td>
<td>50.9</td>
<td>4</td>
<td>50.9</td>
<td>5</td>
</tr>
<tr>
<td>C2.78</td>
<td>206</td>
<td>14</td>
<td>0.2</td>
<td>0.14</td>
<td>316.6</td>
<td>33.1</td>
<td>1.5</td>
<td>33.1</td>
<td>5</td>
</tr>
<tr>
<td>C2.78(T)</td>
<td>207</td>
<td>15</td>
<td>0.26</td>
<td>0.2</td>
<td>319</td>
<td>33</td>
<td>3</td>
<td>33</td>
<td>4</td>
</tr>
</tbody>
</table>

The working load level is assumed to be equal to 0.65 times the ultimate measured load.

Table 5.8: Summary of the Crack Width and Deflection Measurements at Different Load Levels for the Beams in Test Series 'E'.

257
<table>
<thead>
<tr>
<th>Load level (kN)</th>
<th>Adjacent to loading point†</th>
<th>Inside shear span†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical chords</td>
<td>Horizontal chord Upper</td>
</tr>
<tr>
<td>322</td>
<td>219 -20</td>
<td>117 127</td>
</tr>
<tr>
<td>407.4 (Max. load)</td>
<td>241 104</td>
<td>221 132</td>
</tr>
<tr>
<td>108.3 (Failure)</td>
<td>218 159</td>
<td>47 120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load level (kN)</th>
<th>Adjacent to loading point†</th>
<th>Inside shear span†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vertical chords</td>
<td>Horizontal chord Upper</td>
</tr>
<tr>
<td>326.5</td>
<td>950 524</td>
<td>178 -</td>
</tr>
<tr>
<td>383.7 (Max. Load)</td>
<td>769 1208</td>
<td>294 -</td>
</tr>
<tr>
<td>70 (Failure)</td>
<td>698 316</td>
<td>129 -</td>
</tr>
</tbody>
</table>

† For the location of the stirrups refer to Figure 5.5.

Table 5.9: Strains in the Stirrups in Beam Types B2.78 and B2.78(T).
<table>
<thead>
<tr>
<th>Load (kN)</th>
<th>Front face</th>
<th>Back face</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Middle</td>
<td>Bottom</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>123</td>
<td>-370</td>
<td>240</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>-350</td>
<td>240</td>
<td>790</td>
</tr>
<tr>
<td>290</td>
<td>-1010</td>
<td>490</td>
<td>1840</td>
</tr>
<tr>
<td></td>
<td>-950</td>
<td>520</td>
<td>1930</td>
</tr>
<tr>
<td>487</td>
<td>-2100</td>
<td>1270</td>
<td>3850</td>
</tr>
<tr>
<td></td>
<td>-1960</td>
<td>1220</td>
<td>4100</td>
</tr>
</tbody>
</table>

† For the location of the demec buttons refer to Figure 5.6.

Table 5.10: Concrete Strains obtained from Beam Type A1.86.
Figure 5.1: Details of the Beam in Test Series ‘D’.

Figure 5.2: Load-Deflection Curve obtained from the Beam in Test Series ‘D’.
Figure 5.3: Shape of the Beam in Test Series ‘D’ after Failure.
Figure 5.4: Details of the Beams in Test Series 'E'.

262
Figure 5.5: Locations of the Strain Gauges used in Beam Types B2.78 and B2.78(T).

Figure 5.6: Location of the Neutral Axis in the HSC Beam Type B1.86.
Figure 5.7: Load-Deflection Curves obtained from Beam Type A in Test Series 'E'.

264
Figure 5.8: Load-Deflection Curves obtained from Beam Type B in Test Series ‘E’.
Figure 5.9: Load-Deflection Curves obtained from Beam Type C in Test Series 'E'.

266
Figure 5.10: Crack Patterns after Failure for all of the Beams in Test Series 'E'.
Figure 5.11: Shape of Beam Types B2.78 and C1.86 in Test Series ‘E’ after Failure.
Chapter 6

FLEXURAL BEHAVIOUR OF BEAMS RESULTING FROM CONFINEMENT

6.1 INTRODUCTION

The compressive strength of the concrete in the compression regions of beams can be enhanced by the introduction of closely spaced closed stirrups which will also lead to improvements in the ductility of the beams[133]. A large number of confinement models have been developed to predict the stress-strain relationship for the compression concrete and to evaluate the load carrying capacity of tied concrete columns under the action of concentric loads. Some of these models have been modified to account for the presence of strain gradients[175], shear forces[167], loading rates[168, 173], and the magnitude of the applied axial loads[178]. These models have not been verified using experimental data for beams subjected to bending moments with either low or no axial loading.

Test Series A' described in Chapter 3 included under-reinforced beams ($\rho \simeq 1.8\%$) in which closely spaced stirrups were provided. The increase in confinement did not affect the flexural capacity of these beams. The first part of this Chapter therefore examines the flexural capacity of beams when their compression regions
have been confined. The second part of the Chapter examines the fact that over-reinforced beams can be made to behave in a ductile manner when the compression region is confined\[133]\.

### 6.2 EVALUATION OF FLEXURAL CAPACITY DUE TO CONFINEMENT

#### 6.2.1 General

It is possible that the evaluation of the flexural capacity of beams can be based on the same confinement model (Sheikh and Yeh model)\[175, 178]\ as was adopted for the prevention of diagonal failures. However, all the available confinement models including the model which has been adopted in this research programme were based on the results of tests conducted on columns. The following modifications were considered to be necessary in order to make the adopted confinement model applicable to the evaluation of the flexural capacity of beams which were not subjected to axial loadings.

#### 6.2.2 Confinement Enhancement Factor \(K_s\)

The compressive strength of the concrete inside the confined area \(f_{cc}\) because of confinement exceeds its corresponding unconfined value \(f_{cp}\) at ultimate load before spalling of the concrete cover. In this case, flexural theory cannot be used to calculate the depth of the neutral axis which is required for the determination of the strength gain factor \(K_s\) and the geometry of the equivalent concrete compressive block. The effective confined area is therefore found using the configurations of the
stirrups given by equation (4.18) which offers a safer design because it assumes a less effective confined area ($A_{co}$).

### 6.2.3 Concrete Compressive Strain ($\varepsilon_c$)

Laboratory based studies[161, 173, 178] have indicated that the maximum compressive strain at which the first hoop fractures is very high in the case of confined concrete. Scott et al.[173] reported that this value is in the range of 0.02 to 0.038 for concentric loading whereas in the case of eccentric loading the reported value was in the range of 0.061 to 0.0743.

In this investigation, the enhancing confinement effect is intended to be utilised under static loading conditions before spalling of the concrete cover in order to ensure an acceptable level of serviceability. In the case of concentric loading, Scott et al.[173] reported that the concrete cover outside the reinforcement cracks when the compressive strains are of the order of 0.004 or higher. Sheikh and Uzumeri[163] reported that, for concentric loadings, at compressive strain levels of approximately 0.0045 tapping of the specimens gave a hollow sound indicating that the concrete cover had separated. In the case of eccentric loadings, the compressive strain at which the cover concrete started to crack and separate from the core concrete was reported by Scott et al.[173] to be of the order of 0.005.

There are no measurements of the compression strains either when the first hoop fractured or at the point when spalling of the concrete cover occurred in beams under
pure flexure. Nevertheless, it is generally accepted that the strain gradient results in an increase in the compressive strain[154, 158, 173, 179, 180, 181]. Therefore, it can be concluded that for beams not subjected to axial load, the compressive strains will not be less than those found in columns subjected to strain gradients (0.061 to 0.0743 when the first hoop fractures and 0.005 when spalling of the concrete cover occurs).

In order to determine the enhanced flexural capacity of beams at the ultimate limit state the design compressive strain ($\varepsilon_c$) will be taken equal to the minimum strain ($\varepsilon_{s1}$) corresponding to the maximum compressive strength in the confinement model which has been adopted. However, in any case it must not exceed the strain at which spalling of the concrete cover occurs (0.005).

6.2.4 Modified Concrete Compression Block

The resulting concrete compression block parameters $\alpha$ and $\beta$ are determined using the corresponding equations relating to either zone 1 (equations (4.24) and (4.25)) or zone 2 (equations (4.26) and (4.27)) of the stress-strain curve for the confinement model which has been adopted. These equations result in the following parameters:

- For $\varepsilon_c = \varepsilon_{s1}$, $\Omega = 1.0$.

- For $\Omega = 1.0$, $\alpha = 0.75$, and $\beta = 0.889$. 

272
6.3 FLEXURAL CAPACITY OF UNDER-REINFORCED BEAMS

The flexural capacities obtained from the beams included in Test Series 'A' presented in Chapter 3 have been calculated using the approach described above. The measured and the predicted flexural capacities are summarised in Table 6.1. Further details of the calculations can be found in Appendix C.

The prediction shows that although the concrete compressive strength increased by up to 34% there was no corresponding increase in the flexural capacity of the beams \( (\frac{M_{\text{con}}}{M_{\text{u}}})_{\text{average}} = 96.6\% \). This is in agreement with the test results, which also showed no enhancement in the flexural capacity of the beams with increasing confinement \( (\frac{M_{\text{con}}}{M_{\text{u}}})_{\text{average}} = 97.5\% \).

The insignificant influence of confinement on the flexural capacity of beams is attributed to the limited amount of longitudinal reinforcement. The tensile force in the longitudinal reinforcement \( (T) \) is the same for all the beams regardless of the level of confinement i.e. regardless of the concrete compressive strength. It appears that the increase in the lever arm between the internal forces (the tensile force ‘\( T \)’ in the longitudinal reinforcement and the compressive force ‘\( C \)’ in the flexural compression concrete) as a result of the higher concrete compressive strength due to confinement is insignificant. In such cases if \( \varepsilon_c > \varepsilon_{s1} \) is used, the calculated flexural capacities of the beams would not change significantly because of the limited amount of longitudinal reinforcement. This is demonstrated in the example in
Appendix C in which $\epsilon_c$ was assigned a high value equal to 0.0075. The increase in the flexural capacity was only 1.6% compared with that which was calculated using $\epsilon_c = \epsilon_{s1}$. The flexural capacity ($M_{1\text{con}}$) was also calculated assuming that either the whole of the concrete compression area or the confined core balances the tensile force ($T$). The increase in the predicted flexural strength was only 3.6% when the whole section was considered. This also shows the limited effect that the concrete compressive strength has on the flexural capacity of beams for a constant value of tensile force ($T$).

The flexural capacities ($M_{1\text{con}}$) of the test beams shown in Table 6.1 were calculated using the assumption that only the compressive force ($C$) in the confined concrete core balances the tensile force ($T$) in the longitudinal reinforcement. The strain hardening effect need not be taken into account as the results are being used for comparative purposes.

In conclusion, a significant increase in flexural capacity resulting from confinement can only be achieved by increasing the amount of longitudinal reinforcement. In addition, over-reinforced beams can be designed to fail in a ductile manner after reaching their full flexural capacity because of the enhanced ductility of the compression concrete due to confinement.
6.4 FLEXURAL CAPACITY OF OVER-REINFORCED BEAMS

The design approaches adopted by most Codes of Practice require beams to be under-reinforced in order to prevent brittle compression failures. For example the ACI Code of Practice limits the value of the longitudinal reinforcement ratio ($\rho_{\text{max}}$) to 0.75 and 0.5 of $\rho_b$ for static and seismic loadings respectively. When increased ductility due to confinement is provided these limitations are believed to be too restrictive.

In an attempt to utilise the enhancing effects of confinement upon the structural behaviour of beams under the action of static loadings, it is suggested that the value of $\rho$ should be allowed to exceed the value of $\rho_{\text{max}}$. A new balanced-failure condition (cracking of the concrete cover at the yielding of the longitudinal bars) would therefore exist as a result of confinement. These conditions would result in a new balanced-failure longitudinal reinforcement ratio ($\rho'_b$).

6.4.1 New Balanced-Failure Longitudinal Reinforcement Ratio ($\rho'_b$)

The modified concrete compression block used in the confinement model was developed as a modified form of the concrete compression block given by the ACI Code of Practice using a statistical analysis of available results from tests on confined concrete specimens. Therefore, for comparison purposes, the ratio $\rho'_b$ has been derived with reference to the provisions of the ACI Code of Practice. This ratio has been derived in Appendix C and is given below:
\[
\rho'_b = \frac{A_s}{bd} = \frac{\alpha \beta K_s f'_c}{f_y} \left( \frac{\varepsilon_c E_s}{\varepsilon_c E_s + f_y} - \frac{d_1}{d} \right) \frac{b'}{b} + 0.85 f'_c \beta_1 \left( \frac{b'd_1}{bd} - \frac{2b_1}{b} \frac{\varepsilon_c E_s}{\varepsilon_c E_s + f_y} \right)
\]  

(6.1)

where:

\(d'\) is the effective depth of the section measured from the centreline of the stirrup, and not from the top of the section.

\(b'\) is the width of the confined core.

\[d_1 = d - d'.\]

\[b_1 = \frac{1}{2}(b - b').\]

The ratio \(\rho'_b\) in equation (6.1) is a function of the geometry of the section in addition to the characteristics of the steel and the confined concrete. In order to simplify this equation, it is reasonable to assume that all of the compression region is confined. The results from Test Series 'A' have shown that the variation in the concrete compressive strength does not influence significantly the flexural capacity of the beams, hence:

\[
\rho'_b = \frac{A_s}{bd} = \frac{\alpha \beta K_s f'_c}{f_y} \frac{\varepsilon_c E_s}{\varepsilon_c E_s + f_y}
\]

(6.2)

Equation (6.2) indicates that \(\rho'_b\) is a function of the characteristics of the concrete and the steel (similar to \(\rho_b\)) in addition to the confinement characteristics represented here by the factor \(K_s\).
\( \alpha \) and \( \beta \) are material characteristics of the confined concrete and are equal to 0.75 and 0.889 respectively in the case where \( \varepsilon_c = \varepsilon_{n1} = 0.0022K_s \). Equation (6.2) then becomes:

\[
\rho_b' = \frac{A_s}{bd} = \frac{0.667K_s f'_c}{f_y} \frac{0.0022K_s E_s}{0.0022K_s E_s + f_y}
\]  

(6.3)

This relationship has been used for the determination of the longitudinal reinforcement ratio \( \rho_b' \) for the test beams detailed in Tables 6.2 and 6.5.

In order to ensure a good level of serviceability, it is suggested that the value of \( \varepsilon_c \) in equation (6.2) should not be greater than the minimum concrete compressive strain at which the concrete cover starts to crack i.e. 0.005.

In order to ensure reasonable ductile behaviour, a maximum allowable spacing \( s_v \) between the stirrups is proposed beyond which the longitudinal bars would buckle before fracture of the stirrups[223, 224]. A value of \( s_v \) not greater than \( 6 \times d_b \) is recommended[223] where \( d_b \) is the diameter of the longitudinal bars.

### 6.4.2 Relationship Between \( \rho_b' \) and \( \rho_b \)

In the ACI Code of Practice, \( \rho_b \) is given by:

\[
\rho_b = \frac{A_s}{bd} = \frac{0.85\beta_1 f'_c}{f_y} \frac{0.003E_s}{0.003E_s + f_y}
\]  

(6.4)

thus:
\[
\frac{\rho_b'}{\rho_b} = \frac{\alpha \beta K_s \varepsilon_c}{\varepsilon_c E_s + f_y} \frac{0.003E_s + f_y}{0.85 \times 0.003 \beta_i}
\]  

(6.5)

where \(\alpha\), \(\beta\), and \(\varepsilon_c\) are equal to 0.75, 0.889, and 0.0022\(K_s\), respectively.

Equation (6.5) could be expressed in the following simple form for a normal strength confined concrete (\(\beta_1\) is of the order of 0.8) and high strength steel (\(E_s \simeq 200,000\) MPa and \(f_y \simeq 500\) MPa):

\[
\rho_b' = K_s \rho_b
\]  

(6.6)

Equation (6.5) indicates that the ratio \(\frac{\rho_b'}{\rho_b}\) is directly related to \(K_s\). In a comparison with the ACI Code of Practice this equation also indicates that for the same amount of confining stirrups the ratio \(\frac{\rho_b'}{\rho_b}\) increases with increasing concrete strength i.e. with decreasing \(\beta_1\). It should be noted, however, that \(K_s\) decreases as the value of \(f'_c\) increases, equation (4.18). Hence, the net increase in \(\frac{\rho_b'}{\rho_b}\) would not to the same extent be directly proportional to the decrease in \(\beta_1\).

In the case of beam types NA3-1, NB2-1, NB2-2, NB3-1, and NB3-2 in Test Series ‘A’ where the concrete strengths were similar, Table 6.2, the ratio \(\frac{\rho_b'}{\rho_b}\) increased from 0.99 to 1.33 as \(K_s\) increased from 1.08 to 1.32. If the \(\frac{\rho_b'}{\rho_b}\) ratios for beam types NA2-1 and NA2-2 were compared then it can be noted that although \(K_s\) for beam type NA2-2 (1.19) was larger than that for beam type NA2-1 (1.15) the \(\frac{\rho_b'}{\rho_b}\) ratio for beam type NA2-2 (1.11) was smaller than that for beam type NA2-1 (1.14). This is because the value of \(f'_c\) for beam type NA2-2 (35.92 MPa) was smaller than

278
that for beam type NA2-1 (44.64 MPa).

In general, the relationship given by equation (6.6) gave a reasonable estimate of the values of the $\frac{\rho_i}{\rho_b}$ ratios, Table 6.2. The maximum difference was less than $\pm 10\%$. The accuracy of this equation was less conservative for small values of $K_s$ combined with large values of $\beta_1$ (low strength concrete). The results from beam types NB2-1 and NB2-2 in Table 6.2 support this conclusion where the error was 9% (the highest). $K_s$ was equal to 1.08 (the lowest) in the case of these two beams.

It worthwhile noting that when $K_s < 1.1$ the increase in $\rho_b$ was insignificant. It is therefore recommended in this case that the value of $\rho$ should be limited to the value of $\rho_{\text{max}}$.

6.4.3 Design Method for Over-Reinforced Beams

Traditionally, over-reinforced beams are not used in practice because they fail without warning in a brittle manner. Also, since the failure is governed by the characteristics of the compression concrete, failure occurs in the compression concrete before the longitudinal reinforcement reaches its maximum tensile strength which is governed by its yield strength. The tensile capacity of the longitudinal reinforcement and hence the flexural capacity of beams are not fully utilised.

It is proposed in order to overcome the restriction on the use of over-reinforced beams, that the region of the beam subjected to high bending moment is provided
with closely spaced closed stirrups. The longitudinal reinforcement ratio in this case may be increased beyond the maximum ratio allowed by Codes of Practice ($\rho_{\text{max}}$) and even beyond the value of $\rho'_b$ since ductility is ensured by the use of confining stirrups.

The steps in the design process for over-reinforced beams are outlined as follows:

1. The section, in which the overall depth is restricted, is designed for the applied design bending moment ($M$) in the normal way (under-reinforced beam). The longitudinal reinforcement ratio required ($\rho_{\text{req}}$) is then calculated.

2. The values of $\rho_{\text{req}}$ and $\rho_{\text{max}}$ are compared and if $\rho_{\text{req}} > \rho_{\text{max}}$ then the value of $\rho_{\text{req}}$ is used.

