Illumination of Channelised Fluvial Reservoirs Using Geological Well-Testing and Seismic Modelling

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Abstract

Fluvial reservoirs are amongst the most prominent hydrocarbon bearing deposits in the world. Complexity in channel networks, spatial pattern and internal heterogeneities are of the main challenges in characterising these types of reservoir.

In this thesis, a novel geoengineering approach is implemented to integrate the multi-domain information (e.g. outcrop and time-lapse seismic interpretation) and to describe the well-test response of certain fluvial deposits. Comprehensive modelling and numerical well-test simulations have then been employed to study the dynamic behaviour of such systems. These resulted in diagnosing a well-testing family that includes a new generalised "Ramp Effect" model that is presented for the first time. Using systematic geostatistical modelling, the ramp effect is elaborated in terms of spatial statistics. The ramp model has been demonstrated by a few real-life well-test examples in a variety of channelised environments. Sophisticated multi-point statistics modelling are utilized to capture the facies transitions producing the lateral cross-flow transients that result in the ramp effect and to demonstrate how the response can be generated in a meandering and anastomosing fluvial environments.

Internal cross-flow can be confused with external layer cross-flow and with other linear flow responses (e.g., parallel faults, natural or artificial fractures). The non-uniqueness in interpretation of the ramp effect is addressed by employing time-lapse seismic data, which help in detecting the spatial geological heterogeneities and constraining the well-test interpretations. The illuminating power of the time-lapse seismic data is illustrated by synthetic seismic modelling examples.

The implications of a complex fluid (i.e. gas-condensate liquid drop-out) on altering the well-test and seismic responses are also discoursed. A compositional reservoir
simulator is employed to mimic the complex fluid behaviour of the fluvial reservoirs showing the ramp effect. Application of compositional simulations highlights the limitations in current petro-elastic modelling that are unable to take the compositional changes into account. Therefore, a novel approach is also developed to improve the petro-elastic modelling which facilitates the synthetic seismic generation in the presence of continuous composition changes of the fluid. This leads towards a better description of the reservoirs under simultaneous effect of geological and fluid heterogeneities.
To My Lovely Parents...
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Contents

Chapter 1: Introduction ........................................................................................................... 1
  1.1 Geological well-testing and geoengineering workflow ............................................... 2
  1.2 Thesis objectives and approach.................................................................................... 3
    1.2.1 Sophisticated modelling ....................................................................................... 3
    1.2.2 Using dual-domain information .......................................................................... 5
    1.2.3 Synthetic seismic response and complex fluid effects ...................................... 5
  1.3 Thesis outlines................................................................................................................. 6

Chapter 2: Layered Reservoirs with Internal Cross-flow: A Well-Connected Family of Well-Test Pressure Transient Responses ................................................................. 9
  2.1 Analytical interpretation of layered reservoirs............................................................ 10
    2.1.1 Macro cross-flow reservoir .................................................................................. 11
    2.1.2 Commingled reservoir ....................................................................................... 15
  2.2 Geological interpretation of internal cross-flow fluvial reservoirs ........................... 17
    2.2.1 Fluvial Reservoirs .............................................................................................. 17
    2.2.2 Internal Cross-flow in fluvial reservoirs ............................................................. 19
    2.2.3 Geoskin and Geochrome real-life example ...................................................... 25
  2.3 The relations between the analytical and geological interpretation of the layered reservoirs ....................................................................................................................... 27
    2.3.1 Cross-flow systems ............................................................................................ 27
    2.3.2 Commingled systems ......................................................................................... 29
  2.4 Chapter summary ......................................................................................................... 30
Chapter 3: Ramp Effect Characterisation

3.1 Ramp-like phenomena

3.2 How to study the ramp effect

3.3 Statistical Interpretation of the ramp effect

  3.3.1 Ramp effect in 3-D: Effect of vertical permeability

3.4 Analytical-numerical interpretation of ramp

3.5 Numerical interpretation of ramp

  3.5.1 Model set-up

  3.5.2 Effect of vertical permeability

  3.5.3 Effect of correlation length

  3.5.4 Effect of no-flow/leaky boundaries on the ramp effect

  3.5.5 Statistical pressure response

  3.5.5 Particular aspects of commingled and cross-flow systems from numerical experiments

  3.5.6 Production rate of separate layers

3.6 Chapter summary

Chapter 4: Ramp Effect Case studies

4.1 Example 1: The well-testing ramp-like response

  4.1.1 Background

  4.1.2 Well-test interpretation

4.2 Example 2: The well-testing ramp response in a meandering environment

  4.2.1 Background

  4.2.2 Multi Point Statistics (MPS) modelling

  4.2.3 Numerical well-test simulations

  4.2.4 Well-test simulation and validation of the ramp effect

  4.2.5 Analytical modelling

4.3 Example 3: The complex ramp effect response in an anastomosing environment
4.3.1 Background ........................................................................................................... 92
4.3.2 Extended well-test program .................................................................................. 92
4.3.4 Well-test interpretation ....................................................................................... 93
4.3.5 MPS modelling ................................................................................................. 95
4.3.6 Well-test simulations of different facies scenarios ............................................. 101
4.3.7 Facies hybridization ......................................................................................... 106
4.4 Alternative interpretation scenarios ....................................................................... 115
4.5 Implications of misidentification of the ramp model ............................................. 115
4.6 Chapter summary ................................................................................................. 116

Chapter 5: Addressing the Non-Uniqueness by Dual Domain seismic Imaging and
Well-Testing ......................................................................................................................... 117
5.1 Reviewing the seismic wave propagation ............................................................. 117
5.2 Reviewing the pressure diffusion process in well-testing ....................................... 122
5.3 Heterogeneity information from seismic and well-test interpretation ..................... 127
5.4 Time lapse (4-D) seismic technology ..................................................................... 130
5.5 Synthetic 3-D and 4-D seismic modelling through an example: Ainsa II
Channelised Reservoir ................................................................................................. 131
5.5.1 Modelling and simulation set up ........................................................................ 131
5.5.2 Petro-Elastic Modelling (PEM) ....................................................................... 134
5.5.3 Synthetic seismic modelling ............................................................................... 137
5.6 Chapter summary ................................................................................................. 140

