Appendices

A. Derivation of undeformed chip thickness using machining parameters of SPDT

Appendices A is meant to demonstrate the mathematical relations obtained by applying the principles of geometry to derive the critical machining parameters during nanometric cutting.

Figure A1: Tool-workpiece interface during SPDT with a round nose tool [23]

Applying sine law in \( \triangle abc \) of figure A1, we can have:

\[
\frac{\sin(\pi - (\alpha + \beta))}{R} = \frac{\sin \beta}{f} = \frac{\sin \alpha}{R - dc}
\]

\[
\frac{\sin(\pi - (\alpha + \beta))}{R} = \frac{\sin \beta}{f}
\]

\[
\frac{f \sin(\alpha + \beta)}{R} = \sin \beta
\]

Expanding \( \sin(\alpha+\beta) \) and dividing by \( \cos \beta \) on both sides gives the following:

\[
\frac{f \sin \alpha}{R} = \tan \beta \left(1 - \frac{f}{R} \cos \alpha \right)
\]

\[
\tan \beta = \frac{f / R (\sin \alpha)}{\left(1 - \frac{f}{R} \cos \alpha \right)}
\]
Since, $\alpha = 90 - \theta$  => $\sin \alpha = \cos \theta$ and $\cos \alpha = \sin \theta$

$$\tan \beta = \frac{\frac{f}{R} \cos \theta}{1 - \frac{f}{R} \sin \theta}$$

Let $X = \frac{f}{R}$

Now, $\frac{f}{R} \ll \ll 1$  \( \tan \beta = X \cos \theta \)

The above equation can be presented in the form of triangle as:

![Triangle Diagram]

Using (1) \( \frac{\sin \beta}{f} = \frac{\sin \alpha}{R - dc} \)

$$dc = R - \frac{f \cos \theta}{\sin \beta}$$

Substituting value of $\sin \beta$ and replacing $X$

$$dc = R - \frac{f \cos \theta \sqrt{1 + X^2 - 2X \sin \theta}}{X \cos \theta} = R - R \sqrt{1 + \frac{f^2}{R^2} - 2 \frac{f}{R} \sin \theta}$$

Now, $\frac{f}{R} \ll \ll 1$

$$dc = R - R \sqrt{1 - 2 \frac{f}{R} \sin \theta}$$

Applying Taylor’s expansion and by neglecting higher order terms:

since $\theta$ is extremely small

$$dc = f \sin \theta = f \theta$$

Expression for $Z$:

$$\frac{Z}{R} = \cos(\alpha + \beta)$$
Expanding $\cos(\alpha + \beta) = \cos \alpha \sin \beta - \cos \beta \sin \alpha$ and replacing values from $\Delta$

$$Z = \frac{R \sin \theta - X}{\sqrt{1 + X^2 - 2X \sin \theta}}$$

Now, $f/R << 1$ and $\theta$ is small

$$Z = R \left( \sin \theta - \frac{f}{R} \right)$$

$$Z = R \left( \frac{dc}{f} - \frac{f}{R} \right) = R \frac{dc}{f} - f$$

$$dc = \frac{f(z + f)}{R}$$

Figure A2: Geometry to derive transition point

Since, $Y_c$ is the sub-surface damage which is proportional with the feed rate which occurs at the distance $Z_{eff}$ where (figure A2),

$$Z_{eff} = Z_c - \Delta Z$$

$$Z_{eff} = \sqrt{Z_c^2 - 2RY_c}$$

$$Z_{eff}^2 = Z_c^2 - 2RY_c$$

$$Z_{eff}^2 = \left( \frac{Rd_c}{f} - f \right)^2 - 2RY_c$$

$$Z_{eff}^2 - f^2 = R^2 \left( \frac{dc^2}{f^2} - \frac{2d_c}{R} - \frac{2Y_c}{R} \right)$$
\[
\frac{Z_{\text{eff}}^2 - f^2}{R^2} = \frac{dc^2}{f^2} - \frac{2}{R} \left( d_c - Y_c \right)
\]

In order to evaluate critical feed rate, the process limits would be such that \( Z_{\text{eff}} = 0 \)

\[
f_{\text{max}} = \frac{dc}{\sqrt{2(d_c + Y_c)}} \text{ by assuming that } \left( \frac{d_c}{d_c + Y_c} \right)^2 \ll 1
\]

The maximum undeformed chip thickness, \( d_{\text{max}} \), (effective depth of cut) can be calculated according to the cutting tool geometry and cutting conditions as shown in figure 3.

![Diagram](image)

(a): Large feed rate \( \sqrt{(2Ra_0 - a_0^2)} \leq f \)

(b): Small feed rate \( \sqrt{(2Ra_0 - a_0^2)} > f \)

Figure A3: Schematic for maximum undeformed chip thickness [301]

Here, \( a_0 \) is the depth of cut; \( R \) is the nose radius; \( O_1 \) and \( O_2 \) are the centres of two adjacent arc cutting edges, and the distance between \( O_1 \) and \( O_2 \) is the feed rate, \( f \). The maximum undeformed chip thickness \( d_{\text{max}} \) for the two conditions as shown in figure A3 can be calculated as follows:

The maximum undeformed chip thickness for large feed rate while

\[
\sqrt{(2Ra_0 - a_0^2)} \leq f \text{ can be expressed as: } \\
d_{\text{max}} = a_0
\]
The maximum undeformed chip thickness for small feed rate while
\[ \sqrt{(2Ra_0-a_0^2)} > f \] can be expressed as:
\[ d_{\text{max}} = R - \sqrt{R^2 + f^2 - 2f\sqrt{2Ra_0-a_0^2}} \]

When \( R \gg f \) and \( R \gg a_0 \)

Above equation can be finally simplified as:
\[ d_{\text{max}} \approx \frac{f}{R} \sqrt{2Ra_0-a_0^2} \approx f \sqrt{\frac{2a_0}{R}} \]