3. For the value of $\rho_{\text{req}}$, the corresponding value for $K_s$ is calculated using the equilibrium of the internal couple (the tensile force from the longitudinal steel and the compressive force from the modified concrete compression block) which results in the following equations:

$$ c = \frac{A_s f_y}{\alpha b y K_s f'_c} $$

$$ M = A_s f_y \left( d' - 0.5625 \frac{A_s f_y}{K_s b' f'_c} \right) $$

where $c$ is the depth to the neutral axis measured from the centreline of the stirrup.
The contribution of the concrete cover to the flexural capacity of the beam is ignored thus erring on the side of safety.

4. The value of $\rho_b'$ is obtained for the calculated value of $K_s$.

5. If $\rho_{req} > \rho_b'$, the area of the stirrups required is calculated for the larger value of $K_s$ in order to result in $\rho_b' = \rho_{req}$.

6. If the confinement factor required for the prevention of a diagonal failure $K_{s,\text{shear}}$ is larger than the calculated value, the value of $K_{s,\text{shear}}$ is used. It is important to note that the proposed design approach for beams (under the action of shear and flexure) is an interactive one. Therefore, $K_{s,\text{shear}}$ has to be determined from the actual flexural strength, resulting in this case from confinement, which was included in the proposed flexure-shear interaction design model. If $K_{s,\text{shear}}$ controls the design, it is not necessary to re-check the confinement requirements corresponding to the flexural capacity of the section. This is because the change in the flexural capacity due to the increased confinement is not significant, as long as the longitudinal reinforcement content remains unchanged.

7. Practical configurations for the stirrups are assumed and the required spacing of the stirrups is calculated based on the controlling value of $K_s$, using the confinement model which has been adopted.

8. If the calculated spacing is impractical, the longitudinal reinforcement content is modified accordingly and the design procedure is repeated starting from step 3 above.
It should be noted that no material reduction factors were considered in the design procedure described above. The design method for over-reinforced beams can be represented in the form of a flow chart as shown in Figure 6.1.

6.5 TEST SERIES ‘F’: Verification of the design method for over-reinforced beams

6.5.1 Design and Description of Test Beams

This test series included seven over-reinforced beams, Figure 6.2. Beam types T2 and C2 were designed to have an overall length of 2200mm, an effective span of 1400mm, an a/d ratio of 2.0, a longitudinal reinforcement ratio (ρ) of 0.036, and a cross-section of 200mm x 230mm. High-strength deformed steel bars with a nominal diameter of 25mm (A_s = 450.52mm$^2$ and f_y = 477 MPa) were used for the longitudinal reinforcement. The stirrups were fabricated from plain round mild steel bars with a nominal diameter of 10mm (A_s = 74.45mm$^2$ and f_y = 441.5 MPa).

The degree of confinement (spacing of stirrups) required to prevent a compression failure in beam type C2 was determined using the design method for over-reinforced beams detailed in the previous section. Beam type T2 with similar material and geometric properties except that the mid-span region was not provided with stirrups was included for comparison purposes. To ensure yielding of the longitudinal reinforcement before cracking of the concrete cover the longitudinal reinforcement ratio (ρ = 0.036) in beam type C2 was designed to be equal to the maximum allowable ratio resulting from confinement (ρ_c).
The remaining beams were designed to have an overall length of 2500mm, an effective span of 1750mm, an a/d ratio of 3.2, a longitudinal reinforcement ratio ($\rho$) of 0.0465, and a cross-section of $176\text{mm} \times 220\text{mm}$. High-strength deformed steel bars with a nominal diameter of 25mm ($A_s = 483.8\text{mm}^2$, $E_s = 190,000\ MPa$, and $f_y = 526\ MPa$) were used for the longitudinal reinforcement. The stirrups were fabricated from plain round mild steel bars with a nominal diameter of 10mm ($A_s = 74.45\text{mm}^2$, $E_s = 200,000\ MPa$ and $f_y = 441.5\ MPa$). The stirrups used in beam type C3.2-3 had a nominal diameter of 8mm ($A_s = 49.02\ MPa$, $E_s = 191,000\ MPa$, and $f_y = 377\ MPa$). The three type C3.2 beams were designed to have the same level of confinement as used in beam type C2 ($K_s \approx 1.4$). The longitudinal reinforcement ratio for these beams ($\rho = 0.0465$) exceeded the maximum proposed value of ($\rho'_b$) in order to examine the behaviour of highly over-reinforced beams. A different configuration of stirrups was used in beam type C3.2-3 as shown in Figure 6.2. The two type T3.2 beams which were not provided with stirrups in the mid-span region were also included for comparison purposes.

6.5.2 Experimental Work

The majority of the test beams were provided with closely spaced stirrups. The concrete mix used in this investigation, Table 6.3, was designed to have a high workability level (a slump value between 60mm and 180mm) and was made using a relatively small maximum size aggregate (10mm) in order to make the casting operation easier. All the beams, except beam types T2 and C2, were cast in the
horizontal position with the compression concrete face upwards, again to make the casting operation easier. This, however, resulted in variations in the actual effective depths of the beams and the cover at the compression faces. The width of beam types T2 and C2 was larger (200mm), therefore, these beams were cast with the concrete compression face at the base of the shutter because it was possible to insert the vibrator between the main longitudinal reinforcement bars in these beams.

In general, one batch of concrete was used to cast two beams, and six control cubes and cylinders. The test beams and cubes were cast, vibrated, cured, and tested in a similar way to those included in Test Series ‘A’. The concrete strains were measured using demec buttons bonded to the surfaces of the beams. The strains in the longitudinal bars were measured using strain gauges with a gauge length of 10mm bonded to the surfaces of the bars. The gauges used to measure the strains in the stirrups had a gauge length of 5mm. Figure 6.3 shows the locations of the demec buttons and the strain gauges. Appendix A contains additional information on the strain measurement techniques and the data acquisition equipment used in the investigation.

6.5.3 Test Results

The concrete compressive strength, the beam dimensions, the maximum applied total load, and the maximum flexural capacity obtained from the beams are given in Table 6.4. The resulting longitudinal reinforcement ratios for the test beams are summarised in Table 6.5. The maximum flexural capacities of the beams were
compared with those predicted using the proposed method for the evaluation of the flexural capacity of over-reinforced beams. The resistance of the whole of the compression concrete was assumed for the prediction of the flexural capacity of beam type C2. However, the normal and the modified concrete compression blocks were used for the unconfined and for the confined core concrete respectively. This is because in the beams in which $\rho \neq \rho'_b$ the ultimate capacity was reached before cracking of the concrete cover. On the other hand, the resistance of the concrete compression cover in beam type C3.2 was ignored since in these beams the concrete cover cracked before yielding of the longitudinal reinforcement ($\rho > \rho'_b$). The characteristics of the confined concrete in these beams had been obtained from zone 2 of the confinement model with $\varepsilon_c$ taken to be equal to 0.0075 (this is in agreement with the strain measurements obtained from the test beams, Tables 6.8 and 6.9, which show that the longitudinal bars had yielded). The results of the comparisons between the predicted and the actual capacities of the beams are summarised in Table 6.6. Typical crack histories are summarised in Table 6.7 together with the corresponding deflection measurements to permit the evaluation of the performance of the beams at the serviceability limit state and to examine the behaviour of the beams up to failure. Typical strain measurements in the concrete and in the reinforcement are summarised in Tables 6.8 and 6.9 respectively.

The actual load-mid span deflection curves for beam types T2 and C2, beam types T3.2-1 and C3.2-1 and beam types T3.2-2, C3.2-2 and C3.2-3 are shown in Figure 6.4. The load-deflection curves were grouped on the basis of either the ge-
ometric or the material similarities which existed between the beams. The curves obtained from the beams without confinement were included in the corresponding group for comparison purposes. The shape of failure in beam type C3.2-3 is shown in Figure 6.5. Finally, the development of cracks in the confined beam type C2 is shown in Figure 6.6.

6.5.4 Discussion of Results

(a) Modes of Failure

Beam types T2 and T3.2 in which the mid-span regions were not confined failed in a typical brittle manner which is characteristic of the flexural failure of over-reinforced beams. The sudden collapse of these beams occurred as a result of spalling of the compression concrete and buckling of the compression reinforcement in the mid-span regions. In the remaining beams flexural cracks developed and widened prior to spalling of the concrete cover. The flexural cracks under increasing loads continued to widen and it was possible to see the concrete arches forming between the closely spaced stirrups, Figure 6.5. This cracking process continued until testing was stopped on the grounds of safety. A defined failure point was not obvious during the testing of the confined beams. The majority of the beams continued to carry the applied maximum loads without a significant reduction in load carrying capacity up to the completion of testing.
(b) **Flexural Capacity and Ductility**

The unconfined beam types T2, T3.2-1 and T3.2-2 had ultimate flexural capacities of 84.8 kN.m, 55.7 kN.m and 74.1 kN.m respectively, Table 6.4. The concrete and steel strain measurements given in Tables 6.8 and 6.9 indicate that the compression concrete cracked and the failure of the beams occurred before yielding of the longitudinal reinforcement. In addition, as expected the ductilities of these beams were extremely low. The maximum mid-span deflection obtained from these beams ranged from 15mm to 20mm as shown in Figure 6.4. It is, therefore, concluded that excluding over-reinforced beams in practice as recommended by Codes of Practice is fully justified for beams without confinement.

The confined beam type C2 achieved a flexural capacity of 90 kN.m, Table 6.4. The longitudinal reinforcement ratio for this beam ($\rho = 0.0347$) was equal to the maximum allowed ratio resulting from confinement ($\rho'_c$) as shown in Table 6.5. The observations made during the test and the load-deflection curve obtained from this beam, Figure 6.4, showed that the maximum load was reached before cracking of the concrete cover took place. In addition, the flexural crack widths at the various load levels were higher than those obtained from the highly over-reinforced beam type C3.2-2 because of the relatively low value of $\rho$ which was used, Table 6.7. It is interesting to note that this beam achieved 166% of the maximum capacity allowed by the ACI Code of Practice corresponding to the value of $\rho_{max}$. The ductility resulting from the confinement of the compression concrete in this beam was extremely high. The final mid-span deflection obtained exceeded 80mm, Figure 6.4.
and Table 6.7. This deflection is high not only in comparison with the unconfined over-reinforced beam type T2, but also in comparison with the deflection obtained from the traditionally designed under-reinforced beams included in Test Series 'C' and described in Chapter 4.

The highly over-reinforced ($\rho > \rho_b$) beam types C3.2-1, C3.2-2 and C3.2-3 achieved maximum flexural capacities of 64.5 kN.m, 72.4 kN.m and 70.2 kN.m respectively as shown in Table 6.4. The longitudinal reinforcement ratios ($\rho$) in these beams were higher than the corresponding values of $\rho_b$ resulting from confinement, Table 6.5. Spalling of the concrete cover in these beams occurred before yielding of the longitudinal reinforcement took place. The concrete and steel strains obtained from the beams, Tables 6.8 and 6.9 support this conclusion ($\varepsilon_y = \frac{526}{190,000} = 0.0027$). The ductilities resulting from confinement found in the highly over-reinforced beams were also high, Figure 6.4 and Table 6.7, in comparison with beam type C2.

It is concluded that the highly over-reinforced beams can be designed to fail in a ductile manner. However, such beams would not reach their full flexural capacity before spalling of the concrete cover. In this case if the concrete cover is large, a significant reduction in their ultimate capacity may occur. It is believed that further work is still needed to determine the limitations which must be placed on the dimensions of the unconfined concrete compared to the size of the confined concrete in order to prevent a significant reduction in the capacity of such beams.
(c) Serviceability

The main reason for not using over-reinforced beams in practice is that they fail suddenly in a brittle manner if the concrete is not confined. There may be little visible warning of impending failure because the widths of the cracks at the failure section are small, owing to the low steel stress. This implies that one of the serviceability requirements for such beams i.e. control of the crack widths, will be satisfied. The crack widths at the working and maximum load levels, and before unloading found in the test beams, Table 6.7, were small in comparison with those usually obtained from under-reinforced beams.

The other serviceability requirement is the control of the deflection at working load levels. The beam depth is usually considered to be the main factor which control deflection. The allowable deflection of these beams would be approximately 7mm based on BS 8110. Table 6.7 shows that the deflection found in the beams at the working load level satisfy the serviceability requirement for the majority of the beams. An increase in the beam depth should be considered in the cases where reduced deflections were required.

(d) Prediction of Test Results

1. In the design method for over-reinforced beams it was assumed that the maximum concrete compressive strain at cracking of the concrete cover in confined beams would not be less than 0.005. The typical concrete strains which were found in the beams with confinement, Table 6.8, indicated that this is a rea-
sonable assumption. The average concrete strain measured 20mm below the
top face of the beams before cracking of the concrete cover was 0.0054.

2. In the confinement model which has been adopted, it was assumed that the
stress in the stirrups under maximum loading reached its yield value. The
strains obtained from the stirrups which were located in the mid-span region
which was subjected to the maximum bending moment confirmed this assump-
tion, Table 6.9. In some of the beams the strains in the stirrups reached a very
high value which exceeded the operating range of the strain gauge. A maxi-
mum stirrup strain \( \varepsilon_{sv} \) of 0.0043 \( (\varepsilon_y = \frac{377}{19100} = 0.002) \) was obtained in beam
type C3.2-3 which was very well confined as shown in Figure 6.2.

3. The simplified equation (6.6) gave a reasonably accurate prediction of the
value of \( \rho'_b \), Table 6.5. The maximum difference between the predicted values
and those obtained using the more accurate equation (6.3) was 5%.

4. Finally, the prediction of flexural capacity resulting from confinement using
the proposed approach was found to be reasonably accurate, Table 6.6. The
maximum difference between the results was 4%.

6.5.5 Practical Applications

In practical applications, the overall depth of a beam is normally restricted in order
to satisfy aesthetic requirements, etc. If such a beam is to be subjected to high
bending moments, prestressing is occasionally used in order to increase the flexural
capacity of the beam. In situations where the use of prestressed concrete is either
impractical, or uneconomical, an alternative method for increasing the flexural capacity could be to use over-reinforced beams. Over-reinforced beams can also be used in situations where confinement is already available through the provision of either stirrups or some other means e.g. the confinement provided by slabs when slabs are cast monolithically with beams.

The stirrup spacings which were used in the confined beams included in this test series were small. This was because the longitudinal reinforcement ratios used were very large \((\rho_{ave} = 2.5)\) and \((\rho_{max})_{ave} = 3.4\). Also, the cross sections of the beams were small which resulted in limited effectively confined areas requiring large amounts of stirrups in order to provide the required amount of confinement. In practical situations, however, the cross sections of beams are normally larger than those used in this test series. Therefore, it is anticipated that the resulting stirrup spacing would be practical in such situations, particularly if a reasonably large value of \(\rho\) is used.

\[6.6 \text{ SUMMARY}\]

In this Chapter a method for the evaluation of the flexural capacity of under-reinforced beams with confinement has been proposed based on the confinement model which was used previously for the flexural-shear interaction design model. Prediction of the results obtained from the confined beams included in Test Series ‘A’ using the proposed approach was found to be reasonably accurate.
The literature review showed the enhancing influence of confinement on the behaviour of beams. However, no attempt has yet been made to utilise this enhancement in the design of beams under static loadings. It was concluded that over-reinforced beams can be designed to fail in a ductile manner when the concrete compression regions are provided with closely spaced closed stirrups. The method used for the evaluation of the flexural capacity of under-reinforced beams was therefore extended to the design of over-reinforced beams. The beams in Test Series 'F' included seven over-reinforced beams which were designed using the proposed method. The prediction of the test results based on the proposed method was found to be reasonably accurate.

A new balanced-failure longitudinal reinforcement ratio resulting from confinement ($\rho'_{b}$) was also developed in this Chapter. A unique relationship between the value of $\rho'_{b}$ and the value of $\rho_{b}$ given by ACI Code of Practice was found to be a function of confinement characteristics.
where:

\( M_{1n} \) is the predicted flexural capacity of the unconfined section.

\( M_{1\text{con}} \) is the predicted flexural capacity of the confined section.

\( M_{\text{con}} \) is the measured flexural capacity of the confined section.

\( M_n \) is the measured flexural capacity of the unconfined section.

Table 6.1: Flexural Capacities of the Beams in Test Series ‘A’.
where:

\( \frac{\rho'_b}{\rho_b} \) is obtained from equation (6.5).

\( (\frac{\rho'_b}{\rho_b}) \_1 \) is obtained from equation (6.6).

<table>
<thead>
<tr>
<th>Beam type</th>
<th>( f'_c ) (MPa)</th>
<th>( \rho_b )</th>
<th>( \rho_{max} )</th>
<th>( \rho )</th>
<th>( \rho'_{max} )</th>
<th>( \frac{\rho'_b}{\rho_b} )</th>
<th>( (\frac{\rho'_b}{\rho_b}) _1 )</th>
<th>( \frac{\rho'_b - K_s}{\rho_b} )</th>
<th>( \times 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA2-1</td>
<td>44.64</td>
<td>0.0274</td>
<td>0.0206</td>
<td>0.0174</td>
<td>0.0313</td>
<td>1.14</td>
<td>1.15</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>NA2-2</td>
<td>35.92</td>
<td>0.024</td>
<td>0.0180</td>
<td>0.0174</td>
<td>0.0265</td>
<td>1.11</td>
<td>1.19</td>
<td>-7</td>
<td></td>
</tr>
<tr>
<td>NA3-1</td>
<td>39.44</td>
<td>0.0255</td>
<td>0.0191</td>
<td>0.0173</td>
<td>0.0340</td>
<td>1.33</td>
<td>1.32</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>NA3-2</td>
<td>37.5</td>
<td>0.0247</td>
<td>0.0185</td>
<td>0.0175</td>
<td>0.0330</td>
<td>1.34</td>
<td>1.34</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>NB2-1</td>
<td>39.25</td>
<td>0.0254</td>
<td>0.0191</td>
<td>0.0174</td>
<td>0.0250</td>
<td>0.99</td>
<td>1.08</td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>NB2-2</td>
<td>39.04</td>
<td>0.0253</td>
<td>0.0190</td>
<td>0.0175</td>
<td>0.0249</td>
<td>0.99</td>
<td>1.08</td>
<td>-9</td>
<td></td>
</tr>
<tr>
<td>NB3-1</td>
<td>39.9</td>
<td>0.0257</td>
<td>0.0193</td>
<td>0.0174</td>
<td>0.0298</td>
<td>1.16</td>
<td>1.2</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>NB3-2</td>
<td>39.36</td>
<td>0.0255</td>
<td>0.0191</td>
<td>0.0174</td>
<td>0.0294</td>
<td>1.15</td>
<td>1.2</td>
<td>-4</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2: Longitudinal Reinforcement Ratios for the Beams in Test Series ‘A’.