Chapter 6: Using Dual-Domain Heterogeneity Illumination ........................................ 141
6.1 Heterogeneous model response and interpretation ............................................... 142
6.2 Single-domain interpretation ................................................................................. 145
6.2.1 The utility of radial composite inversion ......................................................... 150
6.3 Dual-domain interpretation .................................................................................. 151
6.3.1 Facies Illumination ......................................................................................... 156
6.4 Applicability of dual-domain integration ............................................................. 158
Chapter 7: Additional Complex Fluid Implications .................................................. 172
  7.1 Gas-condensate reservoirs ................................................................. 172
  7.2 Well-test analysis approach in gas-condensate reservoirs ...................... 174
  7.3 Geological model and simulation set up ............................................. 177
  7.4 Native pseudo-pressure derivative response: heterogeneous model single-phase fluid ................................................................. 179
  7.5 Native pseudo-pressure derivative response: homogenous model, two-phase fluid system ............................................................... 180
  7.6 Interfering effect of the geological and the production parameters combined geology vs. fluid signatures ................................................................. 185
    7.6.1 Effect of rate and production time ............................................... 185
    7.6.2 Effect of correlation length ....................................................... 188
    7.6.3 Effect of $k_{v}$ ........................................................................ 190
    7.6.4 Stepwise Homogenization ............................................................ 191
  7.7 Seismic modelling in presence of complex fluid system ......................... 193
    7.7.1 Petro-elastic modelling in gas-condensate reservoirs ................. 193
    7.7.2 Forward seismic modelling .......................................................... 201
  7.8 Chapter summary ............................................................................ 204

Chapter 8: Conclusions and Future Work ..................................................... 206
  8.1 Conclusions .................................................................................. 206
    8.1.1 Illumination and characterization of Ramp Effect ....................... 206
    8.1.2 Interpretation of Ramp Effect in real-case examples .................... 208
    8.1.3 Reducing the non-uniqueness in well-test interpretation using the illuminating power of the frequently acquired time-lapse seismic data .... 209
8.1.4 Complex fluid effects on seismic and well-test responses of the modelled fluvial reservoir ........................................................................................................... 211

8.2 Future research and recommendations .............................................................................. 212

Appendix A: Sensitivity Coefficients ....................................................................................... 214

A.1 He’s method of estimating the time-dependent sensitivity coefficient .................. 214

Appendix B: Petro-Elastic Modelling ..................................................................................... 220

B.1 Gassman’s substitution theory ......................................................................................... 220

B.2 Pore fluid elastic properties .......................................................................................... 222

B.2.1 Elastic properties of oil ......................................................................................... 222

B.2.2 Elastic properties of water ...................................................................................... 225

B.2.3 Elastic properties of gas ......................................................................................... 226

B.3 Effective moduli of multi-mineral and/or multi-fluid media .................................. 227

B.4 Dry frame moduli ........................................................................................................ 229

B.5 Effective density of saturated medium ...................................................................... 230

Bibliography .......................................................................................................................... 231
List of Publications


Chapter 1

Introduction

The classical transient well-test interpretation is based on the analytical solution of the diffusivity equation under various boundary conditions. These analytical solutions are mainly derived for the draw-down cases and are used for the idealized reservoir characterisation based on the estimation of the reservoir parameters. However, many of the analytical solutions are either very complex and computationally costly (Kuchuk, 1996a; Houzé et al., 2011) or may not even be available for most of the geological and reservoir heterogeneities (Zheng et al., 1996; Zheng, 1997; Kuchuk et al., 2010). Therefore, the numerical techniques are implemented to solve the well-test diffusivity equation along with their associated boundary conditions. When the linear diffusivity equation is considered the numerical well-testing behaves as a “super type curve generator” (Houzé et al., 2011). The numerical well-test simulations are proven to be accurate against the analytical simulations that can be used to tackle the non-linearity (e.g. non-Darcy flow, multi-phase flow and non-consolidated formation), complex well situations (e.g. multi-segment and slanted well) and combined complex reservoir heterogeneities (e.g. multi-layered, multi-facies, highly faulted and fractured). The numerical simulations can sometimes lead to useful analytical formulations. One of the earliest studies on the formulation of numerical well-test models was the one by Ayestaran et al. (1989) who modelled the well-test response in the presence of the patches of non-reservoir or impermeable facies within the formation. They have studied the effect of size, distance to the wellbore and the reservoir formation anisotropy and provided a set of well-test diagnosing type curves. The numerical well-testing can also be used in updating the geological and reservoir models based on a history matching approach (Boutaud de la Combe et al., 2005; Kamal et al., 2005).
1.1 Geological well-testing and geoengineering workflow

The term “geological well-testing”, in a broader sense, can be used instead of “numerical well-testing”. This is referred to the numerical simulations of transient tests by setting up the detailed geological models within which different heterogeneity scales are spatially distributed in the model (Massonnat and Bandiziol, 1991; Corbett et al., 1996; Zheng et al., 1996; Corbett et al., 2010). The complex fluid implications can also be deliberated, which gives the unique opportunity to investigative the competing effects of the geology and fluid in altering the dynamic behaviour of the well. This process requires a “geoengineering” workflow in order to integrate the multi-domain information (e.g. Geology, Geophysics and Engineering) and to constrain the well-test modelling and interpretation within a unified framework (i.e. a geological model). Meanwhile, the analytical methods are the pre-steps to numerical well-tests and are still relevant for most of the realistic petroleum reservoirs (Kuchuk et al., 2010; Stewart, 2011).

The well-test interpretation is an inverse problem with non-unique solutions (Bourdet, 2002). This is partly related to sparse data over large 4-D domain (Kuchuk et al., 2010). However, the external information (e.g. well log, core, production log, spatial pressure measurements and seismic data) can be employed to reduce the non-uniqueness nature of the solution (Ehlig-Economides et al., 1990; Robertson et al., 2002; Zheng et al., 2003; Boutaud de la Combe et al., 2005; Gok et al., 2005). Figure 1.1 presented by Corbett et al. (2010) shows a geoengineering workflow for well-test interpretation. This workflow was further extended by Kuchuk et al. (2010) to have an impact on well-test design and execution. Therefore, they have added some operational aspects to the iterative geoengineering loop, which was then called an “outer loop”. Some of these steps include selection of testing hardware and gauges, test design, operation of test and data acquisition and real time interpretation.
1.2 Thesis objectives and approach

1.2.1 Sophisticated modelling

Fluvial reservoirs are important hydrocarbon bearing deposits and their dynamic response (i.e. well-test and 4-D seismic responses) are the main subject of study in this dissertation. It is the fluvial depositional characteristics that give rise to the complex reservoir architectures geometries, spatial distribution patterns, internal heterogeneities petrophysical properties as well as the connectivity between flow units channel sands that combine to give great uncertainty in characterising the effective reservoir properties (Corbett et al., 1998). Therefore, characterisation and interpretation of these reservoirs are great challenges to conquer (Bowman et al., 1993). In this thesis, the geological well-test responses of particular commingled fluvial channelised environments are considered where the pressure derivative curve monotonically increases with time. These reservoirs range from the high-energy laterally deteriorated braided fluvial channels to low-energy anastomosing streams. Different modelling approaches can be used to distribute the spatial heterogeneity in the corresponding geological models.
These include the conceptual models derived from the well data (Johnson and Krol, 1984), stochastic (or geostatistical) models (Strebelle and Journel, 2001) and process-based models (Willgoose et al., 1991). Deficiency and high uncertainty of the conceptual models and difficulty of the time-dependent process-based models lead the geostatistical modelling (i.e. pixel-based and object-based modelling) to be the first choice for modelling of these environments.

In light of geostatistical modelling, some parametric studies are performed to scrutinize the well-test response of geological models along with the acceptable ranges of uncertainty in facies and petrophysical parameters (e.g. variogram type and correlation lengths). These ultimately provide a family of geotype curves (Corbett et al., 2005), that systematically explores the limits of early, middle and late-time responses of particular reservoir environments with any level of fluid and geological heterogeneities (Massonnat et al., 1993; Corbett et al., 1996; Zheng et al., 1996; Corbett, 1997; Zheng, 1997; Sagawa et al., 2000; Ellabad et al., 2001; de Rooij et al., 2002; Robertson et al., 2002; Corbett et al., 2005; Mijinyawa and Gringarten, 2008; Corbett, 2009; Corbett et al., 2010).

Particular fluvial reservoirs containing the reserves in partially connected sandbodies pose special challenges for geological modelling because the influence of isolated fluvial bodies is important when fluid transport and production profiles are calculated (Henriquez et al., 1990). The examples of isolated sandbodies could be that of the low-energy anastomosing and meandering reservoirs where the fluvial deposits are separated by the clay-plug channels. Sparse static conditioning data, limited spatial information, the complex 3-D channels network and non-unique solutions are amongst the main difficulties in producing the reliable reservoir models. In these situations, because the variogram cannot look at spatial continuity between more than two locations at time, pixel-based algorithms give poor representations of the actual facies geometries (Strebelle and Journel, 2001). The object-based models, on the other hand, are difficult to be constrained by the well data. Therefore, the more sophisticated multi-point statistics (MPS) models are employed to construct the complex geological heterogeneities which can better preserve the spatial connectivity of the reservoir facies. Such multiple-point statistics cannot be inferred from typically limited well data but could be read from training images depicting the expected subsurface heterogeneities.
Chapter 1: Introduction

(Strebelle and Journel, 2001). The training image, which carries the spatial relation of the reservoir facies in more than two points in the reservoir, could be a seismic map, an outcrop analogue, a hand-made conceptual model or even a satellite image. The conditioned MPS models are calibrated against the real well-test data and the final matches to the tested data are obtained based on the numerical solutions of the geological testing rather than forcing the simplified and average analytical solutions.

1.2.2 Using dual-domain information

The geological well-test models can be extended to honour the information from other domains (e.g. seismic domain). This further reduces the non-uniqueness of the well-test modelling. The analysis of 4-D seismic data can improve the quality of reservoir characterisation, identify movement of fluid interfaces, and help operators locate bypassed reserves (Fanchi, 1999). The time lapse data can reduce the uncertainty in prediction of the fault transmissibility multiplier, areal sweep and pressure distribution (Stephen and Macbeth, 2006). Therefore, the accurately designed 4-D seismic data convey invaluable information about the lateral reservoir heterogeneities and can be used to reduce the non-uniqueness nature of modelling. This can even result in locating the deterministic geo-objects with the lower degree of uncertainty in the model to appreciate their ramifications in the dynamic behaviour of the modelled system. In a much broader sense (i.e. in long term production) a seismic history matching loop can also be implemented to use the quantitative 4-D seismic information in the reservoir model (Stephen and Macbeth, 2006; Arwini and Stephen, 2011). In this thesis, it is shown that how the 4-D seismic data can be used to reduce the non-uniqueness in the well-test modelling by locating the interpreted lateral heterogeneities.

1.2.3 Synthetic seismic response and complex fluid effects

The fluvial reservoirs are amongst the most common reservoirs in the North Sea where the gas condensate fluid system are not uncommon (e.g. Heron Field (Mckie et al., 2010), Frankin Field (Suiter et al., 2005) and Marnock Field (Fisher and Mudge, 1998)). Therefore, the implications of such complex fluid systems on the well-test behaviour are studied. The fluid complexity can impose a second level of intricacy on the well-test response which complicates the well-test interpretation in presence of the complex geology.
In addition to the well-test response, the geological models can also be used to build the corresponding petro-elastic models (Gassmann, 1951). The petro-elastic models can be considered as a bridge between the geophysics (i.e. seismic response) and the reservoir engineering domain that can link the information of these domains. Although, the petro-elastic modelling of black oil and dry gas systems is common (Batzle and Wang, 1992), this is not widely documented for the gas-condensate systems. To the best of the author’s knowledge, the compositional modelling of the petro-elastic modelling in presence of the complex fluid (i.e. gas-condensate fluid) is formulated for the first time in this thesis. Hence, the oil and gas phase composition changes, which happen during the production history, are accurately taken into account. Implementing the accurate petro-elastic models and considering the actual survey configuration, these models can then be employed to generate the synthetic seismic responses of the geological model at different simulation time-steps (Amini et al., 2011). This provides guidelines when analyzing the real seismic data and helps examine the detection ability of the current seismic practices under the certain level of the fluid and geological heterogeneities. Besides, if the reservoir is monitored by the time-lapse seismic surveys during the well-test (e.g. using permanent sensors), and if the existence and the lateral extension of the reservoir liquid drop-out is confirmed by the 4-D seismic signals, the non-unique well-test interpretations can be subsequently reduced.