294
Table 6.3: Concrete Mix Constituents used in Test Series ‘F’.

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>414</td>
</tr>
<tr>
<td>Water</td>
<td>240</td>
</tr>
<tr>
<td>Sand</td>
<td>785</td>
</tr>
<tr>
<td>Coarse aggregate (10mm)</td>
<td>850</td>
</tr>
<tr>
<td>Water/Cement ratio</td>
<td>0.58</td>
</tr>
<tr>
<td>Slump</td>
<td>60-180 (mm)</td>
</tr>
<tr>
<td>Beam type</td>
<td>a/d</td>
</tr>
<tr>
<td>-----------</td>
<td>-----</td>
</tr>
<tr>
<td>T2</td>
<td>2</td>
</tr>
<tr>
<td>C2</td>
<td></td>
</tr>
<tr>
<td>T3.2-1</td>
<td>3.2</td>
</tr>
<tr>
<td>C3.2-1</td>
<td></td>
</tr>
<tr>
<td>T3.2-2</td>
<td>3.2</td>
</tr>
<tr>
<td>C3.2-2</td>
<td></td>
</tr>
<tr>
<td>C3.2-3</td>
<td>3.2</td>
</tr>
</tbody>
</table>

† Maximum total measured load.

‡ Maximum measured flexural capacity.

Table 6.4: Summarised Results obtained from the Beams in Test Series 'F'.
<table>
<thead>
<tr>
<th>Beam type</th>
<th>$K_s$</th>
<th>$\rho$</th>
<th>$\rho_b$</th>
<th>$\rho_{b1}^{'}$</th>
<th>$\rho_{b2}^{'}$</th>
<th>$\frac{\rho_{b1}^{'} - \rho_{b2}^{'}}{\rho_{b1}^{'}} \times 100$</th>
<th>$\frac{\rho}{\rho_{max}}$</th>
<th>$\frac{\rho}{\rho_{b1}^{'}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2</td>
<td>-</td>
<td>0.0347</td>
<td>0.0219</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.58</td>
<td>2.11</td>
</tr>
<tr>
<td>C2</td>
<td>1.4</td>
<td>0.0347</td>
<td>0.0251</td>
<td>0.0337</td>
<td>0.0351</td>
<td>-4</td>
<td>1.38</td>
<td>1.84</td>
</tr>
<tr>
<td>T3.2-1</td>
<td>-</td>
<td>0.0462</td>
<td>0.0140</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3.30</td>
<td>4.40</td>
</tr>
<tr>
<td>C3.2-1</td>
<td>1.69</td>
<td>0.0473</td>
<td>0.0140</td>
<td>0.0241</td>
<td>0.0243</td>
<td>-1</td>
<td>3.38</td>
<td>4.50</td>
</tr>
<tr>
<td>T3.2-2</td>
<td>-</td>
<td>0.0408</td>
<td>0.0156</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.62</td>
<td>3.49</td>
</tr>
<tr>
<td>C3.2-2</td>
<td>1.62</td>
<td>0.0411</td>
<td>0.0156</td>
<td>0.0252</td>
<td>0.0253</td>
<td>0.0</td>
<td>2.63</td>
<td>3.51</td>
</tr>
<tr>
<td>C3.2-3</td>
<td>1.64</td>
<td>0.0447</td>
<td>0.0168</td>
<td>0.0277</td>
<td>0.0276</td>
<td>0.0</td>
<td>2.66</td>
<td>3.55</td>
</tr>
</tbody>
</table>

† Calculated using equation (6.3).
‡ Calculated using equation (6.6).

Table 6.5: Evaluation of the Longitudinal Reinforcement Ratios for the Beams in Test Series 'F'.

297
<table>
<thead>
<tr>
<th>Beam type</th>
<th>$P_{max}$</th>
<th>$M_{max1}$</th>
<th>$M_{max2}$</th>
<th>$\frac{M_{max2} - M_{max1}}{M_{max2}} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2</td>
<td>452</td>
<td>84.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C2</td>
<td>480</td>
<td>90</td>
<td>88.8</td>
<td>-1</td>
</tr>
<tr>
<td>T3.2-1</td>
<td>196</td>
<td>55.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C3.2-1</td>
<td>227</td>
<td>64.5</td>
<td>65.1</td>
<td>1</td>
</tr>
<tr>
<td>T3.2-2</td>
<td>261</td>
<td>74.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>C3.2-2</td>
<td>255</td>
<td>72.4</td>
<td>69</td>
<td>-5</td>
</tr>
<tr>
<td>C3.2-3</td>
<td>247</td>
<td>70.2</td>
<td>73.2</td>
<td>4</td>
</tr>
</tbody>
</table>

† Maximum total measured load.
‡ Maximum measured flexural capacity.
* Predicted maximum flexural capacity.

Table 6.6: Comparison between the Measured and Predicted Flexural Capacities for the Beams in Test Series ‘F’.
Table 6.7: Summary of the Crack Width and Deflection Measurements at Different Load Levels for the Beams in Test Series ‘F’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>At working load level</th>
<th>At ultimate load</th>
<th>Before unloading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>load (kN)</td>
<td>Defl. crack width (mm)</td>
<td>load (kN)</td>
</tr>
<tr>
<td></td>
<td>(mm)</td>
<td>flex.</td>
<td>diag.</td>
</tr>
<tr>
<td>T2</td>
<td>271</td>
<td>7</td>
<td>0.25</td>
</tr>
<tr>
<td>C2</td>
<td>288</td>
<td>6</td>
<td>0.3</td>
</tr>
<tr>
<td>T3.2-1</td>
<td>118</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>C3.2-1</td>
<td>136</td>
<td>8</td>
<td>0.05</td>
</tr>
<tr>
<td>T3.2-2</td>
<td>157</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>C3.2-2</td>
<td>153</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>C3.2-3</td>
<td>148</td>
<td>7</td>
<td>0.05</td>
</tr>
</tbody>
</table>

† The working load level is assumed to be equal to 0.6 times the actual ultimate load.

Table 6.8: Typical Concrete Strains obtained from the Beams in Test Series ‘F’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>Load (kN)</th>
<th>Strains in concrete† ‡</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Middle</td>
</tr>
<tr>
<td>T3.2-1</td>
<td>174</td>
<td>0.0031</td>
</tr>
<tr>
<td>C3.2-1</td>
<td>178</td>
<td>0.0051</td>
</tr>
<tr>
<td>T3.2-2</td>
<td>241</td>
<td>0.0039</td>
</tr>
<tr>
<td>C3.2-2</td>
<td>238</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

† For the locations of demec buttons refer to Figure 6.3.

‡ Average strains just before cracking of the concrete cover.
Beam type | Load (kN) | Strains (mm/mm $\times 10^6$) obtained from the strain gauges numbers$^\dagger$:
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3.2-2</td>
<td>241$^\dagger$</td>
<td>1552</td>
<td>2338</td>
<td>1547</td>
<td>2387</td>
<td>724</td>
<td>754</td>
<td>415</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>261$^*$</td>
<td>1576</td>
<td>2652</td>
<td>1568</td>
<td>2611</td>
<td>761</td>
<td>864</td>
<td>446</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100$^\triangledown$</td>
<td>167</td>
<td>3178</td>
<td>-61</td>
<td>3041</td>
<td>599</td>
<td>584</td>
<td>356</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C3.2-2</td>
<td>238$^\dagger$</td>
<td>1426</td>
<td>2375</td>
<td>1389</td>
<td>2465</td>
<td>336</td>
<td>1005</td>
<td>286</td>
<td>539</td>
<td>566</td>
<td>722</td>
</tr>
<tr>
<td></td>
<td>259$^*$</td>
<td>1431</td>
<td>3719</td>
<td>1411</td>
<td>4499</td>
<td>1065</td>
<td>1496</td>
<td>-</td>
<td>705</td>
<td>612</td>
<td>802</td>
</tr>
<tr>
<td></td>
<td>232$^\triangledown$</td>
<td>2463</td>
<td>9996</td>
<td>-</td>
<td>7689</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>887</td>
<td>769</td>
<td>910</td>
</tr>
<tr>
<td>C3.2-3</td>
<td>227$^\dagger$</td>
<td>1400</td>
<td>2069</td>
<td>434</td>
<td>265</td>
<td>366</td>
<td>316</td>
<td>409</td>
<td>261</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>247$^*$</td>
<td>7121</td>
<td>10897</td>
<td>1732</td>
<td>1458</td>
<td>2137</td>
<td>390</td>
<td>465</td>
<td>263</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>240$^\triangledown$</td>
<td>-</td>
<td>-</td>
<td>3855</td>
<td>4351</td>
<td>-</td>
<td>394</td>
<td>454</td>
<td>257</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^\dagger$ For the locations of the strain gauges refer to Figure 6.3.

$^\ddagger$ Just before cracking of the concrete cover.

$^*$ At maximum measured load.

$^\triangledown$ Just before unloading.

Table 6.9: Strains in the Reinforcement Bars at various Loading Levels obtained from the Beams in Test Series ‘F’.
Figure 6.1: Flow Chart showing the Design Steps for Over-Reinforced Beams.
Figure 6.2: Detailed of the Beams in Test Series ‘F’.
Figure 6.3: Locations of the Strain Gauges and the Demec Buttons used in the Beams in Test Series 'F'.

303
Figure 6.4: Load-Deflection Curves obtained from the Beams in Test Series ‘F’.
Figure 6.5: Shape of Failure in Beam Type C3.2-3 in Test Series ‘F’.
$c_3 = 2.5$

$p = 240 \text{(Final)}$
Figure 6.6: Development of Cracks in Beam Type C2 in Test Series ‘F’.
Chapter 7

BEAMS WITH CIRCULAR CROSS SECTIONS

7.1 INTRODUCTION

Structural concrete members with a circular cross section are used in many types of structures. They are preferred for bridge columns due to the simplicity of construction and their omnidirectional strength characteristics under wind and seismic loads. Bridge columns are usually subjected to lateral loads arising from impacts or the effects of a vehicle braking on the bridge in addition to wind and earthquake loadings. Circular sections are also frequently used as impact barriers and bollards. Although, less common in buildings because of difficulties of detailing beam-column connections, glazing, and installing curtain walls, they currently appear to enjoy an architectural revival. Circular piles are used extensively in the foundations of buildings. In addition they are used in the form of secant piling to form diaphragm walls. In many cases piles are subjected to lateral loads, particularly when they are being used as retaining walls.

Despite the prevalence of circular sections in practice, their behaviour under transverse loading is still not fully understood. This is mainly due to the general lack of understanding of the mechanisms of diagonal failure and the relatively
limited research work which has been conducted into the behaviour of circular sections under predominantly transverse loadings.

Although shear failures in members with a circular cross section have been common in recent earthquakes[149], their shear strength has not yet received as much attention as their flexural strength and ductility. This has been attributed to the following reasons:

1. Shear failures were related to poor detailing, such as overlapping shear reinforcement in the concrete cover, which spalled during earthquakes.

2. It was much easier to adopt more conservative design provisions to ensure against shear failures since the actual shear strength of circular sections was not known precisely, rather than conducting more research.

3. Shear failures in many cases occurred as a result of underestimating the design shear force. During earthquakes, the prediction of the design shear force requires an accurate knowledge of the actual flexural strength. This means that members should be designed for shear forces higher than those theoretically required to develop the maximum flexural strength at the designated locations of plastic hinges. Design for shear corresponding to conservative provisions for flexural capacity, seems to be one of the main reasons for diagonal failures during earthquakes.
7.2 PREVIOUS RESEARCH ON BEAMS WITH CIRCULAR CROSS SECTIONS

7.2.1 Capon and de Cossio (Mexico)

In order to evaluate the shear capacity of circular sections, a test programme\cite{226} consisting of 25 beams was conducted by Capon and de Cossio in Mexico in 1965. The beams were tested in the horizontal position and loaded with either a single load at mid-span or with a load at each of the third span points. The corresponding shear span to depth ratios were 2.4 and 4.2. The majority of the beams were tested under pure bending and without web reinforcement. Two beams were subjected to combined bending moment and axial force. Four beams were provided with circular hoops at different spacings. Longitudinal reinforcement including different percentages and distributions which covered almost all possible practical configurations were investigated.

The authors concluded from the test results that the shear strength is dependent on the concrete strength, the gross sectional area and to a lesser extent on the percentage of longitudinal steel (which was recommended to be uniformly distributed within the members) and the a/d ratio. They also confirmed that expressions proposed by the ASCE-ACI Committee 326\cite{12} were adequate for the evaluation of the shear strength. They proposed the introduction of minor modifications to account for the increased shear strength due to the presence of axial loads. They confirmed with respect to the contribution of stirrups to shear strength that the Mörsch analogy\cite{6} adequately estimated the strength. Their test programme, how-
ever, included only four beams with web reinforcement which may have impaired their findings. Nevertheless, to date this work is still regarded as the basis for shear design for such members in many Codes of Practice[8, 9, 10, 11, 55].

7.2.2 Khalifa and Collins (Canada)

A test programme[227] consisting of five 445mm diameter circular beams was carried out at Toronto University in 1981. The beams were subjected to double bending and an axial loading of 1000 kN. One beam was subjected to load reversals. All beams failed in shear.

Khalifa and Collins reported that the shear strength was 20% higher than that predicted by the ACI-Code equations[8]. They argued that the modified compression field theory[119] gave a more reliable prediction of the shear strength of beams with circular cross sections.

Obviously, due to the small number of test beams, the results of their research cannot be widely applied. The limitations of the compression field theory (refer to Chapter 2) has resulted in the shear provisions given by the Canadian Code of Practice being based on the strut and tie models recently developed in Europe as well as on the compression field theory[2].
7.2.3 Yan, Masahide, and Kenji (Japan)

In order to prevent the diagonal failure of structural concrete members with circular cross sections under seismic loads, a new method called the Super-Reinforcing Method [228] was introduced in Japan 1986. The diagonal failure was prevented by confining the critical regions with steel tubes. The test programme consisted of seven specimens having a shear span to depth ratio of 1.0. The specimens were provided with hoops, cast in steel tubes and then tested by inducing deformation corresponding to double bending. The test specimens were fixed at their ends. Four out of the seven specimens were tested with axial loading. The remaining specimens were subjected to transverse cyclic deformations in order to investigate the behaviour of beams under seismic loading.

The authors concluded from the test results that ductility, shear and flexural strengths, and energy absorption capacities can be significantly improved by confining the critical regions of members in steel tubes.

The proposed method of confinement proved to be very efficient in preventing diagonal failure at critical regions under load reversals. It showed the effect of confinement on enhancing strength, serviceability and ductility of members. However, the research was restricted to a limited number of test specimens and variables.
Twenty five 400mm diameter structural concrete members were tested under cyclic reversals of lateral inelastic displacements[149]. The test variables included the axial load level, the longitudinal and transverse reinforcement ratios and distributions and the a/d ratios which varied between 1.5 and 2.5. The members were tested as vertically positioned cantilevers. A lateral displacement was applied to the free end. Eleven members were also subjected to axial load. The contribution of the concrete to shear strength was measured at the onset of diagonal cracking. The contribution of the transverse reinforcement to shear strength was measured assuming that the contribution of the concrete remained unchanged as the total shear increased, with the balance being carried by the 45° truss action.

The authors concluded from their tests that the shear strength was dependent on the axial load level, the a/d ratio, the amount of transverse reinforcement, and the flexural ductility. They proposed design recommendations and suggested the introduction of modifications to the relationships for shear in the both ACI[8] and New Zealand[11] Codes of Practice.

This research could be regarded as being comprehensive for the prediction of the shear behaviour of members with circular sections during earthquakes. The application of the proposed recommendations is, however, limited to the range of variables investigated in the test programme. It is believed that the findings were impaired by the use of the 45 degree-truss analogy which ignores the influence of
confining stirrups on the contribution of the concrete to the overall resistance to loadings.

7.2.5 Clarke and Birjandi (UK)

A test programme[229] including fifty beams with circular cross-sections was carried out recently (March 1993) at the British Cement Association. Fourteen of the specimens were without web reinforcement. The remaining beams were provided with either links or spirals as shear reinforcement. The variables included the diameter of the beams (152mm, 300mm and 500mm), the concrete strength \( f_{cu} = 25 \text{ MPa}, 35 \text{ MPa} \) and 50 \text{ MPa} \) and the longitudinal reinforcement ratio \( \rho = 0.9\% \) to 5.6\%.

The authors concluded from the results of their test programme that the same approach which is used for the shear design of rectangular sections in BS 8110 can be used for circular sections. The geometrical properties were defined for the circular section as follows:

- the area of tension reinforcement is the area of the steel below the mid-depth of the section.
- the effective depth \( d \) was taken as the distance from the extreme compression fibre to the centroid of the tension reinforcement.
- the term ‘bd’ was taken as the area of concrete from the extreme compression fibre down to the depth ‘d’.
The authors admitted that they did not include all the possible variables in their test programme. They stated that hundreds of tests would be needed for that purpose. The aim of their tests was to compare the actual behaviour of these beams with that of a corresponding rectangular section. They assumed that if good correlation could be obtained, the behaviour of specimens outside the range tested could be predicted with confidence.

7.2.6 Summary and Conclusions

A review of previous research on circular sections [149, 226, 227, 228, 229] and Code provisions [8, 9, 10, 11, 99], resulted in the following points:

1. Most Codes of Practice, do not put enough emphasis on the shear strength of circular sections. They usually relate it to that of an equivalent rectangular section assuming that the empirically based shear relationships are still applicable. Considerable concerns had been expressed by piling specialists, who felt that the current situation was unsatisfactory and that a design method for circular sections had to be developed [229].

2. The enhancing effect of confinement upon the strength and the ductility of concrete was recognised and utilised in the design of columns subjected to seismic loading. However, in the design of beams, the enhancing effect of confinement was overlooked.

3. To explain the mechanism of shear and thus predict the shear strength, a number of theories have been put forward [1, 4, 5, 18, 67, 119]. Despite the
prominence given to these recently developed techniques they have failed to produce a rigorous general solution for all static and geometric conditions. Nevertheless, only a few of these techniques have ever been applied to circular sections[119].

4. Earlier research did not place much emphasis on the examination of the mechanisms of diagonal failures experienced in tests on rectangular sections, however, it was suspected that the main contributor to shear strength at the ultimate limit state was the compression zone[5, 106]. The main aim of the work on members with circular cross sections was to compare their behaviour with that predicted by Codes of Practice with respect to strength and ductility.

5. The research work did not cover the wide range of values applicable to each variable which might affect the behaviour of such a section e.g. the aspect ratio, the axial load level, the percentage and distribution of the longitudinal reinforcement, size effect and the effect of the transverse reinforcement configuration.

6. A considerable amount of the research work has investigated the effect of seismic loading on shear strength. Therefore, the results were confined to their respective practical applications.

7.3 AIMS OF THE PROPOSED TEST PROGRAMME

Historically, most of the diagonal failures in members with circular cross sections occurred during earthquakes. Consequently, previous research has mainly concen-
trated on post-ultimate behaviour. Little emphasis has been put on either serviceability requirements under monotonic loadings or on the examination of the basic structural behaviour of these beams under transverse loading. There is little point in conducting more tests using more specific loading configurations in an effort to refine the existing empirically based shear equations. These equations are empirical and as a result neither explain the shear behaviour, nor adequately predict the shear strength for most geometric and static loading conditions. On the other hand, a better understanding of the basic shear mechanism will assist in predicting the behaviour of members irrespective of the loading conditions thus leading to the establishment of a basis for a rational and unified design method to prevent brittle diagonal failures in all members regardless of the shape of their cross section.