1.3 Thesis outlines

In Chapter 2, the multi-layered cross-flow and commingled reservoirs are briefly reviewed. The three extrema variants that are potentially present in fluvial systems are presented. These well-test family members include the geoskin, geochoke and a new generalised “ramp effect” response that is the core subject of this thesis.

In Chapter 3, the well-testing “ramp effect” response is fully characterised by statistical, analytical-numerical and numerical approaches. Thousands of black-oil simulations are performed and the ramifications of the statistical and the geoestatistical parameters on the ramp responses are excogitated. A realistic high net:gross commingled braided-fluvial reservoir is used to illustrate the appearance of the ramp effect on the well-test response and a set of geotype curves are generated.
In Chapter 4, few real-life well-test examples in different fluvial environments are studied. The examples range from the geological interpretations of the ramp effect in a short test conducted in a high energy braided stream reservoir to a very long extended build-up test in a low energy anastomosing system. In presence of the sparse well data and complicated geology, the complex multi-point statics modelling were employed within a geoengineering framework to match the real test data with the model response.

In Chapter 5, the joint well-test and seismic approach (i.e. a dual domain) is introduced to address the non-uniqueness nature of the problem. In this chapter, the pressure diffusion of well-tests and the propagation of the seismic waves are discussed. A fundamental comparison between these two domains is made and different approaches of information integration are reviewed. The petro-elastic modelling and the 4-D seismic response are introduced as sources of information that can be used to reduce the uncertainty in well-test interpretation.

In Chapter 6, the frequent time lapse seismic maps are used to reduce the non-uniqueness modelling in well-test interpretations. This is illustrated through a ramp-effect example in commingled fluvial reservoir with further external boundaries effect. The time lapse seismic data convey the valuable information about the existence and location of the lateral heterogeneities (e.g. permeability baffles) that are likely to be hidden in common 3-D seismic routines and be ignored during the test interpretations. The feasibility of these frequent 4-D seismic gatherings is also conferred from reservoir engineering viewpoint. Moreover, as a step forward, the direct permeability extraction form the pressure maps (inverted from 4-D seismic data) is examined.

In Chapter 7, the implications of gas condensate fluid on the well-test and time lapse seismic response are addressed. This is conducted through compositional simulations of a real rich gas-condensate fluid system. This aims at exploring the competing effects of the fluid complexity (i.e. liquid drop-out) and the geological complexity (i.e. ramp effect) on altering the native single-phase well-test response. The fluid complexity also reflected in the synthetic seismic response of the model. A workflow is developed for the first time to include the compositional variations of the fluid phases in the petro-elastic model that is eventually manifested in the synthetic seismic response.
Chapter 1: Introduction

Thesis conclusions and further work are discussed in Chapter 9. Appendix A presents the mathematical framework to calculate the well pressure sensitivity coefficient that is used in Chapter 3, and the Appendix B includes the Petro-elastic modelling approach for common black oil and dry gas reservoirs.
Chapter 2

Layered Reservoirs with Internal Cross-Flow: A Well-Connected Family of Well-Test Pressure Transient Responses

Cross-flow reservoirs are reservoirs where flow occurs between units within the reservoir. This has been long considered for reservoir with flow between layers and results in a typical V-shape pressure derivative response. The role of layers is emphasised; “If the layers are in hydraulic communication within the reservoir (as well as through the wellbore) the layers are said to experience cross-flow” (Horne, 1995). Bourdet’s (2002) index has “Cross-flow: See Interlayer Flow sic Layered Reservoirs”. The mathematical intricacies associated with analytical well-test modelling of these reservoirs result in some simplified geometries (i.e. “layer-cake models”) where some associated cross-flow phenomena are ignored or misinterpreted. In particular, this issue will be further highlighted in the complex network of layered fluvial channels whenever the geological processes impose more complexity in the well-test interpretation. This chapter starts with a brief introduction to layered reservoirs with the related well-test concepts. A geoengineering approach is used to describe three transient response extrema variants in architecture that are potentially present as a continuum in fluvial systems and thus the pressure responses can be assembled as a family of geotype curves. These transient response extrema are the geoskin, the geochoke and the ramp effect.
There will be a review of the first two that have already been published (Corbett et al., 1996; Corbett et al., 2005). However, the ramp response is a newly identified member and is the core subject of this thesis. This will be briefly introduced in here, to be followed by the detailed discussion and characterisation of ramp effect will be presented later in the coming chapters.

2.1 Analytical interpretation of layered reservoirs

The classical (i.e. analytical) well-test model of the layered reservoirs is commonly based on an idealized mathematical model (e.g. isotropic homogenous layer-cake model), where the reservoir consists of horizontally homogeneous and isotropic layers with different permeabilities, porosities, thicknesses or skin factors. Despite the model simplicity, the mathematical solution of the associated partial differential equation is somewhat complicated and cumbersome.

The reservoir layers experience cross-flow when they are in hydraulic communication within the reservoir. Figure 2.1 shows a cross-flow reservoir where the layers can communicate within the reservoir and the wellbore. Because the cross-flow happens in macro scale (i.e. between the layers and/or reservoir zones), these types of reservoirs can be classified as “macro cross-flow” reservoirs.

On the other hand, the fluid production is commingled at the wellbore when the layers are separated by some impervious interfaces (or layers). Figure 2.2 shows a commingled reservoir where the reservoir layers can only communicate within the wellbore.

There are some economic preferences in the cross-flow reservoirs compared to the commingled reservoirs; the cross-flow reservoirs have a shorter operating life, higher
ultimate recovery, a reduced perforating and completing cost and requires less engineering time for interpretation (Chaudhry, 2004). Therefore, there might be some growing interest in converting commingled reservoirs into the cross-flow reservoirs using some particular techniques like hydraulic fracturing.