Tests[149, 178] to investigate the effect of confinement on flexural behaviour have shown that the flexural strength of columns subjected to axial loads and bending moments was significantly enhanced by confinement irrespective of the shape of the cross section of the members. However, tests on rectangular beams subjected to only transverse loading have not shown similar improvements (Test Series 'A'). In the case of beams with circular cross sections subjected to only transverse loading, conclusions regarding the effect of confinement upon their flexural strength cannot be made from the available test data.

Therefore, the main objectives of testing beams with circular cross sections in this investigation are:
1. To investigate the contribution of the compression zone to the resistance to transverse loading (shear) thus validating the Compressive Force Path (CFP) concept with respect to the circular section.

2. To determine the role of stirrups in the prevention of diagonal failures.

3. To examine the effect of confinement upon the flexural behaviour of beams.

4. To compare the effect of confining the compression concrete on the behaviour of beams with circular cross sections to that of rectangular beams.

5. To investigate the applicability of the proposed models for the prevention of diagonal failures and for the evaluation of the flexural capacity of beams with circular cross sections.

7.4  TEST SERIES 'G': Influence of confinement on the behaviour of beams with circular cross sections

A series of tests was undertaken on simple beams with circular cross sections which were reinforced with conventional stirrups and stirrups which did not extend down the full depth of the beam. In the latter case, the stirrups had the sole purpose of confining the concrete in the compression region.

7.4.1  Description of Test Beams

A total of eight 200mm diameter structural concrete beams were cast and tested. All the beams had an overall length of 2000mm and an effective span of 1600mm. The shear span to depth ratio was 4.0. The beams are shown in Figure 7.1. Figure 7.2
shows the geometry of the cross section of the beams as well as the stress and strain diagrams used in the design. The only variable investigated was the effectiveness of the lateral reinforcement and hence the beams were identical except for the lateral reinforcement. Eight 12mm nominal diameter longitudinal deformed steel bars with a yield strength of 515 MPa were uniformly distributed around the perimeter of the beams. The stirrups were fabricated from 8mm nominal diameter plain round mild steel bars with a yield strength of 377 MPa. Circular and a combination of circular and semi-circular shaped stirrups were used for the traditionally detailed beams and for beams detailed to confine the concrete compression region respectively. For some beams, the spacing of the stirrups, which was based on the provisions of BS 8110, was reduced by 50% in order to investigate the effect of confinement on the strength and the ductility of the beams.

It can be deduced from the above description of the test beams that only the design of the traditional beams was required. All of the other beams were detailed in accordance with the above design procedure and the shape of the CFP. The design approach for the traditional beams is given in Appendix D.

7.4.2 Experimental Work

The concrete mix used in the investigation is given in Table 7.1 and the resulting compressive strengths for the various beams are given in Table 7.2. The beams were cast in vertically positioned PVC tubes. Two beams were cast from each batch of concrete. The beams and the control cubes were cast, compacted and remained in
their tubes/moulds in the laboratory under ambient conditions until testing. The top cast surfaces of the specimens were covered with damp hessian over which polythene sheeting was tightly wrapped. The beams were removed from the PVC tubes prior to testing and whitewashed for early identification of cracks under loading.

A more detailed description of the programme of experimental work is given in Appendix A.

7.4.3 Test Results

The measured loads were corrected in order to allow for variations in the materials and the dimensions using the same method as applied in Test Series ‘A’ The corrected measured loadings are shown in Table 7.2. Table 7.3 gives the values of the crack widths at 50% of the ultimate load, ultimate load, and when the mid-span deflection reached a value of 60mm. Figure 7.3 shows the loading arrangement which was used in the testing of the beams with the circular cross sections. Figure 7.4 shows the typical crack patterns for the following three types of beams just before removal of the load.

- Beams unreinforced for shear, beam type B1.

- Beams conventionally reinforced for shear, beam type B2.

- Beams detailed to provide confinement to the compression concrete, beam type B6.
The load-deflection curves show the maximum load capacity, stiffness, and ductility of the beams. The test results have been divided into two groups for ease of discussion. However, for comparative purposes, beam type B1, which was not reinforced for shear, was included in the two groups. The first group, Figure 7.5.a, consisted of beams having conventional circular shaped stirrups in the shear span region (beam type B2) and the other beams in which the stirrups were also placed in the mid-span region (beam types B3 and B4). The second group, Figure 7.5.b, consisted of beams in which the CFP was confined with reinforcement (beam types B5 and B6). Finally, Figure 7.5.c presents comparative results for all beams. Testing of beam types B4-B7 was stopped when the mid-span deflections exceeded a value of 80mm on the grounds of safety.

7.4.4 Discussion of Test Results

(a) Modes of Failure

Beam type B1 which was unreinforced for shear behaved as expected. A diagonal crack developed in the shear span, resulting in the failure of the compression zone near to the loading point. In all the other types of beams, the diagonal cracks extended up to the compression region and were prevented from propagating further by the presence of the transverse reinforcement. Flexural failures occurred in beam types B2-B7 at load levels at least equal to the anticipated maximum load capacity of the section and were accompanied by large deflections and spalling of the compression zone in the mid-span region. Figure 7.4 shows typical crack patterns for the various beams at failure. In the case of the beams in which the spacing of the
transverse reinforcement was 50mm spalling of the concrete cover took place after the ultimate load was reached. In the case of these beams, it was possible to see the concrete arches[154] which had formed between the circular stirrups. On the other hand, in the beams in which the spacing of the stirrups was 100mm, the compression steel buckled and spalling of the concrete in the compression zone occurred because of the lack of confinement due to the relatively large spacing of the stirrups.

In accordance with the concepts underlying conventional shear design, the failure mode of beam types B5 and B6 should have been similar to that of beam type B1. However, the test results indicated that the diagonal cracks in the two beams were restrained near the neutral axis. Both beams exhibited strengths and ductilities which were either similar to or greater than those found in the conventionally detailed beam type B2. The widths of the flexural and the diagonal cracks in beam types B5 and B6 satisfied the crack width requirements of the serviceability limit state. They were similar to those found in the conventionally detailed beam type B2 at 50% of the ultimate load, Table 7.3. The combination of circular and semi-circular shaped stirrups in beam type B7 succeeded in significantly reducing the diagonal cracks as shown in Table 7.3.

(b) Load Capacity and Ductility

Beam type B1 had an ultimate load carrying capacity of 75 kN (76% of the full flexural capacity of beam type B2), which compares closely with the results from Kani’s Valley[18]. The ultimate shear capacity, as predicted by the Code provisions,
was expected to be 65.8 kN \((=1.25v_{cd})\). Alternatively, if the calculated strength was based on the model developed by Bobrowski et al.\[87\] the predicted capacity would have been 65.7 kN.

The conventionally detailed beam type B2, reached an average transverse load of 98.6 kN. This was 18\% higher than the flexural load capacity predicted by the Code of Practice.

All beams exhibited a typical flexural ductile behaviour except beam type B1 which was unreinforced for shear. Beam type B3 had a similar strength to beam type B2. However, when the spacing of the stirrups was reduced to 50mm in beam type B4, the resulting strength was 17\% higher. Beam types B3 and B4 had ductilities at least one and a half times that exhibited by beam type B2. The enhanced strength exhibited by beam type B4 can only be attributed to the closer spacing of the transverse reinforcement. Similarly, the increased ductility is attributed to the presence of the transverse reinforcement in the mid-span region[106]. The strengths and ductilities of beam types B1-B4 are illustrated in Figure 7.5.a.

Beam types B5 and B6, based on the present Code provisions, were expected to have the same strength as beam type B1 since the truss structure required to transfer the "shear" load to the support could not have been present. The test results have indicated that those beams achieved higher strengths i.e. 1.29 and 1.74 times the corresponding strength of beam type B1 respectively. The two beams exhibited
high ductilities before failure i.e. approximately four times that exhibited by beam type B1. The results from the tests on beam types B1, B5 and B6 are summarised in Figure 7.5.b.

The comparative results for beam types B1-B7 are summarised in Figure 7.5.c. The comparison shows that for beams in which the transverse reinforcement was spaced at 100mm, regardless of the stirrup configurations, strengths were higher than those predicted by the Code of Practice. The ductility was significantly higher when the transverse reinforcement was extended across the mid-span region. In the case of the beams in which the transverse reinforcement was spaced at 50mm, the strength and ductility were enhanced significantly. The flexural strength was increased by up to 1.36 times the strength of beam type B2. In similarly detailed rectangular beams (Test Series ‘A’) the increases in strength were found to be insignificant. However, in the case of the rectangular section, the tensile force in the longitudinal bars was limited \( T = \Sigma A_s f_y \). The increase in the lever arm \( (jd) \) between the internal forces (the tensile force in the longitudinal bars and the compression force in the concrete) was small and the corresponding increase in the flexural capacity \( T \cdot jd \) was insignificant. On the other hand, for the circular section, the ductility which was found allowed an increase in the tensile forces in the second and third layers of the longitudinal bars from the upper face of the section. This resulted in an increase in the tensile forces \( T = \Sigma A_s f_y \) thus leading to a significant increase in the flexural capacity of the section. This explanation shows why beam types B6 and B7 exhibited large increases in their flexural capacities (32% and 36% respectively)
when compared to the corresponding value from beam type B4 (17%) as shown in Table 7.2. The second layer of longitudinal reinforcement from the top in these beams was 34mm lower than the corresponding layer in beam type B4. This resulted in an increase in both the lever arm and in the tensile force in that layer thus leading to the relatively large increases in the flexural capacities of beam types B6 and B7.

7.4.5 Contribution of Compression Zone to Shear Capacity

The detailing used in beam types B5 and B6 tests the hypothesis that the compression zone plays a more significant role than is normally assumed in resisting shear forces. These beams achieved strengths and ductilities at least comparable to those found in beam type B2 which was reinforced in accordance with Code provisions.

The detailing used in beam types B5 and B6 assumes the same mechanism for resisting moment and shear, irrespective of the level of loading. When the concrete capacity is exceeded, the transverse reinforcement is intended to enhance the strength of the concrete in the compression zone, which is the main element in the beam structure which resists the imposed loadings (axial, shear and bending moment). The adoption of the same approach for the provision of the resistance to applied loads would lead to a unified design approach for structural concrete members.

The analytical work presented in Appendix D was based on the proposed models for the prevention of diagonal failures and for the evaluation of the flexural capacity of beams due confinement, presented in Chapters 3, 4, and 6. The results of this
work have shown that the volumetric ratio of the transverse reinforcement, which was based on the design recommendations in the Code of Practice[9], was sufficient to offset the reduction in concrete strength ($\Delta f_c$), but was not sufficient to result in an increase in flexural capacity (beam type B3) as shown in Table 7.4. The spacing of the stirrups in this beam was 100mm, which is larger than the recommended spacing required to prevent buckling of the longitudinal bars ($6d_o[223]$). Indeed, the longitudinal bars buckled and initiated a failure thus leading to the conclusion that the confinement provided by the stirrups was ineffective. However, when the spacing was reduced to 50mm in beam types B4, B6 and B7, the amount of transverse reinforcement was high enough to increase the ductility and compressive strength to levels above the unconfined strength thus resulting in an increase in the flexural capacity of the beams. For example, beam types B4 and B6 had flexural capacities 17% and 32% higher than beam type B2 which attained its full flexural capacity. The corresponding ratios predicted by the proposed model were found to be in close agreement (15.7%, and 30.8% respectively).

Therefore, the rigorous design of a beam structure should take into consideration the interactive relationship between shear and bending moment in order to prevent possible brittle diagonal failures during earthquakes.

7.4.6 Conclusions

1. The uncracked concrete in the compression regions makes a larger contribution to shear resistance than is usually assumed by Codes of Practice, particularly
for beams with web reinforcement. Confinement of the compression region in beams with a circular cross section was found to increase the capacity of the beam structure to resist transverse (shear) loadings. The reduction in the flexural capacity due to shear stresses is offset by the increase in strength of the compression concrete due to confinement.

2. Confinement of the compression zone in the mid span region of beam types B3-B7, where there was no variation in the bending moment, increased the ductility of the beams by at least one and a half times compared to that of beam type B2 which was detailed in accordance with current Code provisions. Also, in the case of beam types B4, B6 and B7 where the spacing of the transverse reinforcement was reduced to 50mm, the flexural capacity increased on average by 28% compared to the results obtained from beam type B2. Such a significant increase in flexural carrying capacity has not been found in tests conducted on equivalent rectangular shaped beams.

3. The detailing approach used in beam types B5 and B6 which had previously been found to prevent diagonal failures in rectangular beams, was also found to be applicable to beams with circular cross sections. The respective strengths of beam types B5 and B6 were found to be either similar to or higher than the strength obtained from beam type B2. The ductility of beam types B5 and B6, was also at least one and a half times that found in beam type B2.

4. The proposed flexure-shear interaction model adequately predicted the amount of transverse reinforcement required to ensure a ductile flexural failure in the
beams. Also, according to the proposed method for the evaluation of the flexural capacity of the member due to confinement, the prediction of the enhanced flexural capacity resulting from confinement compared favourably with the experimental results obtained from the laboratory based test programme.

5. The development of truss action as assumed in Codes of Practice is not necessarily required to transmit lateral loading (shear) to the supports. It is concluded that the circular stirrups resisted shear loads as a result of the confinement they provided to the compression region. They thus restrained the development of the transverse tensile stresses which cause failure and also compensated for the reduction in the concrete compression strength. However, the circular stirrups proved to be more effective than the semi-circular stirrups in restraining the growth of diagonal cracks. Circular stirrups will also be necessary to provide the confinement required for members subjected to load reversals.

7.5 SUMMARY

In this chapter the concept of preventing diagonal failures in beams by confining the concrete compression regions with closed stirrups were validated experimentally for beams with circular cross sections. The test results obtained were also used to prove the applicability of the proposed flexure-shear interaction design model and the proposed method for the evaluation of the flexural capacity due to confinement of the circular section. The effects of other factors which usually influence the
behaviour of structural concrete members with a circular cross section such as the concrete strength, the presence of axial loads, etc, require further investigation.
<table>
<thead>
<tr>
<th>Material</th>
<th>Weight (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>429</td>
</tr>
<tr>
<td>Water</td>
<td>235</td>
</tr>
<tr>
<td>Sand</td>
<td>798</td>
</tr>
<tr>
<td>Coarse Aggregate (10mm)</td>
<td>798</td>
</tr>
<tr>
<td>Water/Cement ratio</td>
<td>0.55</td>
</tr>
<tr>
<td>Slump</td>
<td>60-180 (mm)</td>
</tr>
</tbody>
</table>

Table 7.1: Concrete Mix Constituents used in Test Series 'G'.
### Table 7.2: Test Results.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Concrete Strength 'f&lt;sub&gt;cu&lt;/sub&gt;' (MPa)</th>
<th>Beam size (mm)</th>
<th>Correction factor (F)</th>
<th>Corrected load (P&lt;sub&gt;r&lt;/sub&gt;) (kN)</th>
<th>( \frac{P_{r}}{P_{trad}} \times 100 )</th>
<th>( \frac{P_{r}}{P_{theo}} \times 100 )</th>
<th>% reduction in load on completion</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>43.46</td>
<td>194.0</td>
<td>1.0134</td>
<td>75</td>
<td>76</td>
<td>100</td>
<td>90</td>
</tr>
<tr>
<td>B2*</td>
<td>36.87</td>
<td>194.85</td>
<td>1.0720</td>
<td>98.6</td>
<td>100</td>
<td>118</td>
<td>33.5</td>
</tr>
<tr>
<td>B3</td>
<td>30.22</td>
<td>192.7</td>
<td>1.2023</td>
<td>97.4</td>
<td>99</td>
<td>117</td>
<td>16</td>
</tr>
<tr>
<td>B4</td>
<td>33.05</td>
<td>195.7</td>
<td>1.1236</td>
<td>115.7</td>
<td>117</td>
<td>138</td>
<td>4</td>
</tr>
<tr>
<td>B5</td>
<td>43.46</td>
<td>195.9</td>
<td>1.0027</td>
<td>96.3</td>
<td>98</td>
<td>115</td>
<td>23</td>
</tr>
<tr>
<td>B6</td>
<td>33.05</td>
<td>193.0</td>
<td>1.1444</td>
<td>130.5</td>
<td>132</td>
<td>156</td>
<td>1</td>
</tr>
<tr>
<td>B7</td>
<td>30.22</td>
<td>193.0</td>
<td>1.1990</td>
<td>134.3</td>
<td>136</td>
<td>160</td>
<td>-2(rise)</td>
</tr>
</tbody>
</table>

† \( P_{trad} \) is the corrected measured load for beam type B2.

‡ \( P_{theo} = 83.8 \) kN is the calculated flexural load for the assumed dimensions and material properties.

* Average of two identical beams.
<table>
<thead>
<tr>
<th>Beam</th>
<th>Crack width (mm)</th>
<th>Defl. (mm)</th>
<th>Crack width (mm)</th>
<th>Defl. (mm)</th>
<th>Crack width when midspan defl.=60mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flexure</td>
<td>Diagonal</td>
<td>Flexure</td>
<td>Diagonal</td>
<td>Flexure</td>
</tr>
<tr>
<td>B1†</td>
<td>0.2</td>
<td>4</td>
<td>0.8</td>
<td>0.3</td>
<td>12</td>
</tr>
<tr>
<td>B2</td>
<td>0.2</td>
<td>0.05</td>
<td>5</td>
<td>3.0</td>
<td>0.2</td>
</tr>
<tr>
<td>B3</td>
<td>0.2</td>
<td>0.05</td>
<td>5</td>
<td>2.3</td>
<td>0.2</td>
</tr>
<tr>
<td>B4</td>
<td>0.2</td>
<td>0.05</td>
<td>5</td>
<td>1.9</td>
<td>0.2</td>
</tr>
<tr>
<td>B5</td>
<td>0.25</td>
<td>0.05</td>
<td>6</td>
<td>1.3</td>
<td>0.7</td>
</tr>
<tr>
<td>B6</td>
<td>0.3</td>
<td>0.1</td>
<td>6</td>
<td>1.7</td>
<td>0.5</td>
</tr>
<tr>
<td>B7</td>
<td>0.25</td>
<td>0.05</td>
<td>6</td>
<td>1.5</td>
<td>0.2</td>
</tr>
</tbody>
</table>

† Beam type B1 failed at a mid-span deflection of 19mm.

Table 7.3: Crack History.
The second layer of longitudinal bars was positioned at a distance of 106.5mm from the compression face of the beam (see Figure 7.1).

Table 7.4: Enhanced Flexural Capacities.
Figure 7.1: Detailing of the Beams.
Figure 7.2: Cross-Section of the Beams.

Figure 7.3: Loading Arrangement.
Figure 7.4: Crack Patterns and Failure Modes.
Figure 7.5: Load-Midspan Deflection Curves for: (a) Beam Types B1, B2, B3 & B4.
(b) Beam Types B1, B5 & B6. (c) All Beams in Test Series ‘G’.
Chapter 8

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

8.1 CONCLUSIONS

8.1.1 Current Design Approaches to Structural Concrete Beams

(a) Flexure Design

1. In the absence of stirrups the flexural failure of beams occurs as a result of the development of a multiaxial state of stress resulting from dilation of the concrete in a localised region within the concrete compression zone. However, it is considered sufficient for practical purposes to assess the flexural capacity on the basis of the simplified rectangular stress block given by Codes of Practice.