![Figure 2.2: A commingled reservoir where the producing layers are separated by some impervious layers. The producing layers can only communicate within the wellbore.](image)

It should be noted that besides the reservoir structure, the detailed well-test interpretation of the multi-layered reservoirs depends upon the testing procedures, as well. In the conventional testing procedures for instance, when all the layers are tested simultaneously, only the average permeability and skin factor of the system are estimated. However, if the individual layer rates are available the layers permeabilities and the skin factors can be accordingly determined. Getting at this can be done either by testing each individual layer separately or using some techniques of monitoring the down-hole pressure and the rate of layers (Kucuk et al., 1986; Ehlig-Economides and Joseph, 1987; Larsen, 1994). For example, Kucuk et al. (1986) presented a testing procedure for multi-layered reservoirs, which is based on measuring the pressure and the rate at the top of individual layers using a PLT tool. This involves moving the testing device up and down the well with multiple drawdown testing procedures and can provide an estimation of the individual layer permeabilities and skin factors.

### 2.1.1 Macro cross-flow reservoir

The macro cross-flow occurs after some time of production where the high permeable layer depletes faster. This provokes a vertical pressure difference between the high and low permeable layers. The flow (that is defined by the interlayer cross-flow coefficient, $\lambda$) occurs from the low to high permeable layers and could either happens under a pseudo-steady state (Gao, 1984; Bourdet, 1985; Ehlig-Economides and Joseph, 1987;
Park and Horne, 1989; Kuchuk, 1996b) or a transient state (Chen et al., 1990). Studies have shown that if the permeability contrast of layers were greater than 100 then the cross-flow direction would be almost vertical (Sabet, 1999).

When testing all reservoir layers together, the pressure derivative response frequently looks a normal single layer reservoir (Horne, 1995). However, when there is a high contrast in the layer thicknesses, skin factors and the permeabilities, the response can bear a resemblance to the double porosity reservoir where a V-shape signature appears on the derivative curve.

This V-shape response is characterised based on the layers capacity contrast, \( \omega \), the layers transmissivity contrast, \( \kappa \), and the interlayer cross-flow coefficient (or the exchange term), \( \lambda \) (Horne, 1995; Bourdet, 2002). For an equivalent two-layer system and using a semi-permeable wall resistance between the layers (Gao, 1984), these parameters are defined as follows

\[
\omega = \frac{(\varphi h c_i)_1}{(\varphi h c_i)_1 + (\varphi h c_i)_2} \quad (2.1)
\]

\[
\kappa = \frac{(k h)_1}{(k h)_1 + (k h)_2} \quad (2.2)
\]

\[
\lambda = \frac{r_w^2}{(k h)_1 + (k h)_2} \frac{2}{2 h'_z + \frac{h'_i}{k'_1} + \frac{h'_z}{k'_2}} \quad (2.3)
\]

where, \( \varphi \) is porosity, \( c_i \) is total isothermal compressibility coefficient in “1/psi”, \( \varphi h c_i \) is the storage capacity of each layer in “ft/psi”, the \( k h \) is the layer transmissivity in “md×ft”, \( r_w \) is the wellbore radius in “ft”, \( k_{z1} \) and \( k_{z2} \) are the vertical permeabilities of the layers in “md” and \( k'_z \) and \( h'_z \) are the vertical permeability and thickness of the semi-permeable membrane separating the reservoir layers. The high permeable layer is considered as layer 1. The smaller the capacity contrast (\( \omega \)) or the larger the transmissivity contrast (\( \kappa \)), the deeper the valley amplitude. However, the capacity contrast or the transmissivity are not always independent. In the sandstone reservoirs for example, there are usually some relations between the porosity and the permeability. This means that the permeability and porosity are inter-related and the high permeable
layer usually has a higher porosity. In this situation, the thickness (and isothermal compressibility factor) of the layers can be considered as the controlling parameters of V-shape appearance on the derivative curve.

The interlayer cross-flow coefficient on the other hand, affects the time at which the valley appears on the derivative curve. The smaller the $\lambda$ and/or the larger the $\omega$, the later the V-shape appearance on the Log-Log diagnostic plot. If the high permeable layer has a negligible storage, then the cross-flow duration could continue throughout the life of the well. This is while the pressure can rapidly stabilize whenever the vertical permeability of the high permeable layer is greater than its horizontal permeability and the horizontal permeability contrast of the layers is of order of 10 (Streltsova, 1988).

It should be noted that the Log-Log diagnostic plot consists of two separate curves: $\Delta P$ and $\Delta P'$ (or the derivative). and they are defined as follows

$$\Delta P = P_i - P_{wf}(t)$$

$$\Delta P' = \frac{d(\Delta P)}{d \ln(t)}$$

for a drawdown test, and

$$\Delta P = P_{ws}(\Delta t) - P_{wf}(\Delta t = 0)$$

$$\Delta P' = \frac{d(\Delta P)}{d(\ln(\frac{t_p + \Delta t}{\Delta t}))}$$

for a build up test, in which $P_i$ is the initial pressure, $P_{wf}$ and $P_{ws}$ are the wellbore pressure during the draw-down or shut-in periods respectively, $P_{wf}(\Delta t=0)$ is the wellbore pressure at the time of shut-in, and $t_p$ is the total production time before the shut-in period.

For more general cases with multi rate history or continuous down-hole rate and pressure measurements the derivative is taken with respect to a more general function that is called superposition function. This function is a complex function of the rate and
time and is usually different for build-up and draw-down responses (refer to the well-testing text books: Horne, 1995, Bourdet, 2002, Hoze et al., 2011 or Stewart, 2011). An example of the superposition derivative will be presented in Chapter 3 where the superposition derivatives of layer pressures are empirically related to the constant rate draw-down derivative response of the well pressure.

A geological example for the applicability of the double permeability model would be that of the carbonate reservoir where thick limestone layers, with low permeabilities, are interbedded with very thin, high permeable dolomites. Here, the transmissivity ratio could be remarkably close to unity, which results in a well-test behaviour resembling the fissured reservoir (Sabet, 1999).