2. Test results[133] have indicated that confinement of the compression concrete with closed stirrups improves the ductility of beams. Furthermore, it has been shown that it is possible for over-reinforced beams to fail in a ductile manner. Therefore, it is concluded that the limitations on the longitudinal reinforcement ratio ($\rho$) imposed by Codes of Practice (e.g. in the ACI Code of Practice $\rho \leq 0.75\rho_b$) are too restrictive when confinement is present.
(b) **Design of Beams Under the Combined Action of Shear and Flexure**

1. At present, there is an urgent need for the establishment of an unified approach for the analysis and design of the entire range of structural concrete members.

2. Rational design models for the prevention of a diagonal failure should account for the reduction in the flexural capacity of beams due to the influence of shear. The determination of the amount of stirrups required to prevent diagonal failures in beams should be related to the magnitude of the flexural capacity of the beams (flexural-shear interaction design approach).

3. The relative ultimate flexural capacity of beams \( \left( \frac{M_u}{M_f} \right) \) is a more realistic indicator of the load carrying capacity of beams at diagonal failure than the nominal shear strength \( \left( \frac{V}{b_d} \right) \). It should thus be used as the basis for design models for the prevention of diagonal failures.

### 8.1.2 Flexural Behaviour of Beams with Confinement

A method for the evaluation of the flexural capacity of beams in which confinement is present has been proposed in Chapter 6. The method was extended to the design of over-reinforced beams in order that they could be made to behave in a ductile manner. The applicability of the proposed analytical methods were investigated experimentally on beams with rectangular cross sections (Test Series ‘A’ and Test Series ‘F’) and beams with circular cross sections (Test Series ‘G’). The conclusions from this part of the investigation are given below:
1. It is possible to design over-reinforced beams so that they fail in a ductile manner and achieve their full flexural capacity.

2. The flexural capacity of under-reinforced beams is mainly influenced by the characteristics of the longitudinal reinforcement rather than the degree of confinement of the compression concrete. On the other hand, for over-reinforced beams the confinement characteristics have a major influence on behaviour.

3. The ductility of all types of beams increases as the confinement is increased.

4. The serviceability of over-reinforced beams satisfied Code requirements.

5. As a result of confinement a new longitudinal reinforcement ratio ($\rho_b'$) under balanced-failure conditions was found to exist. A unique relationship between $\rho_b'$ and the value given by Codes of Practice ($\rho_b$) was found to be a function of the confinement characteristics.

6. The prediction of the flexural capacity resulting from confinement based on the proposed method for all types of beams which have been investigated was found to be reasonably accurate.

8.1.3 Flexure-Shear Interaction Design Model

The development of the proposed flexure-shear interaction design model for structural concrete beams was described in Chapters 3 and 4. The model was then verified experimentally using normal-size beams (Test Series 'C'), large beams (Test Series 'D'), beams made from high strength concrete with cube strength up to 80 MPa
(Test Series 'E') and beams with circular cross section (Test Series 'G'). The main conclusions obtained from this part of the research programme is given below:

1. The loads applied to beams are transmitted to the supports along a compressive force path which consists of horizontal and inclined leg regions. The actual structural behaviour of beams under the action of lateral loading is influenced by the critical state of stress which exists in either the horizontal or the inclined leg regions. In type II beams, the most critical stress condition exists in the horizontal leg region. However, restraining these stresses by confining only the horizontal leg with closed short stirrups forces the critical state of stress to occur in the inclined leg regions. Confinement of both leg regions is therefore required in order to prevent diagonal failures in these types of beams. On the other hand, the critical tensile-compressive state of stress exists in the inclined leg regions in the case of type III beams. The confinement of the inclined leg regions (shear spans) alone enables beams to achieve their full flexural capacity.

2. The proposed flexure-shear interaction model is more realistic because:

(a) The model is based on only one load carrying mechanism i.e. when the capacity of the concrete alone is exceeded the confining stirrups are intended to enhance the strength of the compression concrete which is the principal resisting element.

(b) The model was derived from a better understanding of the behaviour of concrete at the material level and of the actual behaviour of structural
concrete beams. It is recognised that failures in beams occur as a result of the development of transverse stresses in the concrete compression regions. Therefore, the presence of confining stirrups in this region will restrain these stresses and delay failure.

(c) The load carrying capacity of beams without stirrups are determined from the relative flexural capacity of beams \( \frac{M_u}{M_f} \) rather than the nominal shear strength \( \mu = \frac{V}{b_d} \) which was considered not to give a true representation of the load carrying capacity of beams at diagonal failure.

3. The confined concrete compression regions in the beam structure make a larger contribution to the load carrying capacity of beams than is normally assumed in traditional design methods. It was confirmed in this investigation that the confined concrete compression region is the main contributor to the resistance of beams to applied loading. The diagonal failure of the beams was prevented by confining the concrete compression regions, as defined by the compressive force path concept, with closed stirrups. The increase in the concrete compressive strength resulting from the presence of the confining stirrups was aimed at offsetting the reduction in the load carrying capacity of beams because of the presence of shear. Therefore, the presence of truss action assumed by Codes of Practice is not necessarily required to enable beams to carry loads in excess of the capacity of the concrete.

4. The proposed flexural-shear interaction design model which is based on a single behavioural mechanism and takes into consideration the interactive relation-
ship between shear and flexure, adequately predicted the amount of transverse reinforcement required to ensure ductile flexure behaviour in the range of beam types included in this programme of research.

5. The serviceability of the beams designed and detailed based on the proposed approach was similar to that obtained from the traditionally designed and detailed beams.

6. The proposed flexure-shear interaction design model resulted in a more economical design for the following reasons:

(a) The use of short stirrups in the horizontal leg regions.

(b) The amount of stirrups required for the majority of the beams, particularly that required for the inclined legs, is normally smaller than that required by the traditional design approaches.

(c) The anticipated reduction in the partial safety factors resulting from the increased confidence in the design approach.

7. The design approach proposed for the prevention of the diagonal failures in beams is not restricted to the adoption of the detailing arrangement. It can readily be used in situations in which conventional full length stirrups are more practical. The confinement requirements in that case are calculated using the corresponding effectively confined area provided by the conventional stirrups.
8.1.4 Miscellaneous

1. The proposed new detailing arrangement is anticipated to be more efficient in the case of the upgrading and the maintenance of existing structural concrete members such as beams or slabs in existing bridges. The approach could be applied to such structures to enable them to sustain more demanding loading levels. In this case, the enhancement of the major parts of the members (the horizontal leg regions) can easily be achieved since only short stirrups need be provided. It is only the inclined leg regions which require treatment over the full depth of the members.

2. The confinement model developed by Sheikh et. al. [178] which was derived from results of tests on columns was modified to make it applicable to beams. The prediction of the results obtained from beams in this research programme based on the modified confinement model was found to be reasonably accurate.

8.2 RECOMMENDATIONS FOR FUTURE WORK

1. In order to implement the flexure-shear interaction design model it was necessary to evaluate the ultimate relative flexural capacity for the horizontal and the inclined leg regions \( \frac{N_f}{M_f} \). The leg capacity curves, Figures 4.3 and 4.4-Chapter 4, were based on a limited number of beam tests. It is therefore recommended that a more general model for the determination of the relative ultimate flexural capacity of the legs \( \frac{N_f}{M_f} \) is developed. The model could be developed either theoretically based on the CFP concept, or experimentally
using a statistical analysis of large number of beam tests covering all the major factors.

2. The effects of axial and prestressing forces on shear strength can be related to the effect these have on controlling the state of stress which exists in the region of the compressive force path. Axial tensile forces adversely affect the critical state of stress but axially applied compression or prestressing forces enhance the state of stress. It is recommended that the implication of the presence and the magnitude of axial loads on the proposed flexure-shear interaction model, particularly for circular sections which are usually required to carry axial loads is investigated. It is also recommended that the models which have been developed are extended to prestressed concrete beams.

3. Currently, high-strength concrete with cube compressive strengths exceeding 150 MPa are available for use in the construction industry[215]. The applicability of the flexure-shear interaction design model was investigated for concrete strengths up to 80 MPa. It is therefore recommended to investigate the applicability of the model for concrete with a higher compressive strength.

4. Highly over-reinforced beams were included in Test Series 'F'. The results obtained from these beams indicated that a significant reduction in capacity may occur as a result of cracking of the concrete cover. It is therefore recommended that further research work is conducted in order to impose limitations on the size of the unconfined concrete cover in comparison with the size of the confined core.
5. The proposed detailing approach for stirrups is anticipated to have a significant advantage over the traditional approaches to detailing in the maintenance and upgrading of existing structural concrete members. It is recommended that the feasibility of enhancing the strength and ductility of existing structural concrete elements to enable them to carry more demanding loading levels is studied. The underlying approach to this particular problem is in the possibility of casting a new concrete layer to act as a compression zone in the beam structure. In this case, only the inclined leg regions require major modification.

6. Codes of Practice normally place a limit on the maximum applied shear stress \( v = \frac{V}{M} \) in order to prevent web crushing in beams e.g. the provisions of BS 8110 limit \( v \) to the smaller of either \( 0.8\sqrt{f_{cu}} \) or 5.0 MPa. The enhancing effect on the concrete compressive strength and the ductility which results from confinement of the concrete web with closed stirrups is generally ignored. In this investigation the shear stress in several of the beams with web reinforcement (e.g. beam type A2 in Test Series 'C') exceeded the maximum value allowed by the Code of Practice by more than 30% but they still failed in flexure. It is recommended that a more general relationship is developed for the concrete compressive strength in the webs of the beams which accounts not only for the adverse effects of the cracks but also for the possible enhancement resulting from confinement.
Appendix A

EXPERIMENTAL PROCEDURES

A.1 MATERIALS

The quality and performance of concrete depends to a large extent on the characteristics and proportions of its constituent materials. It was therefore important that the quality of the material remained consistent throughout this programme of research. A brief description of all the materials used is included in the following sections.

A.1.1 Cement

Ordinary Portland cement (Blue Circle) was used throughout the investigation. The cement was stored in bulk in a purpose built silo with a capacity of 10 tonnes.

A.1.2 Aggregates

Washed and dried uncrushed coarse and fine aggregates were used. The different sizes of the aggregates were stored in bulk in an air-dry condition in separate compartments inside the concrete batching plant.
(a) Coarse Aggregates

The uncrushed coarse aggregates which were used had a maximum size of either 10mm or 20mm and were rounded in appearance. The aggregates comprised particles from a number of rock sources and were free from any impurities which might have adversely affected the bond between the aggregates and the cement paste. The concrete mix design method[230] used did not require the grading of the coarse aggregates. However, by inspection it was possible to judge that they were well graded.

(b) Fine Aggregates (Sand)

The fine aggregate was locally available washed concrete sand. Details of the grading of the sand used is shown in Table A.1. The results from the sieve analysis indicated that the sand conformed to a zone 2 classification[230].

A.1.3 Reinforcement

Two types of reinforcement: high tensile strength deformed bars with nominal diameters of 12, 20, 25, and 32mm were used for the longitudinal reinforcement and plain round mild steel bars with nominal diameters of 8 and 10mm were used for the stirrups and the secondary reinforcement.

In order to determine the tensile strength of the reinforcing bars, at least three tensile tests were carried out on each type of steel in accordance with the procedure detailed in BS EN 10 002 : 1990[231]. Typical results from these tests are shown in

347
Table A.2. The resulting average yield strengths were used in the design calculations. Typical stress-strain curves for the mild steel (8mm nominal diameter) and the high strength (25mm nominal diameter) steel bars are shown in Figure A.1. To obtain the stress-strain curve for the large diameter steel bars (nominal diameters larger than 12mm), two 10mm electrical resistance strain gauges were bonded to each bar. A typical 25mm nominal diameter steel bar under such a test is shown in Figure A.2.

A.1.4 Storage Conditions

When the coarse and fine aggregates were delivered they were passed through a fluidised bed drier to remove any moisture. The aggregates were then stored in separate compartments inside a covered silo which was also used to store the cement. The reinforcing steel bars were stored on racks inside the laboratory.

A.2 CONCRETE MIX DESIGN

A.2.1 Normal-Strength Concrete (NSC)

A recognised concrete design method[230] was used to determine the required proportions of the mix constituents. Two trial mixes were normally made to confirm the design for each of the required concrete strengths ($f_{cu}$). The slump was measured in accordance with BS 1881 Part 102 and a total of approximately eighteen 100mm cubes were cast, cured, and tested after 3, 7, and 28 days in accordance with BS 1881 Parts 108 and 116 for each trial mix. The cube test results indicated that the control on quality was acceptable for the majority of the concrete mixes.
The mixes used in the investigation were designed to give an average cube compressive strength at 28 days ($f_{cu}$) equal to the specified strength i.e. the target mean strength was taken to be equal to the characteristic strength.

The mix proportions used, the results from the slump tests, and the compressive strengths obtained from the control cubes and cylinders for each beam in all the test series are detailed in the corresponding sections in the thesis.

**A.2.2 High-Strength Concrete (HSC)**

The design compressive strength of the concrete cubes for all the HSC beams was 80 MPa. The level of strength required in the mixes was achieved using Ordinary Portland cement, a very low water-cement ratio (0.28), and careful quality control in the production of the mix. The required workability was achieved using a water-reducing admixture i.e. a superplasticiser. The superplasticising admixture used was CONPLAST 430 which complies with BS 5075 and ASTM C4994 Type F and was supplied by FOSROC Limited.

The design method outlined in the Concrete Society Current Practice Sheet Nos. 94[232] and 95[233] was used for the design and production of the trial mixes. Six 100mm cubes and six (6 inch (152mm) × 12 inch (305mm)) cylinders were cast and stored in water at constant temperature of approximately 21° C until testing. The measured slump was 110mm. The control specimens were either tested after 14
days or after 28 days. The mix constituents were subsequently modified to achieve the required strength ($f_{cu} = 80 \ MPa$). The actual concrete mix proportions used in the high strength concrete beams are detailed in the relevant section in the thesis.

A.3 MANUFACTURE OF THE SPECIMENS

A.3.1 Steel Cages

(a) Rectangular Beams

High-yield strength deformed steel bars were used as main reinforcement in the beams. Normally either two 8mm or 10mm diameter mild steel bars were placed in the concrete compression zone to assist in the assembly of the reinforcement cage and not to contribute to the flexural capacity of the beams. The steel bars were cleaned to remove any traces of oil, paint, or loose scale i.e. surface rust, in order not to weaken the bond with the concrete. To prevent anchorage failure of the longitudinal steel bars, the beams were extended beyond the supports. The end zones of the beams were reinforced with either 8mm or 10mm diameter mild steel stirrups. The stirrups were securely tied to the main reinforcement bars using soft wire ties. Plastic spacers were fixed on either the main bars or on the stirrups to avoid any movement of the reinforcement cage during compaction of the concrete and to ensure that the required cover distances were maintained. Typical steel cages are shown in Figure A.4. The details of the individual steel cages are included in the relevant sections throughout the thesis.
(b) **Beams with Circular Cross-Sections**

The longitudinal steel bars were cut and bent in the normal way. The circular shaped stirrups were made from a single bar. The ends were overlapped and welded together. The semicircular shaped stirrups were made from one circular and one straight steel bar. The straight bar was bent at the ends, overlapped, and welded to the semicircular bar in order to prevent anchorage failure.

Two circular wooden templates were used to assist in the assembly of each steel cage in order to ensure that the longitudinal bars were placed in the correct position inside the steel cage. Slightly oversized holes were drilled around the perimeter of the templates into which the longitudinal bars were placed in their correct position and fixed to the stirrups.

**A.3.2 Mixing**

Concrete was mixed in a Liner Cumflow (pan) mixer with a maximum capacity of approximately 0.3 cubic metre. The required amounts of the constituent materials were weighed and then fed into the mixer in the sequence detailed below. The required quantity of water was weighed and added to the mix by hand. The following sequence was used in the production of the concrete mixes:

1. Coarse and fine aggregates were mixed for one minute approximately.

2. Cement was added, and mixed for a further minute.
3. Tap water was added and mixing continued for about 2 minutes. (mixing was in fact continued until the concrete was of uniform consistency and colour). Some of the water was added prior to this stage in the case of very large mixes in order to ease the mixing operation.

The required amount of the superplasticiser was added to the mix water for the high strength concrete.

A.3.3 Casting

(a) Casting of Rectangular Beams

The beams were cast in shutters made from structural steel channel sections. The inside surfaces of the shutters, were lightly coated with a layer of release agent before casting. The cage was then placed inside the shutter. Plastic spacers were used to ensure the required cover spacing was obtained.

Each beam together with its own set of control specimens (cubes and/or cylinders) normally required only one batch of concrete. A concrete batch was sufficient to cast two of the small-size beams. Four batches of concrete were used to cast the large beam (230mm x 700mm x 6300mm) which was included in Test Series 'D'. The consistency of the concrete used to cast the large beam was achieved by ensuring good quality control with respect to the batching and mixing of the mix constituents also by monitoring the resulting workability of the individual mixes. In addition, four control specimens (two cylinders and two cubes) were taken from each of the four batches of concrete used to cast the large beam in order to determine the re-
sulting strength of each batch of concrete.

In general, the beams were cast with the compression face of the concrete at the base of the shutter. This was to ensure that any bleeding of the concrete, etc did not occur in the concrete compression region. The concrete was placed in the shutters in two layers. Each layer of concrete was compacted using a poker vibrator. The vibration was terminated when air bubbles stopped appearing at the top surface of the concrete. Three control cubes and cylinders were normally cast with each test beam. The control specimens were compacted using a standard electrically operated vibrating table for a period of approximately 90 seconds.

(b) Casting of Beams with Circular Cross Section

The beams with a circular cross section were cast in the vertical position inside 200mm nominal inside diameter PVC tubes with a wall thickness of approximately 3mm. The base of each tube was sealed using wooden blocks to prevent the escape of the concrete mortar. A highly workable mix was used to facilitate casting (slump ranged from 110mm to 145mm). Each batch of concrete was shared equally between two beams. The concrete was placed in through the top of each tube and compacted in 5 layers. These beams and the control specimens were compacted using a similar procedure to that adopted for the rectangular beams.
A.3.4 Curing

The beams and the control specimens were cast and stored in their moulds inside the laboratory under ambient conditions. They were normally covered with damp hessian over which polythene sheeting was tightly wrapped.

A.3.5 Preparing the Specimens for Testing

The beams were removed from the shutters before testing and whitewashed to enable the early identification of cracks under loading. A grid consisting of horizontal lines and vertical lines was drawn on the surface of each beam to act as a reference for the cracks.

A.4 TESTING

A.4.1 Testing Machine

All beams were loaded using a servo-controlled universal test machine which had a vertical load capacity of up to 2000 kN with a resolution of 1 kN. The total load applied was displayed on a digital indicator on the control panel of the test machine. A Tonipact 3000 cube crushing machine which has a minimum vertical load capacity of 3000 kN with a resolution of 1 kN was used to test the control cubes and cylinders.