Even if the double permeability parameters are in favour of having a V-shape derivative response, this may not appear on the Log-Log diagnostic plot. This happens, for instance, where the high permeable layer is not perforated and the limited production is coming from low permeable layer. Hence, the derivative response of the cross-flow reservoir is similar to that of the limited entry partial perforation well.

Figure 2.3 shows the Log-Log diagnostic plot of a double permeability reservoir with different transmissivity ratios. When two layers are producing to the well, the sequence of flow regimes on the Log-Log derivative plot is an “early stabilization”, a “V-shape” signature, and a final equivalent homogeneous stabilization. Usually, the early stabilization is not a typical feature and only the late time stabilization can be analyzed to provide a thickness-weighted homogeneous permeability and an average skin factor (Bourdet, 2002). However, when only one layer is producing into the wellbore, the derivative response differs from this and the V-shape derivative signature may disappear. The individual layers skin factors can also considerably affect the Log-Log diagnostic plot of a double permeability reservoir (refer to Bourdet (2002) or Houzé et al. (2011)).
2.1.2 Commingled reservoir

Under the assumption of the infinite homogenous reservoir, the well-test response of a commingled reservoir entirely looks like a single layer response (Lefkovits et al., 1961) but with a higher amplitude of response than an equivalent homogenous system (Bourdet, 2002). This provides with an average permeability and a skin factor. The average permeability is the thickness-weighted average of the layers permeabilities. This is only exact whenever the layer diffusivities and the skin factors are equal. The unequal skin factors may create a considerable underestimation (up to 50\%) of the total flow capacity of system from the early time Semi-Log straight lines (Larsen, 1981). On the other hand, the skin factor obtaining from Semi-log straight line is a function of a small pseudo-skin factor (Larsen, 1981), due to layering, and also an average mechanical skin factor of the present individual layers (Tariq and Ramey, 1978; Prijambodo et al., 1985).

This can be shown as follows

\[
S_I = \frac{1}{2} \sum_{i=1}^{n} \frac{(kh)_i}{(kh)_i} \ln \left( \frac{(kh)_i}{(kh)_i} \frac{\phi \mu c_i}{\phi \mu c_i} \right) + \sum_{i=1}^{n} \frac{(kh)_i}{(kh)_i} S_i
\]  

(2.8)

in which, \(S_i\) is the mechanical skin factor of layer \(i\), and \(S_I\) is the skin factor including the pseudo-skin and average mechanical skin factor. It is worth noting that when the layers are bounded and have different drainage areas, the well-test response might show some
different signatures than a homogenous-like behaviour. In a particular case of a two-layer reservoir, when the high transmissive layer is bounded, the early stabilization is followed by a “middle time” derivative rise. Wherever the permeability contrast is greater than 10, the derivative rise takes a unit slope trend, which shows the high permeable layer is depleting (Stewart, 2011). This causes the deliverability of the well to decrease with time (Sabet, 1999). The analysis of this unit slope trend can provide the volume of the high permeable layer. If the test duration is long enough, the derivative response stabilizes on a secondary stabilization, which corresponds to the radial flow in low permeable layer. It should be noted that the time to reach the total pseudo-steady state condition for a commingled system could be extremely long and may take years (Lefkovits et al., 1961; Kazemi, 1970; Cobb et al., 1972). This would be a stumbling block in determining the average pressure from the short build-ups or draw-downs.

Figure 2.4 shows the draw-down comparison for two commingled systems of a two-layer reservoir at which one of them has a bounded high permeable layer. The single-layer homogeneous-like behaviour of the unbounded system (i.e. a single uniform stabilization corresponds to the arithmetic average of permeabilities $k=550$ md) is dramatically changed when the high permeable layer is bounded (i.e. there is a secondary stabilization corresponds to low permeable layer $k=100$ md). More to the point, the build-up response of such a system is essentially the same as the draw-down unless there is usually a production time effect before the unit slope flow regime. This generally happens when the pseudo-steady state regime is not reached during the preceding draw-down period. The influence of production time on the build-up response is manifested as a sudden reduction of the derivative curve before the middle time unit slope trend.
Figure 2.4: The draw-down responses of two commingled systems of a two-layered reservoir at which one of them has a bounded high permeable layer. The black dotted curves are the well-test response of infinite layers, and the grey curves are the test response where the high permeable layer is bounded.

2.2 Geological interpretation of internal cross-flow fluvial reservoirs

2.2.1 Fluvial Reservoirs

Fluvial reservoirs are the ones related to the transport and deposition of sediments in the channels that are highly variable in terms of geometry (sinuosity and shape), internal characteristics and spatial distribution (connectivity). The channel pattern can vary from a single ribbon channel or isolated point-bar to even more complex multi-story channels (Corbett et al., 1998; Zheng et al., 1998). The meandering and braided channel deposits are two prominent sub-elements of the fluvial environments. The meandering reservoirs are high sinuosity channels that provide the environment for deposition of sediments in the river point-bars (mainly sands deposits) or in the oxbow-lakes (mainly silty and clay deposits) (Richardson et al., 1987). The high net:gross (vertically and areally) braided fluvial reservoirs (e.g. Triassic Sherwood Formation of the Wytch Farm Field, the Lower Jurassic Statfjord sandstones or Brent Field) were the result of the deposition of the sediments of the low sinuosity rivers (Richardson et al., 1987) on a continental environment. The braided river deposits are of good quality with porosity up to 30 % and the permeability up to thousands millidarcies (Serra, 1985). The coarse-grained sediments deposit in the base of the channels leading to higher quality sands in the base while the silty and shaly sediments are found in the upper part of the channels. The overbank flow results in broadening of the channels and creating of the fine-grained
sediments (i.e. the levees). The braided fluvial deposits have excellent to good lateral connectivity and poor (e.g. layered flood plain) to good (e.g. massive braid plain) vertical connectivity (Richardson et al., 1987). The anastomosing deposits are another important type of the fluvial environments that might have multiple channel belts of which individual channel belts can be braided, meandering or straight channels (Bart, 2001). However, most of anastomosing rivers consist of straight channels (Smith, 2009) and are characterised by complicated overbank deposits (Weerts, 1996). In this environment, the reservoir deposits are in the form of some elongated (and possibly isolated) sidebars with high quality sands that are deposited in one side of the channel belt.

The transient well-test response of the fluvial environments is commonly associated with a middle time linear flow regime on the derivative curve. This is a response of a simplified model of a pair of infinite parallel no-flow boundaries. Therefore, this could be interpreted as an interface between the faulted (or fractured) and channelised reservoirs. This simplified geometry, enables us to derive an analytical solution for the linear flow regime, and estimate the channel width. Although, this estimated channel width seems to be a deterministic and engineering-based quantity, it can also be used in some stochastic reservoir modelling workflows. For example, Massonnat et al (1993) used an object modelling algorithm with simplified shapes (i.e. half pipes and discs) to represent the channels and levees in the geological modelling of Gerry field. In their study, the analytical well-test interpretation provided a channel width of 60 m which was directly assigned as the constant channel width in the stochastic model. In this approach, the number of the channels (i.e. the volumetric proportion of channels in the model) was the controlling parameter to acquire the reasonable match. Another example was presented by Holden et al.(1995) who used the well-test average permeability and the nearest distance to the channel boundary in the stochastic reservoir modelling. In their approach the estimated distance, L, including some uncertain bounds (i.e. $L \pm \Delta L$), was used to condition the generated channel structures to the analytical quantities and to eventually match the overall test response. However, it should be noted that, if the channel width in the geostatistical model is not constant, and if the permeability and porosity are not uniform inside the channel, then the computed channel width from a well-test will be related to the width in the region near the well, but the relationship will probably not be straightforward for complex models (Dean S. Oliver Pers.Com.).
Clearly, the geological complexities related to the lateral and vertical stacking of the channels may impose the further complexities associated with multi-layered environments. Hence, it is always necessary to employ the geological interpretation from additional sources of data like outcrop studies. Therefore, the scope of well-testing has been enhanced from mainly a parameter estimation technique to a more sophisticated discipline from which different levels of reservoir heterogeneity can be analysed (Du and Stewart, 1994).

Geological interpretation of the well-testing has long been recognized by many authors (Massonnat and Bandiziol, 1991; Corbett et al., 1998; Zheng et al., 2003). The well-testing can have a principal role in choosing the right analytical well-test model. This is usually applied when the underlying geology is not too complex. Moreover, the well-testing disciplines help diagnose the effect of the multi-scale heterogeneities on the well-test response. This is commonly required because the complex geological heterogeneities cannot be simply presented in terms of average analytical well-test models which can be misleading. For example, the calculation of the distance to the no-flow boundaries is disturbed by the presence of complex channel networks or presence of levees (Massonnat et al., 1993). The well-test response of well centred within two parallel no-flow boundary is associated with a middle time \( \frac{1}{2} \) slope in the pressure derivative response where the channel width can be estimated from analysis of this straight line (Stewart, 2011).

### 2.2.2 Internal Cross-flow in fluvial reservoirs

Certain fluvial reservoir types have extreme property variations at various scales within the reservoir layers/units. Flow occurs laterally within the layers with varying degrees of vertical communication. These (micro-) cross-flow effects manifest themselves as either negative geoskin, a geochoke or sustained by-passed cross-flow (described here for the first time as a “Ramp Effect”). The ramp effect is a new well-testing response that is one of cross-flow – not between layers – but within layers. The ramp phenomenon can occur when there is no effective vertical permeability.

Cross-flow can occur various ways in these systems. Where there is good lateral and vertical communication, where there is restricted communication laterally and vertically
and where there is only restricted lateral variability (Figure 2.5). These result in the geoskin, gechoke and the new ramp pressure response, respectively.

Early well-testing studies on the fluvial systems attempted to find semi-analytical solutions (Sagawa et al., 2000) for internal heterogeneities. However, these proved too cumbersome for practical use and the numerical simulators were used to generate type curves for diagnostic uses (Corbett et al., 1996; Sagawa et al., 2000). Numerical well-testing using black oil simulators (with improved gridding solutions) became the preferred option for tackling heterogeneous reservoirs (Robertson et al., 2002) and this approach has also been described as “geological well-testing” (Kuchuk et al., 2010).

**Geoskin:** Negative geoskin, arising from high permeability elements, was first described from numerical well-testing models (Corbett et al., 1996) and will occur in high net-gross reservoirs with good vertical and lateral communication (Figure 2.6: top). This is a fairly large skin value that often occurs in stimulated wells where high conductive zones control the production. Although, these conductive zones could be due to the presence of natural or induced fracture, it has been observed that certain types of the reservoirs (e.g. high net:gross reservoir) show a highly negative skin value. This negative skin—“geoskin”- can provide an indication of high permeability lenses of limited extent (less than 10% of the depth of investigation) originated from the minor channels intersecting the well. Since these channels have the well-testing response normally associated with fractures, they have been termed “Pseudo-fracture channels (PFC)” (Corbett et al., 1996).
Figure 2.5: Three variants of fluvial reservoir architecture with subtle differences. In each of these systems net:gross would be relatively high (greater than 50%) so these would be known as high net:gross fluvial systems. These are often braided in nature but can also contain meandering elements. In the latter case there is high net:gross over the lateral domain. Channels often have width thickness ratio of order 1:30. These images have channel aspect ratios 1:6 suggesting a vertical exaggeration of 5. Top two models are more typical braided systems whilst the lower one is more meandering. Top: A moderately thick unit of well stacked vertically and laterally channels. Internal properties of channels are variable – leading to patches of high permeability intra-channel elements. Middle: A lower net to gross system where channels are less well connected. Lower: A fluvial sequence containing a number of layers with low vertical connection. The layer average permeability may decrease away from well, and the connectivity of the channel fill deposits within the layers can be moderate or poor.

This means that there is commonly a half-slope derivative trend in the early time response. Corbett et al. (1996) performed several sensitivity analyses and found out that the level of geoskin is sensitive to radius, thickness and permeability contrast of the PFC’s and the background matrix, while it is insensitive to number or location of PFC’s intersecting the wellbore. Corbett et al. (1996) further explained that the geoskin can be
used along with the core analysis and production logs to predict the extent of the PFC’s. They provided some geological knowledge that allows the well-test to be used more quantitatively in the description of fluvial reservoirs. The contrast in permeability between the pseudo-fracture channel (1:100, 1:1000) and the effective radius of the elements (less than 10% of the depth of investigation of the well-test) were noticeable limitations. There must be little impediment to vertical flow for the response to be seen. Clearly, recognition of – or the expectation of – negative geoskin is a measure of the quality of the completion. In some circumstances, a well with zero skin might still be damaged with resulting reduced productivity. Misidentification of these pseudo-fracture elements can result in the false interpretation of a fractured reservoir (Ellabad et al., 2001).