A.4.2 Loading Arrangement

The test beams were loaded using either a four point or a three point loading arrangement. The load was distributed from the test machine to the test beams using
a structural steel spreader beam. A hinge, designed to ensure that the load was centrally applied and to permit rotation of the spreader beam was used at the connection point between the test machine and the spreader beam.

The loading arrangement used in the tests on the beams with a circular cross section differed slightly from that used in the case of the rectangular shaped beams.

(a) Loading Points for the Rectangular Beams

The beams were supported at one end on an assembly consisting of a roller sandwiched between two steel plates. The width of the plate which was in contact with the beam was 100mm in order to prevent bearing failures in the concrete. At the other end a rocker arrangement was used to prevent accidental lateral movement of the beams, however, the bearing plate was free to rotate during testing. The localised areas of the beams which were in contact with the loading and the support assemblies were normally smooth enough to permit the plates to be in direct contact with the beams. In cases where this was not possible an epoxy resin free-flow grout was placed between the plates and the beams. The testing machine and the loading arrangements used to test rectangular beams are shown in Figure 5.3 in Chapter 5.

(b) Loading Points for the Beams with Circular Cross Section

The test beams were supported at the each end on steel saddles which were designed to prevent localised bearing failures. The saddles were clad with a short length of the PVC tubing which had been used in the casting of the beams to ensure a smooth
contact surface. At one end, the saddle was placed on a roller which was welded to a base plate resting on a plinth. The other support was similarly detailed, except that the roller was not welded to the base plate. Similar arrangements were used at the loading points. The loading arrangements for the beams with a circular cross section is shown in Figure 7.3 in Chapter 7.

The loading arrangement described above, permitted rotational and small lateral movements to take place at the supports and the loading points thus ensuring equal load distribution and stability of the test arrangement.

A.4.3 Loading Procedures

All the beams were tested under displacement control at a predetermined rate of 2mm/minute. The displacement rate was automatically controlled and checked against the readings from the deflection gauges positioned at the centre of the beams.

In general, the data generated during each test (loads, crack widths, displacements, etc.) was recorded after each 2 mm increments of displacement after which the load was kept constant. The test sequences were continued until either the beams failed or they were stopped on the grounds of safety because of excessive beam deflections. The beams were usually unloaded in two equal steps. The time required to record a complete set of readings at each load stage varied between 10 to 30 minutes. The overall testing time of a beam varied from 2 to 5 hours.
A.5 MEASUREMENTS

A.5.1 Load

The total load (P) applied to each beam was continuously displayed on the control unit of the test machine. The accuracy of the load readings was checked and found to be correct using a load cell which had been calibrated using a reference test machine.

A.5.2 Deflection

Linear Variable Differential Transducers (LVDT's) were used to measure the deflection at the supports (in several cases dial gauges were used at these points), the loading points, and at the mid-span of each beam. However, during the tests on the first beam (B2) with a circular cross-sections it was noted that rotation of the middle saddle was taking place. A transducer was positioned on the top of the saddle. The saddle was subsequently removed and the mid-span deflection was measured using a dial gauge resting on the base of the test machine. The dial gauge had a resolution of 0.01mm. Dial gauge readings equivalent to the initial transducer readings for this beam were estimated from the equivalent set of readings obtained from an identical beam.

The displacement transducers were mounted on an independently supported frame and connected to a series of general purpose electronic indicators (μPR685 Dynisco R). The displacement transducers were calibrated over a span of approxi-
mately 200mm. The resolution of the indicators was approximately 0.01mm.

A.5.3 Strain Measurement

(a) Concrete Strains

The longitudinal strains in the concrete were measured at predetermined positions in several of the beams. This was carried out to determine the strain at which the concrete cover started to spall and also to determine the location of the neutral axis in the beams made from high strength concrete. The strain in the concrete was measured using a Demec gauge and an arrangement of Demec buttons bonded to the external surfaces of the beams using an epoxy adhesive. The positions of the Demec buttons on the surfaces of the beams are shown in the relevant sections in the thesis.

(b) Steel Strains

The strains in the longitudinal steel bars and in the stirrups in several of the beams were measured in order to calculate the level to which they were stressed. The strains were also measured in the steel bars used in the tests to determine the tensile stress-strain relationships of the steel.

Two types of electrical resistance strain gauges (ERSG's) manufactured in Japan by KYOWA Electronic Instruments Co., Ltd. were used in the investigation. Strain gauges with gauge lengths of 10mm (Type KFG-10-120-C1-11) and 5mm (Type KFG-5-120-C1-11) were used to measure the strain in the 25mm nominal diameter steel
bars and in the 8mm nominal diameter stirrups respectively.

Parts of the ribs on the longitudinal steel bars were removed and the bars were locally flattened at each of the strain gauge positions to facilitate the placement of the strain gauges. At those locations the bars were rubbed down with emery paper and cleaned with a water-based degreasing surface cleaner. Finally, the gauges were bonded to the bars using an epoxy adhesive. Once the epoxy had hardened, three layers of protective coating were applied to waterproof the installation. The gauges were wired in such a way that each gauge formed a quarter-bridge circuit.

An ORION Delta (3531D) data acquisition system (designed and manufactured by the Solartron Electronic Group Ltd.) was used for recording the strain gauge readings. It had a display with front panel controls, a built in printer and an Epson SMD-280H, 3.5 inch micro floppy disk drive. The control and programming of the unit was accomplished using the front panel keys in conjunction with “prompt” messages which appeared on the integral display. The data acquisition program was already available and it was only necessary to input the specified characteristics of strain gauges. The built-in disk drive could either be used for the storage of the recordings directly or to replay previously logged data. The printer gave instant printouts of the recordings. Conversion of the measurements to strain values was carried out automatically. In the case of the strain gauge measurements the initial resistance and bridge imbalances were measured and stored for subsequent use in the strain calculations.
A.5.4 Cracking

After each load increment, the beams were inspected for cracks. Two crack width microscopes were used in the investigation. One with a resolution of 0.1mm manufactured by FLUBACHER & Co. This one was used in Test Series ‘A’, ‘B’, ‘C’ and ‘G’. The other one had a resolution of 0.02mm and was manufactured by Wexham Developments and used in Test Series ‘D’, ‘E’, and ‘F’.

The cracks were marked on each face of the test beams normally at load levels close to 50% and 100% of the maximum anticipated loads and just prior to unloading. The development of each crack was recorded on the beam surfaces with the corresponding applied load level. The crack patterns were photographed and hard copy sketches were also made. Figure A.3 shows typical crack patterns at various loading levels obtained from beam type B5 which was included in Test Series ‘G’ and described in Chapter 7. On the completion of a test, the beam was photographed to record the final deflected shape.

A.5.5 Beam Testing Results

At each load stage, the following recordings were made:

1. The displacement increment which was shown in millimetres on the display panel on the control unit of the test machine (resolution of 0.1mm).

2. The total applied load (P) in kN.

3. The displacement readings on the dial gauges in millimetres.
4. The LVDT readings in voltages shown on the electronic indicators.

5. The flexural and the diagonal crack widths in millimetres.

6. Comments on the physical state of the beam.

A typical set of test results (beam type NB3-1 used in Test Series ‘A’) is given in Table A.3.

Additionally, the control cubes were also tested and the concrete compressive strengths were recorded at the time of the corresponding beam test.

A.6 PROCESSING OF THE TEST RESULTS

A.6.1 Loads

It was necessary for comparison purposes to correct the measured loads using the actual geometric properties of the test beams and the compressive strength of the concrete. The corrected loads \( P_R \) were obtained by multiplying each of the test loadings \( P \) by a correction factor \( F \). The factor \( F \) was determined for each beam as follows:

\[
F = \frac{M_1}{M_{\text{actual}}}
\]

where:

\( M_1 \) is the theoretical ultimate bending moment capacity which was calculated using the geometric and material properties used in the design of the beams.
$M_{\text{actual}}$ is the theoretical ultimate bending moment capacity based on the actual measured geometric and material properties of the beams.

A.6.2 Deflection

Normally, the deflections were recorded at three points i.e. at the centre of the two loading plates and at the mid-span of each beam. The method used to calculate the deflections is summarised below.

1. The total deflections at the different points in the beams (including the support points) were determined either using the calibration relationships obtained for each transducer or directly from the readings on the dial gauges.

2. The deflection at the three points for each loading increment was found by subtracting the deflection corresponding to the movement at the supports from the total deflection calculated during the previous load increment.

A computer program written in Fortran was used to perform these calculations.

A typical processed set of test results (beam type NB3-1 used in Test Series \textquoteleft A\textquoteleft) is given in Table A.4.
<table>
<thead>
<tr>
<th>Sieve Size (mm) or μm</th>
<th>Retained (g)</th>
<th>Passing (g)</th>
<th>% Passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.0</td>
<td>500&lt;sup&gt;1&lt;/sup&gt;</td>
<td>100.0</td>
</tr>
<tr>
<td>5</td>
<td>14.4</td>
<td>485.6</td>
<td>97.1</td>
</tr>
<tr>
<td>2.36</td>
<td>104.7</td>
<td>380.9</td>
<td>76.2</td>
</tr>
<tr>
<td>1.18</td>
<td>84.7</td>
<td>296.2</td>
<td>59.2</td>
</tr>
<tr>
<td>300</td>
<td>148.2</td>
<td>148.0</td>
<td>29.6</td>
</tr>
<tr>
<td>150</td>
<td>70.8</td>
<td>77.2</td>
<td>15.4</td>
</tr>
<tr>
<td>Base</td>
<td>73.5</td>
<td>3.7</td>
<td>0.7 (loss)</td>
</tr>
</tbody>
</table>

Table A.1: Sieve Analysis of Fine Aggregate.

<sup>1</sup>Mass of the sample = 500 g
<table>
<thead>
<tr>
<th>Type</th>
<th>Nominal Diameter (mm)</th>
<th>Measured Diameter (mm)</th>
<th>Measured Area (mm²)</th>
<th>Yield Strength (MPa)</th>
<th>Ultimate Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Φ32</td>
<td>32</td>
<td>30.93</td>
<td>751.4</td>
<td>527</td>
<td>665</td>
</tr>
<tr>
<td>Φ25</td>
<td>25</td>
<td>24.82</td>
<td>483.8</td>
<td>526</td>
<td>621.6</td>
</tr>
<tr>
<td>Φ20</td>
<td>20</td>
<td>19.74</td>
<td>306.04</td>
<td>532.6</td>
<td>646.3</td>
</tr>
<tr>
<td>Φ12</td>
<td>12</td>
<td>11.667</td>
<td>106.9</td>
<td>514.5</td>
<td>629.9</td>
</tr>
<tr>
<td>Φ10</td>
<td>10</td>
<td>9.61</td>
<td>72.53</td>
<td>453.6</td>
<td>530.8</td>
</tr>
<tr>
<td>Φ8</td>
<td>8</td>
<td>7.90</td>
<td>49.02</td>
<td>377</td>
<td>596</td>
</tr>
</tbody>
</table>

Table A.2: Tensile Strength of Reinforcing Steel Bars.
Test Date: 14 August 1991

<table>
<thead>
<tr>
<th>Control Displacement (mm)</th>
<th>Load P (kN)</th>
<th>Displacement Readings (Voltage)</th>
<th>Crack Width (mm)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1  2  3  4  5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>18510</td>
<td>15960 15120 5.961 6.842</td>
<td></td>
<td></td>
</tr>
<tr>
<td>88.5</td>
<td>40</td>
<td>18460 15700 15020 5.690 6.740</td>
<td></td>
<td></td>
</tr>
<tr>
<td>86.4</td>
<td>69</td>
<td>18290 15500 14820 5.640 6.660</td>
<td></td>
<td></td>
</tr>
<tr>
<td>84.1</td>
<td>99</td>
<td>18120 15300 14620 5.569 6.586</td>
<td>0.15 0.15</td>
<td></td>
</tr>
<tr>
<td>81.9</td>
<td>124</td>
<td>17920 15100 14440 5.541 6.530</td>
<td>0.2 0.25</td>
<td></td>
</tr>
<tr>
<td>79.7</td>
<td>148</td>
<td>17750 14900 14250 5.512 6.487</td>
<td>0.3 0.65</td>
<td></td>
</tr>
<tr>
<td>77.6</td>
<td>169</td>
<td>17570 14690 14090 5.431 6.431</td>
<td>0.35 0.7</td>
<td></td>
</tr>
<tr>
<td>75.0</td>
<td>197</td>
<td>17360 14400 13850 5.397 6.370</td>
<td>0.4 1.4</td>
<td></td>
</tr>
<tr>
<td>72.5</td>
<td>220</td>
<td>17130 14100 13670 5.325 6.302</td>
<td>0.45 1.95</td>
<td></td>
</tr>
<tr>
<td>70.1</td>
<td>247</td>
<td>16930 13800 13450 5.298 6.253</td>
<td>0.5 2.4</td>
<td></td>
</tr>
<tr>
<td>67.8</td>
<td>253</td>
<td>16740 13500 13220 5.248 6.220</td>
<td>0.85 2.55</td>
<td></td>
</tr>
<tr>
<td>65.6</td>
<td>256</td>
<td>16570 13200 13050 5.239 6.197</td>
<td>1.25 2.6</td>
<td></td>
</tr>
<tr>
<td>63.4</td>
<td>261</td>
<td>16380 12900 12750 5.211 6.172</td>
<td>1.85 2.9</td>
<td></td>
</tr>
<tr>
<td>60.9</td>
<td>263</td>
<td>16170 12590 12530 5.189 6.146</td>
<td>2.4 2.9</td>
<td></td>
</tr>
<tr>
<td>58.7</td>
<td>258</td>
<td>16030 12300 12200 5.153 6.132</td>
<td>4.0 3.0</td>
<td></td>
</tr>
<tr>
<td>55.3</td>
<td>258</td>
<td>15750 11890 11830 5.120 6.121</td>
<td>4.2 3.0</td>
<td></td>
</tr>
<tr>
<td>51.6</td>
<td>257</td>
<td>15420 11400 11430 5.074 6.097</td>
<td>4.2 3.0</td>
<td></td>
</tr>
<tr>
<td>46.6</td>
<td>257</td>
<td>15000 10800 10990 4.905 6.075</td>
<td>4.2 3.1</td>
<td></td>
</tr>
<tr>
<td>39.3</td>
<td>259</td>
<td>14510 10000</td>
<td>4.751 6.057 4.5 3.1</td>
<td></td>
</tr>
<tr>
<td>30.9</td>
<td>257</td>
<td>13920 9000</td>
<td>4.646 6.055 5.0 3.25</td>
<td></td>
</tr>
<tr>
<td>23.4</td>
<td>258</td>
<td>13330 7960</td>
<td>4.453 6.055 5.05 4.1</td>
<td></td>
</tr>
<tr>
<td>16.8</td>
<td>257</td>
<td>12830 7000</td>
<td>4.147 6.073 5.1 5.6</td>
<td></td>
</tr>
<tr>
<td>9.6</td>
<td>261</td>
<td>12260 6000</td>
<td>3.959 6.173 5.45 6.25</td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>262</td>
<td>11640 5030</td>
<td>3.688 6.178 5.9 7.2</td>
<td></td>
</tr>
<tr>
<td>-7.1</td>
<td>270</td>
<td>10950 3970</td>
<td>3.353 6.237 6.25 8.0</td>
<td></td>
</tr>
<tr>
<td>-26.4</td>
<td>255</td>
<td>8920 1570</td>
<td>1.522 6.330 6.35 9.1 4</td>
<td></td>
</tr>
</tbody>
</table>

**Unloading**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>1  2  3  4  5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.76</td>
<td>125</td>
<td>9700 2600</td>
<td>3.083 6.450</td>
</tr>
<tr>
<td>4.3</td>
<td>11</td>
<td>10950 4000</td>
<td>3.894 6.587</td>
</tr>
</tbody>
</table>

Notes:
1- Cracking in concrete near the loading points.
2- Spalling of the concrete cover near the loading points.
3- Spalling of concrete cover in the mid-span region.
4- Loading was stopped on the grounds of safety.

Table A.3: Test Records for Beam Type NB3-1.
Test Date: 14 August 1991

<table>
<thead>
<tr>
<th>Load (P) (kN)</th>
<th>Corrected Load ( P_r = P \times F )† (kN)</th>
<th>Deflection (mm)</th>
<th>1 (Mid-Span)</th>
<th>2</th>
<th>3 (Mid-Span)</th>
<th>4</th>
<th>5 (Mid-Span)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>38.3</td>
<td>0.270</td>
<td>2.429</td>
<td>0.820</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>69</td>
<td>66.0</td>
<td>1.947</td>
<td>4.376</td>
<td>2.817</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>94.7</td>
<td>3.607</td>
<td>6.316</td>
<td>4.802</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>118.7</td>
<td>5.612</td>
<td>8.286</td>
<td>6.615</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>148</td>
<td>141.6</td>
<td>7.312</td>
<td>10.262</td>
<td>8.535</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>169</td>
<td>161.7</td>
<td>9.070</td>
<td>12.306</td>
<td>10.109</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>197</td>
<td>188.5</td>
<td>11.171</td>
<td>15.176</td>
<td>12.534</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>210.5</td>
<td>13.443</td>
<td>18.124</td>
<td>14.315</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>247</td>
<td>236.4</td>
<td>15.450</td>
<td>21.104</td>
<td>16.543</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>253</td>
<td>242.1</td>
<td>17.339</td>
<td>24.080</td>
<td>18.866</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>256</td>
<td>245.0</td>
<td>19.060</td>
<td>27.082</td>
<td>20.600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>261</td>
<td>249.8</td>
<td>20.969</td>
<td>30.073</td>
<td>23.659</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>263</td>
<td>251.7</td>
<td>23.086</td>
<td>33.016</td>
<td>25.899</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>258</td>
<td>246.9</td>
<td>24.480</td>
<td>36.060</td>
<td>29.266</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>258</td>
<td>246.9</td>
<td>27.304</td>
<td>40.163</td>
<td>33.047</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>257</td>
<td>246.0</td>
<td>30.625</td>
<td>45.057</td>
<td>37.124</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>257</td>
<td>246.0</td>
<td>34.763</td>
<td>50.998</td>
<td>41.535</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>259</td>
<td>247.9</td>
<td>39.627</td>
<td>58.960</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>257</td>
<td>246.0</td>
<td>45.553</td>
<td>68.966</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>258</td>
<td>246.9</td>
<td>51.408</td>
<td>79.332</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>257</td>
<td>246.0</td>
<td>56.257</td>
<td>88.846</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>261</td>
<td>249.8</td>
<td>61.930</td>
<td>98.862</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>262</td>
<td>250.7</td>
<td>68.028</td>
<td>108.487</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>258.4</td>
<td>74.797</td>
<td>119.012</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>255</td>
<td>244.0</td>
<td>94.008</td>
<td>141.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† The correction factor \( F = 0.9570 \)

Table A.4: Processed Test Results for Beam Type NB3-1.
Figure A.1: Typical Stress-Strain Curves for Steel Reinforcement Bars. (a) 8mm Nominal Diameter Bars. (b) 25mm Nominal Diameter Bars.
Figure A.2: Typical Steel Bar of Nominal Diameter of 25mm Used in a Tensile Test.
Figure A.3: Crack Patterns at Various Load Levels for Beam Type B5 in Test Series 'G'.
Figure A.4: Typical Steel Cages inside shutters.
(a) Beam Type C2.78 (HSC) (Test Series 'E')

(b) Beam Type C3.2-3 (Test Series 'F')

(c) Beam Type B7 (Test Series 'G')
Appendix B

CONFINEMENT REQUIREMENTS TO PREVENT DIAGONAL FAILURES

B.1 EVALUATION OF $K_e$ FOR THE BEAMS IN TEST SERIES ‘A’

B.1.1 Prototype Example: Beam Type NA2-1

Input Data:

$f_{cp} \simeq 0.8f_{cu} = 44.6 \text{ MPa}$

$\phi_{10}$ Steel

$A_s = 72.53 mm^2, \quad f_y = 453.6 \text{ MPa}$

$\phi_{20}$ Steel

$A_s = 306.04 mm^2, \quad f_y = 532.6 \text{ MPa}$

$\rho = 0.0174$

$s = s_v = 70 mm$

$\rho_t = 0.0286$

The geometry of the cross section is shown in Figure B.1. The location of the neutral axis was based on the provisions of the ACI Code of Practice.
(a) Evaluation of the Confinement Required for the Inclined Leg Near the Point where the Load Changes Direction \((K_{\text{provided}})\)

\[
A_{\text{con}} = 150 \times 82.66 = 12399 \text{ mm}^2
\]

\[
P_{\text{occ}} = 12399 \times 44.64(10)^{-3} = 553.5 \text{ kN}
\]

\[
\Sigma c_i^2 = (130)^2 + 2 \times (72.66)^2 = 27458.95 \text{ mm}^2
\]

Substituting into equation (4.30):

\[
K_s = 1 + \frac{12399}{140 \times 553.5} \left[ \left( 1 - \frac{27458.95}{5.5 \times 12399} \right) \left( 1 - \frac{0.5 \times 70}{150} \right) \left( 1 - \frac{0.25 \times 70}{82.66} \right) \right] / \sqrt{0.0286 \times 453.6}
\]

\[
K_s = 1 + 0.16\left[0.597 \times 0.767 \times 0.79\right] 3.6 = 1.21
\]

\[
K_{\text{provided}} = K_s = 1.21
\]

(b) Evaluation of the Confinement Required for the Inclined Leg \((K_{\text{required}})\)

\[
r_{t,\text{measured}} = \frac{M_t}{M_f} = 0.77 \quad \text{(Figure 4.3)}
\]

From equations (4.12) and (4.13),

\[
r_{u,\text{(old, model)}} = \frac{M_u}{M_f}
\]

\[
r_{u,\text{(old, model)}} = \zeta \left( \frac{0.83(0.018)^{\frac{1}{2}}(32)^{\frac{1}{2}} \times 2.0 + 206.9(0.018)^{\frac{1}{2}}(2.0)^{-\frac{1}{2}}}{532.6 \times 0.018 \left( 1 - \frac{0.018 \times 532.6}{1.7 \times 32} \right)} \right)
\]

\[
r_{u,\text{(old, model)}} = 0.811 \left( \frac{2.46 + 2.57}{7.9} \right) = 0.52
\]
Substituting into equation (4.7):

\[ r_{new} = 0.77 + 0.23 \frac{0.54 - 0.52}{1 - 0.52} = 0.78 \]

From equation (4.3),

\[ K_s = 1 + (1 - 0.78) = 1.22 \]

\[ K_{s,required} = K_s = 1.22 \]

B.1.2 Remaining Beams

The results of the calculations for the confinement requirements as well as the input data for the remaining beams are summarised in Table B.1.

B.2 DESIGN OF THE BEAMS IN TEST SERIES 'C'

B.2.1 Prototype Example: Beam Type B1.5

Design Data

Beam size = 200mm x 400mm

\( a/d = 3.2 \)

Longitudinal reinforcement = 1Φ25 (\( A_s = 483.8\,mm^2 \), \( f_y = 526\,MPa \)) + 2Φ20
\[(A_s = 2 \times 310.35 = 620.7 \text{mm}^2, \ f_y = 500 \text{ MPa})\]

\[f_{\text{ave}} \text{ for longitudinal reinforcement} = 513 \text{ MPa}\]

Stirrup: \(\Phi 8 \ (A_s = 49.9 \text{mm}^2, \ f_y = 420.8 \text{ MPa})\)

Concrete strength: \(f_{\text{cu}} = 40 \text{ MPa}, \ f'_c \simeq 0.8f_{\text{cu}} = 32 \text{ MPa}\)

**Traditional Design**

The required stirrup spacing is 155mm based on the provisions of BS 8110. The same spacing is required for the whole of the shear span.

**Design Based on the Proposed Flexure-Shear Interaction Design Model**

(a) **Design of the Horizontal Leg**

\[r_{\text{t,measured}} = \frac{M_t}{M_f} = 0.95 \quad (\text{Figure 4.4})\]

From equations (4.12) and (4.13),

\[
\begin{align*}
    r_{u(\text{old, model})} &= \frac{M_u}{M_f} \\
    &= \zeta \left( \frac{0.83(0.018)^{\frac{3}{2}}(32)^{\frac{3}{2}} \times 3.2 + 206.9(0.018)^{\frac{3}{2}}(3.2)^{-\frac{3}{2}}}{532.6 \times 0.018 \left(1 - \frac{0.018 \times 532.6}{1.7 \times 32}\right)} \right) \\
    &= 0.811(3.94 + 1.271) \quad 7.9 = 0.535
\end{align*}
\]

\[
\begin{align*}
    r_{u(\text{new, model})} &= \zeta \left( \frac{0.83(0.015)^{\frac{3}{2}}(32)^{\frac{3}{2}} \times 3.2 + 206.9(0.015)^{\frac{3}{2}}(3.2)^{-\frac{3}{2}}}{513 \times 0.015 \left(1 - \frac{0.015 \times 513}{1.7 \times 32}\right)} \right) \\
    &= 0.763(3.707 + 1.092) \quad 6.6065 = 0.554
\end{align*}
\]

Substituting into equation (4.7):
\[ r_{t_{\text{new}}} = 0.95 + 0.05 \frac{0.554 - 0.535}{1 - 0.535} = 0.952 \]

From equation (4.3),

\[ K_s = 1 + (1 - 0.952) = 1.048 \]

The geometry of the cross section is shown in Figure B.2.a.

\[ A_{\text{con}} = 14151.2 \text{mm}^2 \]

\[ P_{\text{occ}} = 452.8 \text{ kN} \]

\[ \Sigma c_i^2 = 32101.6 \text{mm}^2 \]

\[ \rho_t = \frac{1.24}{s} \]

Substituting into Equation (4.30):

\[ 1.048 = 1 + \frac{14151.2}{140 \times 452.8} \left[ \left( 1 - \frac{32101.6}{5.5 \times 14151.2} \right) \left( 1 - 0.5 \times s \right) \left( 1 - 0.25 \times s \right) \right] \sqrt{\frac{1.24}{s} \times 420.8} \]

\[ s_v = s = 175 \text{mm} \]

(b) Design for the Inclined Leg

\[ r_{t_{\text{measured}}} = \frac{M_t}{M_f} = 0.94 \quad (\text{Figure 4.3}) \]

\[ r_{t_{\text{new}}} = 0.94 + 0.06 \frac{0.554 - 0.535}{1 - 0.535} = 0.942 \]

\[ K_s = 1.058 \]
(b.1) Near the Horizontal Leg

The geometry of the cross section is shown in Figure B.2.b.

\[ A_{\text{con}} = 14151.2 \text{mm}^2 \]
\[ P_{\text{occ}} = 452.8 \text{ kN} \]
\[ \Sigma c_i^2 = 32101.6 \text{mm}^2 \]
\[ \rho_t = \frac{0.94}{s} \]

Substituting into equation (4.30):

\[ 1.058 = 1 + \frac{14151.2}{140 \times 452.8} \left[ \left( 1 - \frac{32101.6}{5.5 \times 14151.2} \right) \left( 1 - \frac{0.5 \times s}{152} \right) \left( 1 - \frac{0.25 \times s}{93.1} \right) \right] \]
\[ \sqrt{\frac{0.94}{s} \times 420.8} \]
\[ s_v = s = 160 \text{mm} \]

(b.2) Near the Supports

The geometry of the cross section is shown in Figure B.2.c.

\[ A_{\text{con}} = 17799 \text{mm}^2 \]
\[ P_{\text{occ}} = 569.6 \text{ kN} \]
\[ \Sigma c_i^2 = 27424.8 \text{mm}^2 \]
\[ \rho_t = \frac{0.94}{s} \]

Substituting into equation (4.31):
B.2.2 Design of the Remaining Beams

The results from the design calculations as well as the input data for the remaining beams are summarised in Table B.2.

\[
1.058 = 1 + \frac{17799}{140 \times 569.6} \left[ \left( 1 - \frac{27424.8}{5.5 \times 17799} \right) \left( 1 - \frac{0.5 \times s}{152} \right) \right] \sqrt{\frac{0.94}{s} \times 420.8}
\]

\[s_v = s = 220\text{mm}\]
Table B.1: Confinement Requirements to Prevent Diagonal Failures in the Beams in Test Series ‘A’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>ρ</th>
<th>Cross section (mm x mm)</th>
<th>Required $K_s$</th>
<th>Stirrup spacing $l$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA1-1</td>
<td>2.0</td>
<td>35</td>
<td>202.5 x 305</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NA1-2</td>
<td>2.5</td>
<td>35</td>
<td>202.5 x 305</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NB1-1</td>
<td>3.2</td>
<td>60</td>
<td>203.5 x 310</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NB1-2</td>
<td>3.2</td>
<td>60</td>
<td>203.5 x 310</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Stirrup diameter was 10mm for beam types A1.5 and A2 and 8mm for the remaining beams.

Required spacing near the horizontal leg.

Table B.2: Prevention of Diagonal Failures in the Beams in Test Series ‘C’.

<table>
<thead>
<tr>
<th>Beam type</th>
<th>a/d</th>
<th>ρ</th>
<th>Cross section (mm x mm)</th>
<th>Required $K_s$</th>
<th>Stirrup spacing $l$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.5</td>
<td>1.75</td>
<td>1.5</td>
<td>200 x 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>1.75</td>
<td>1.5</td>
<td>200 x 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1.5</td>
<td>3.2</td>
<td>1.8</td>
<td>200 x 300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1.8(T)</td>
<td>2</td>
<td>1.8</td>
<td>200 x 300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1.5</td>
<td>3.9</td>
<td>1.5</td>
<td>200 x 400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1.8(T)</td>
<td>4.0</td>
<td>1.8</td>
<td>200 x 300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>3.9</td>
<td>2</td>
<td>200 x 400</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Stirrup diameter was 10mm for beam types A1.5 and A2 and 8mm for the remaining beams.
‡ Required spacing near the horizontal leg.
* Required spacing near the supports.
### Strains in stirrups in beam type B1.8 \( (\text{mm/mm} \times 10^6) \)

<table>
<thead>
<tr>
<th>Load level (kN)</th>
<th>Vertical leg</th>
<th>Horizontal leg</th>
<th>Inside shear span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adjacent to loading point</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Upper</td>
</tr>
<tr>
<td>0.0</td>
<td>4.4</td>
<td>-4.5</td>
<td>3.1</td>
</tr>
<tr>
<td>67</td>
<td>13.0</td>
<td>-20.3</td>
<td>40.1</td>
</tr>
<tr>
<td>98</td>
<td>7.1</td>
<td>491.0</td>
<td>65.7</td>
</tr>
<tr>
<td>125</td>
<td>5.0</td>
<td>472.2</td>
<td>93.4</td>
</tr>
<tr>
<td>150</td>
<td>44.0</td>
<td>480.8</td>
<td>139.7</td>
</tr>
<tr>
<td>175</td>
<td>158.8</td>
<td>516.0</td>
<td>182.4</td>
</tr>
<tr>
<td>199</td>
<td>196.5</td>
<td>534.5</td>
<td>215.9</td>
</tr>
<tr>
<td>223</td>
<td>262.3</td>
<td>591.3</td>
<td>251.7</td>
</tr>
<tr>
<td>249</td>
<td>370.9</td>
<td>710.4</td>
<td>308.2</td>
</tr>
<tr>
<td>266</td>
<td>425.6</td>
<td>794.8</td>
<td>356.5</td>
</tr>
<tr>
<td>282</td>
<td>458.3</td>
<td>901.5</td>
<td>399.0</td>
</tr>
<tr>
<td>286</td>
<td>463.5</td>
<td>948.2</td>
<td>426.4</td>
</tr>
<tr>
<td>287</td>
<td>461.2</td>
<td>971.5</td>
<td>440.3</td>
</tr>
<tr>
<td>291</td>
<td>470.5</td>
<td>998.6</td>
<td>458.5</td>
</tr>
<tr>
<td>260</td>
<td>454.9</td>
<td>1013.8</td>
<td>447.1</td>
</tr>
<tr>
<td>92</td>
<td>350.9</td>
<td>895.3</td>
<td>287.2</td>
</tr>
</tbody>
</table>

### Strains in stirrups in beam type B1.8(T) \( (\text{mm/mm} \times 10^6) \)

<table>
<thead>
<tr>
<th>Load level (kN)</th>
<th>Vertical leg</th>
<th>Horizontal leg</th>
<th>Inside shear span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adjacent to loading point</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vertical</td>
<td>Horizontal</td>
<td>Upper</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.3</td>
<td>1.3</td>
<td>-</td>
</tr>
<tr>
<td>89</td>
<td>1.3</td>
<td>63.2</td>
<td>-</td>
</tr>
<tr>
<td>143</td>
<td>23.1</td>
<td>508.3</td>
<td>-</td>
</tr>
<tr>
<td>195</td>
<td>512.2</td>
<td>1098.4</td>
<td>-</td>
</tr>
<tr>
<td>243</td>
<td>818.1</td>
<td>1345.6</td>
<td>-</td>
</tr>
<tr>
<td>271</td>
<td>1004.1</td>
<td>1457.7</td>
<td>-</td>
</tr>
<tr>
<td>273</td>
<td>1025.0</td>
<td>1457</td>
<td>-</td>
</tr>
<tr>
<td>275</td>
<td>1067.4</td>
<td>1453.5</td>
<td>-</td>
</tr>
<tr>
<td>266</td>
<td>1061.1</td>
<td>1423.8</td>
<td>-</td>
</tr>
<tr>
<td>91</td>
<td>556.3</td>
<td>788.0</td>
<td>-</td>
</tr>
</tbody>
</table>

Table B.3: Strains in the Stirrups in Beam Types B1.8 and B1.8(T) in Test Series ‘C’.
Figure B.1: Cross Section Geometry for Beam Type NA2.1.

Figure B.2: Cross Section Geometry for Beam Type B1.5. (a) The Horizontal Leg. (b) The Inclined Leg near the Horizontal Leg. (c) Near the Supports.
Appendix C

FLEXURAL CAPACITY RESULTING FROM CONFINEMENT

C.1 PREDICTION OF THE FLEXURAL CAPACITY OF THE BEAMS IN TEST SERIES 'A'.

C.1.1 Prototype Example: Beam NA2-1

Input Data:

\[ f_{cp} \approx 0.8 f_{cu} = 44.64 \text{ MPa} \]

\( \Phi_{10} \text{ Steel} \)

Diameter = 9.61 mm, \( A_s = 72.53 \text{ mm}^2 \), \( f_y = 453.6 \text{ MPa} \)

\( \Phi_{20} \text{ Steel} \)

Diameter = 19.74 mm, \( A_s = 306.04 \text{ mm}^2 \), \( f_y = 532.6 \text{ MPa} \)

\[ s_v = 70 \text{ mm} \]

\[ \rho_t = 0.0286 \]

Details of the beam are given in Figure B.1.

Confinement Enhancement Factor (\( K_s \))

\[ A_{co} = 150 \times 140 = 21000 \text{ mm}^2 \]
\[ P_{occ} = 21000 \times 44.64 \times 10^{-3} = 937.44 \text{kN} \]

\[ K_s = 1 + \frac{21000}{937.44} \left[ \left( 1 - \frac{130^2 + 120^2}{5.5 \times 21000} \right) \left( 1 - \frac{0.5 \times 70}{150} \right) \left( 1 - \frac{0.5 \times 70}{140} \right) \right] \]

\[ K_s = 1.15 \]

\[ f_{cc} = 1.0 \times 1.15 \times 44.64 = 51.34 \text{MPa} \]

**Predicted Confined Flexural Strength (M1_{con})**

(i) Assume that the tensile force (T) is balanced by the compressive force (C) in the confined core.

\[ C = T \]

\[ \beta f_{cc} \alpha x b = \Sigma A_s f_y \]

\[ 0.889 \times 51.34 \times 0.75 x \times 150 = 3 \times 306.04 \times 532.6 \]

\[ x = 95.23 \text{mm} \]

\[ M1_{con} = \Sigma A_s f_y \left( d' - \frac{\alpha c}{2} \right) \]

\[ M1_{con} = 3 \times 306.04 \times 532.6 \times 10^{-6} \left( 255 - \frac{0.75 \times 95.23}{2} \right) \]

\[ M1_{con} = 107.23 \text{kN.m} \]

(ii) Consider the case where \(\varepsilon_c = 0.0075\).

This is to show that the increase in the flexural capacity of beams is not influenced
significantly as a greater proportion of the concrete compression block reaches its ultimate confined strength \( f_{cc} \).

\[
\Omega = \frac{0.0075}{0.0022 \times 1.15} = 2.964
\]

For \( \varepsilon_{s2} > 0.0075 \),

\[
\alpha = \frac{6 \times 2.964^2 - 4 \times 2.964 + 1}{2 \times 2.964(3 \times 2.964 - 1)} = 0.895
\]

\[
\beta = \frac{2(3 \times 2.964 - 1)^2}{3(6 \times 2.964^2 - 4 \times 2.964 + 1)} = 0.992
\]

\[
C = T
\]

\[
0.992 \times 51.34 \times 0.895x \times 150 = 3 \times 306.04 \times 532.6
\]

\[
x = 71.83 \text{ mm}
\]

\[
M_{1\text{con}} = 3 \times 306.04 \times 532.6(10)^{-6} \left(255 - \frac{0.895 \times 71.83}{2}\right)
\]

\[
M_{1\text{con}} = 108.98 \text{ kN.m}
\]

(iii) Assume that the tensile force (\( T \)) is balanced by the compressive force (\( C \)) over the whole compression area (confined core and cover).

\[
T = C
\]
\[ T = C_{\text{core}} + C_{\text{side cover}} + C_{\text{top cover}} \]

\[ 3 \times 306.04 \times 532.6 = 0.889 \times 51.34 \times 0.75x \times 150 + 0.85 \times 44.64 \times 0.726x \times 52.5 \]

\[ + 0.85 \times 44.64 \times 5 \times 150 \]

\[ x = 69.98 \text{ mm} \]

\[ C_{\text{core}} = 359.3 \text{ kN} \]

\[ C_{\text{side cover}} = 101.2 \text{ kN} \]

\[ C_{\text{top cover}} = 28.46 \text{ kN} \]

Let \( d' = d - 5\text{mm} \), then:

\[ M_{1\text{con}} = C_{\text{core}} \left( d' - \frac{\alpha x}{2} \right) + C_{\text{side cover}} \left( d - \frac{\beta_1 x}{2} \right) + C_{\text{top cover}} \left( d - \frac{5}{2} \right) \]

\[ M_{1\text{con}} = 359.3(10)^{-3} \left( 255 - \frac{0.75 \times 69.98}{2} \right) + 101.2(10)^{-3} \left( 260 - \frac{0.726 \times 69.98}{2} \right) \]

\[ + 28.46(10)^{-3} \left( 260 - \frac{5}{2} \right) \]

\[ M_{1\text{con}} = 113 \text{ kN.m} \]

C.1.2 Calculation of the Flexural Capacity of the Beams in Test Series 'A'.

The predicted flexural capacities of the test beams are summarized in Table C.1.

C.2 DERIVATION OF \( \rho'_b \)

(i) \( \rho'_b \) is derived from a consideration of the actual characteristics of the core and the cover concrete. The concrete compression block given by the ACI Code of Practice is used for the cover concrete and the modified concrete compression block shown in Figure C.1 is used for the core concrete.
From Figure C.1:

\[
\frac{f_s}{E_s} = \frac{d - c_b}{c_b}
\]

\[
c_b = \frac{\varepsilon_c E_s}{E_s + f_y} d
\]  
(C.1)

Equating tensile and compressive forces:

\[
T = C
\]

\[
A_s f_y = \alpha \beta K_s f_c'(c_b - d_1)b' + 0.85 f'_c \beta_1(b'd_1 + 2b_1 c_b)
\]  
(C.2)

Substituting \(c_b\) from equation (C.1) in equation (C.2):

\[
\rho'_b = \frac{A_s}{bd} = \frac{\alpha \beta K_s f_c'}{f_y} \left( \frac{\varepsilon_c E_s}{E_s + f_y} - \frac{d_1}{d} \right) \frac{b'}{b}
\]

\[
+ 0.85 f'_c \beta_1 \left( \frac{bd_1}{bd} - \frac{2b_1}{b} \frac{\varepsilon_c E_s}{E_s + f_y} \right)
\]  
(C.3)

where:

\(d'\) is the effective depth of the beam cross section measured from the centreline of the stirrup and not from the top of the section.

\(b'\) is the width of the confined core.

\(d_1 = d - d'\)

\(b_1 = \frac{1}{2}(b - b')\)
(ii) $\rho'_b$ is determined on the assumption that the modified concrete compression block is applicable to the whole compression concrete including the cover,

Equating tensile and compressive forces:

$$T = C$$

$$A_s f_y = \alpha \beta K_s f'_c c_b b$$  \hspace{1cm} (C.4)

Substituting $c_b$ from equation (C.1) into equation (C.4):

$$\rho'_b = \frac{A_s}{bd} = \frac{\alpha \beta K_s f'_c}{f_y} \frac{\varepsilon_c E_s}{\varepsilon_c E_s + f_y}$$  \hspace{1cm} (C.5)
<table>
<thead>
<tr>
<th>Beam Type</th>
<th>$s_v$ (mm)</th>
<th>$\rho_t$</th>
<th>$K_s$ (flexure)</th>
<th>$f'_{c}$ (MPa)</th>
<th>$f_{cc}$ (MPa)</th>
<th>$M_{con}$ (kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA2-1</td>
<td>70</td>
<td>0.0286</td>
<td>1.15</td>
<td>44.64</td>
<td>51.34</td>
<td>107.23</td>
</tr>
<tr>
<td>NA2-2</td>
<td>70</td>
<td>0.0286</td>
<td>1.19</td>
<td>35.92</td>
<td>42.75</td>
<td>103.72</td>
</tr>
<tr>
<td>NA3-1</td>
<td>35</td>
<td>0.0572</td>
<td>1.32</td>
<td>39.44</td>
<td>52.06</td>
<td>107.47</td>
</tr>
<tr>
<td>NA3-2</td>
<td>35</td>
<td>0.0572</td>
<td>1.34</td>
<td>37.5</td>
<td>50.25</td>
<td>106.9</td>
</tr>
<tr>
<td>NB2-1</td>
<td>120</td>
<td>0.0167</td>
<td>1.08</td>
<td>39.25</td>
<td>42.39</td>
<td>103.54</td>
</tr>
<tr>
<td>NB2-2</td>
<td>120</td>
<td>0.0167</td>
<td>1.08</td>
<td>39.04</td>
<td>42.16</td>
<td>103.43</td>
</tr>
<tr>
<td>NB3-1</td>
<td>60</td>
<td>0.0334</td>
<td>1.2</td>
<td>39.9</td>
<td>47.88</td>
<td>105.97</td>
</tr>
<tr>
<td>NB3-2</td>
<td>60</td>
<td>0.0334</td>
<td>1.2</td>
<td>39.36</td>
<td>47.23</td>
<td>105.71</td>
</tr>
</tbody>
</table>

Table C.1: Confined Flexural Capacities of the Beams in Test Series 'A'.
Figure C.1: Modified Stress and Strain Diagrams Under Balanced-Failure Conditions.
Appendix D

BEAMS WITH A CIRCULAR CROSS SECTION

D.1 TRADITIONAL BEAM DESIGN

In this case it was important to follow a formal design procedure similar to that which was adopted for the rectangular beams in order to obtain an acceptable and comparable design solution. Unfortunately, most Codes of Practice do not place much emphasis on the design of beams with a circular cross section. In the interests of consistency, wherever possible, the provisions of BS8110 have been adopted, however, other sources of information[149, 229, 234, 235, 236] have been used. The load and the capacity reduction factors which are present in Codes of Practice to safeguard against failure were ignored. It should be noted that in the case of beams designed to fail in flexure a relatively large load factor was used in the design for shear in order to prevent shear failures. This was to allow for the uncertainties involved in the design of beams with a circular cross section.

D.1.1 Flexural Design

The application of the basic principles of equilibrium and compatibility offers the most direct approach for the determination of the ultimate strength design of reinforced concrete members. However, in the case of circular sections where the
reinforcement bars are located at discrete points around the perimeter of a circle, the equations of equilibrium are complex and no explicit solutions can be determined. It was, therefore, necessary to use time consuming trial and error techniques on beams of various sizes and with different steel configurations and material properties. The beams which are discussed here are those which were selected for inclusion in the test programme. The evaluation of the flexural capacity of the test beams was carried out assuming that the contribution from the compression longitudinal reinforcement was either ignored or included.

The steps which were followed in the determination of the flexural capacity of the test beams are summarised below:

(a) Ignoring the Longitudinal Compression Steel

The beam cross section is shown in Figure 7.2.

1. The tensile force in the longitudinal reinforcement bars \( T \) is found by assuming an initial value for the steel tensile stress:

\[
T = \sum_{\text{all tensile steel}} A_s f_s \quad \text{(D.1)}
\]

2. The compressive force \( C \) in the compression concrete is found by equating the compression and tensile forces across the section:
\[ C = T \]  

(D.2)

3. The compression concrete area \((A_{cc})\) is obtained from the following relationship:

\[
A_{cc} = \frac{C}{0.67f_{cu}} = \frac{(D)^2}{8} (2\alpha - \sin(2\alpha)) 
\]  

(D.3)

4. Dimensional details of the section \((\alpha, e, s, \text{and} \ x)\) are subsequently obtained.

5. Strains in the steel are found from the following relationships:

\[
\epsilon_s = \frac{0.0035}{x} (d1 - x) 
\]  

(D.4)

\[
\epsilon_s = \frac{0.0035}{x} (d2 - x) 
\]  

(D.5)

6. The steel stress is obtained from the following relationship:

\[
f_s = \epsilon_s E \neq f_y 
\]  

(D.6)

7. The steel stresses are checked and if they are found not to be as assumed in step 1, all the steps described above are repeated until a satisfactory agreement is reached.

8. The distance to the centroid of the compressive concrete from the middle of the section \((Y)\) is obtained from the following relationship:
$$Y = \frac{2}{3} \left( \frac{D \sin^3 \alpha}{2 \alpha - \sin 2\alpha} \right)$$  \hspace{1cm} (D.7)

9. The ultimate moment capacity is calculated from the following equation:

$$M_1 = \sum f_s A_s (Y + K1, \text{ or } K2)$$  \hspace{1cm} (D.8)

The results obtained from the design calculations are given in the following section.

(b) Considering the Longitudinal Compression Steel

The beam cross section is shown in Figure 7.2.

1. The location of the neutral axis is assumed.

2. The geometrical properties of the section ($\alpha, e, c, x$) are subsequently obtained.

3. $A_{cc}$ is obtained using the geometrical properties of the section.

4. The longitudinal steel stresses ($f_{s1}, f_{s2}, f_{s3}, f_{s4}$) are hence calculated.

5. The tensile steel force ($T$) is determined from the following relationship:

$$T = \sum_{\text{all tensile steel}} f_s A_s$$

6. The compressive force in the concrete ($C_c$) which corresponds to $A_{cc}$ is calculated.
7. Hence the compressive concrete area \( A_{cc} \) is calculated.

8. The values of \( A_{cc} \) from steps 3 and 7 are compared and if they are not identical the steps described above are repeated until a satisfactory agreement is reached.

9. The location of the centroid of the compressive concrete (\( Y \)) is determined.

10. The ultimate bending moment capacity \( M_{\text{design}} \) is obtained.

The results from the calculations are found in the following section (D.2).

**D.1.2 Design for Shear**

The provisions of BS8110 assume that the applied shear force (\( V \)) is resisted, if necessary, by the concrete and the steel. It requires that a portion of the shear stress (\( v_s \)) in excess of the concrete shear capacity (\( v_c \)) is to be carried by the steel stirrups. While the determination of the required amount of stirrups is based on the 45° truss analogy approach, the design concrete stress is given empirically in the Code of Practice[9]. The concrete contribution (\( V_c \)) was taken[8, 11] to be equal to \( v_c \) bd. It was necessary to re-define the parameters \( b \) and \( d \) for the circular section. The effective depth (d) could not be less than the distance from extreme compression fibre to the centroid of the tension reinforcement in the opposite half of the member[8, 229]. The parameter ‘\( b \)’ was taken as the diameter of the member[149]. The diameter and spacing of the stirrups were determined using equation (D.9) which was derived or circular sections by Ghee et. al.[149].
\[ V_s = \frac{\pi(2A_{sv}f_{yu})d_s}{4s_v} \]  

(D.9)

where:

- \( V_s \) is the shear force in excess of the concrete shearing capacity which is required to be resisted by the stirrups.
- \( A_{sv} \) is the cross-sectional area of one leg of a stirrup.
- \( f_{yu} \) is the yield strength of the stirrup.
- \( s_v \) is the spacing of the stirrups.
- \( d_s \) is the stirrup overall diameter (centre to centre).

The results of the shear design calculations for the test beams are detailed in the following section.

The following steps were included in the design process of the test specimens to prevent unwanted failures.

- To prevent anchorage failure of the longitudinal bars, the beams were extended 150 mm beyond the centreline of the supports. The effective lengths of the bars were in excess of that required by BS 8110. In addition, the ends of the bottom four bars were bent through an angle of 90° for a distance of at least 100 mm.
• To prevent bearing failure inside bends, the minimum bend radius specified by the provisions of BS 8110 was adopted.

• The ends of the stirrups were overlapped and welded to prevent anchorage failure.

The details of the calculations for the test beams and a complete description of this test programme have been published elsewhere[237].

D.2 DESIGN CALCULATIONS FOR THE BEAMS IN TEST SERIES ‘G’

D.2.1 Assumed Geometrical Properties

The following dimensions were obtained from Figures 7.1 and 7.2.

\[k1 = 66 \sin(67.5) = 60.98 \text{mm}\]
\[k2 = 66 \sin(22.5) = 25.26 \text{mm}\]
\[d_1 = 100 + 60.98 = 160.98 \text{mm}\]
\[d_2 = 100 + 25.26 = 125.26 \text{mm}\]

The effective depth \((d) = 143.12 \text{ mm} \) (to the centroid of the tensile steel)

or,
\[d = 0.8D = 160.0 \text{ mm}\]

For shear calculations, the lower value was adopted.
D.2.2 Assumed Material Properties

(a) Concrete

\( f_{cu} = 40 \text{ MPa} \)

(b) Steel

\( \Phi 8 \)

Diameter = 7.9 mm

Area = 49.02 mm\(^2\)

\( f_y = 377 \text{ MPa} \)

\( T12 \)

Diameter = 11.667 mm

Area = 106.9 mm\(^2\)

\( f_y = 514.5 \text{ MPa} \)

D.2.3 Flexural Capacity

(a) Ignoring the Effect of the Compression Steel

All longitudinal steel was found to have yielded.

\[
T = C = 220000.2 \text{ N}
\]

\[
A_{\infty} = 8208.96 \text{ mm}^2
\]
\[ \alpha = 67.38 \]
\[ e = 100 \cos(\alpha) = 38.46 \text{mm} \]
\[ Y = 63.87 \text{mm} \]
\[ M = M_1 = 23.54 \text{ kN.m} \]

(b) Influence of the Compression Steel

\[ \alpha = 64.36 \]
\[ e = 43.27 \text{mm} \]
\[ c = 56.73 \text{mm} \]
\[ x = 63.03 \text{mm} \]
\[ \epsilon_{s1} = 1.333 \times 10^{-3} \]
\[ f_{s1} = 266.65 \text{ MPa (Compression)} \]
\[ \epsilon_{s3} = 6.5 \times 10^{-4} \]
\[ f_{s3} = 130.05 \text{ MPa (Tension)} \]
\[ \epsilon_{s2} = 3.456 \times 10^{-3} \]
\[ f_{s2} = f_{s1} = 514.5 \text{ MPa (Tension)} \]
\[ T = 247804.89 \text{ N} \]
\[ C_c = 196524.96 \text{ N} \]
\[ A_{cc} = 7333.02 \text{ mm}^2 \]
\[ Y = 66.625 \text{mm} \]
\[ M = M_{\text{design}} = 25.14 \text{ kN.m} \]
### D.2.4 Design for Shear

\[ V = \frac{P}{2} = \frac{M}{a} = \frac{25.14}{0.6} = 41.9 \text{ kN} \]

\[ V_{\text{design}} = 1.5V = 62.85 \text{ kN} \]

\[ v_{\text{applied}} = 2.196 \text{ MPa} < 5 < 0.8\sqrt{f_{\text{cu}}} = 5.06 \text{ MPa} \implies \text{ok.} \]

\[ v_c = 0.92 \text{ MPa} \]

\[ v_s = 1.276 \text{ MPa} \]

\[ V_s = 36524 \text{ N} \]

\[ s_v = 120.8 \text{ mm} \]

\[ s_{v_{\text{max}}} = 0.75d = 107.34 \text{ mm} \]

Use \( s_v = 100 \text{ mm} \).

### D.2.5 Anchorage Length

\[ l_{d_{\text{required}}} = 12\Phi = 144 \text{ mm} \text{ (beyond the centre-line of the support).} \]

\[ l_{d_{\text{provided}}} = 250 \text{ mm} \text{ (actual length) \implies ok. or,} \]

\[ l_{d_{\text{required}}} = 12\Phi + d/2 = 216 \text{ mm} \text{ (beyond the face of the support).} \]

\[ l_{d_{\text{provided}}} = 300 \text{ mm} \text{ (actual length) \implies ok.} \]

### D.3 Amount of Confinement Required to Prevent Diagonal Failures

Comparing the experimental results obtained from beam types B1 and B2

\[ V_f = \frac{P_{\text{traditional}}}{2} = \frac{P_{\text{pa}}}{2} = 49.3 \text{ kN} \]

(for beam type B2 in Table 7.2)

and
\[ V_i \approx V_u = \frac{P_{u1}}{2} = 37.5 \text{ kN} \quad \text{(for beam type B1 in Table 7.2)} \]

From equation (4.5):

\[ K_{s,req.} = 1 + \frac{49.3 - 37.8}{49.3} = 1.24 \quad \text{........................\quad (a)} \]

Alternatively, using Kani's Valley for an equivalent rectangular beam in which

\[ b = 194.85 \text{ mm}, \quad d = 150 \text{ mm}, \quad \rho = 1.5\%, \quad \text{and} \quad a/d = 4.0, \]

\[ \frac{M_u}{M_f} \approx 0.8, \quad \text{and} \quad \frac{M}{M_f} = 0.20 \]

Therefore from equation (4.3):

\[ K_{s,req.} = 1 + 0.20 = 1.20 \quad \text{........................\quad (b)} \]

The values of \( K_{s,req.} \) in (a) and (b) are in close agreement.

It should be noted that in the case of the rectangular beams where \( a/d = 4.0 \), Figures 4.3 and 4.4, the inclined leg only requires nominal stirrups. On the other hand, the corresponding relative flexural capacity of the horizontal leg \( \frac{M_u}{M_f} \) is slightly larger than the relative ultimate flexural capacity of the beam \( \frac{M}{M_f} \). Therefore, for the circular section, it is anticipated that the actual value of \( K_{s,req.} \) would be less than the calculated value if \( \frac{M_u}{M_f} \) (i.e. \( V_i \)) was considered instead of \( \frac{M}{M_f} \) (i.e. \( V_u \)).

Equation (4.7) which was developed for circular columns [155] was used to relate the value of \( K_s \) to the characteristics of the transverse reinforcement.

Hence,

\[ K_{s,prov.} = 1 + \frac{377 \times 49.02}{150 \times 100 \times 29.5} = 1.34 \]
Alternatively, using the model developed by Mander [172]:

\[ K_{sprov.} = 1.37. \]

Therefore, it is concluded that the confinement provided \((K_{sprov.})\) was in excess of the required level of confinement \((K_{req.})\).

**D.4 FLEXURAL CAPACITY**

In calculating the flexural capacity of the test beams, the equivalent concrete compression block was used. In the evaluation of the enhanced flexural capacity due to confinement, the modified equivalent concrete compression block developed by Sheikh and Yeh [178] was adopted. The calculated flexural capacities, based on equilibrium and compatibility requirements, for beam types B2, B3, B4, and B6 are shown in Table D.1.
Beam Type | $K_s$ | $f_{cp}$ | $f_{cc}$ | $M_f$ (kN.m) | % of enhancement
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPa</td>
<td>MPa</td>
<td>Calculated</td>
<td>Observed</td>
<td>Calculated</td>
</tr>
<tr>
<td>B2</td>
<td>1.0</td>
<td>36.87</td>
<td>36.87</td>
<td>21.96</td>
<td>29.58</td>
</tr>
<tr>
<td>B3</td>
<td>1.42</td>
<td>30.22</td>
<td>42.91</td>
<td>22.09</td>
<td>29.22</td>
</tr>
<tr>
<td>B4</td>
<td>1.75</td>
<td>33.05</td>
<td>57.84</td>
<td>25.4</td>
<td>34.71</td>
</tr>
<tr>
<td>B6†</td>
<td>1.75</td>
<td>33.05</td>
<td>57.84</td>
<td>28.72</td>
<td>39.15</td>
</tr>
</tbody>
</table>

† The second layer of the longitudinal bars was positioned at a distance of 106.5mm from the compression face of the beam (see Figure 7.1).

Table D.1: Enhanced Flexural Capacities.
Bibliography


404


tract CEB Bulletin 126).


416


419


422


426