*Geochoke:* The geoskin is an early time phenomenon and is developed when the pressure response extends beyond the high permeability patches connected to the well. However, the presence of additional patches beyond the restriction leads to expansion of flow which has been termed “geochoke” (Corbett et al., 2005). The geochoke phenomenon occurs as a ‘hump’ on the derivative curve (Corbett et al., 2005). The hump is a combination of a negative skin, a short radial flow and later expanding flow (expanding $k \times h$) in sequence. The hump is caused by an effective restriction (choke) in the near well bore region caused by ineffective communication between geological bodies (hence geochoke) (Figure 2.6: middle). Channels that intersect the well are readily depleted. Recharge takes time to be effective and this slow recharge results in a short period of restricted flow.

*Ramp:* The generalised ‘ramp’ response is a further extension of the geoskin and geochoke family of test responses occurring when there is an extended reduction of $k \times h$ due to constrained flow in highly heterogeneous layers. It is termed a ramp because it may have slopes that approximate $\frac{1}{2}$ in extreme cases. The ramp is manifested in draw-down or build-up response of commingled systems as a monotonic increase of pressure derivative over at least one logarithmic cycle. The ramp should eventually ‘plateau off’ at the effective transient $k \times h$ of the system (Figure 2.6: lower and Figure 2.7). This effective $k \times h$ is, for example, the exact geometric average in 2-D systems or 3-D random systems.
The ramp effect has been observed in some high- (e.g. braided) or low- (e.g. meandering or anastomosing) energy fluvial environments. In the braided streams, for example, the ramp is regularly due to lateral reduction of the permeability away from the well, which is a continuous “petrophysical property”. However, in the meandering or anastomosing streams the ramp effect may be either due to the reduced proportion of the high-quality “facies” within the larger investigated volumes, or due to flow between the patches of high quality sandbodies that are separated by the low quality silts or shales (sand communication within some semi-transmissible interface). There might also be some more complex well-test signature in highly heterogeneous and non-stationary environments. In these situations, there might be a combination of the “reduced k×h” of a smaller scale (still can be distinguished by well-test response) and the “sand patches connectivity” issue in much larger scale. The latter needs an extended well-test program. The ramp effect is a general response that might also be seen in very heterogeneous carbonate reservoirs (Corbett et al., 2010; Stewart, 2011).

The ramp effect will be discussed in details in the next chapters of this thesis and some case studies will also be presented.

It should be highlighted that in the real world there might be some situations with a complex architecture that show a transition between these three well-test extrema. For instance, if there reservoir geology contains some meandering or braided elements, the vertical and lateral connectivity variations within the reservoir layers could result in a situation with a transient pressure response between the geoskin, the gechoke and the ramp effect.
Figure 2.6: Representation of the well-test responses in the fluvial systems shown in Figure 2.5. **Top:** Initial flow will come from the higher permeability elements WITHIN the channels. These elements are small relative to the depth of investigation and will show up in the well-test response as negative geoskin. **Middle:** Channel flow is experienced in early time followed by channel recharge which can be effective in 3-D giving rise to a choking effect. **Lower:** Where there is no vertical permeability the recharging comes from the poorer quality reservoir left behind the depletion 'radius' in the high-permeability channels. In this case, there is intra-layer cross-flow from bypassed elements. Apparent flow away from the well is connected to the well through a 3-D network (this is a simplified illustration of the process and the arrows indicate the flow directions).
2.2.3 Geoskin and Gechoke real-life example

Previous work has considered the testing of two wells (Wells A and B) in the Wytch Farm Oilfield. The Sherwood Sandstone reservoir in the Wytch Farm Field is a braided fluvial reservoir and these two wells are particularly interesting to study as they have different well-test responses, despite being in the same reservoir unit (Toro-Rivera et al., 1994; Zheng et al., 2007).

A Drill Stem Test (DST) in Well A was conducted over a 65 m (231 ft) interval of Sherwood sandstone and was producing oil with GOR 221 scf/STB. After 12 hours of drawdown at an average constant rate of 980 STBO/day, the well was shut-in for a period of 18 hrs. The build-up period was then analysed on a Log-Log diagnostic plot (pressure derivative response) for identification of the geological heterogeneities, flow regimes and the analytical interpretation (Figure 2.8). The pressure derivative response of well A is divided into three regions Early Time Region (ETR), Middle Time Region (MTR) and Late Time Region (LTR). Each region conveys different types of information about the well, the reservoir and the boundary conditions, which require special geological and statistical knowledge for interpretation.
Figure 2.8: Well-A pressure response in Wytch Farm field representing the geochoke. The ½ slope lines show the early linear flow regime show negative geoskin due to presence of minor channels intersecting the well (Toro-Rivera et al., 1994). The middle time region is the radial flow period represents matrix flow.

The ETR matching of the pressure derivative response with the ½ slope line shows the existence of a linear flow regime. This linear trend has been interrupted by a small hump followed by another ½ slope line in ETR. This was due to a sudden pressure change as a result of a change in hydrostatic pressure reference (Toro-Rivera et al., 1994). The well-test analysis shows the average permeability of 44 md and a relatively large negative skin value of -3.6. This negative skin (i.e. “geoskin”) provides an indication of high permeability lenses of limited extent (less than 10% of the depth of investigation, in this case 50 to 500ft) originated from the minor channels straddling the well.

The middle time interpretation of the pressure derivative response provides the well-test effective permeability of 44 md. This value was found to be close to the geometric average of the air core permeability of 42.4 md. This value might justify the randomness nature of the medium as the geometric average of permeability is close to the effective permeability (Fanchi, 2000; Corbett et al., 2010). The position of well respect to PFC’s creates different types of middle time radial flow regime. For instance, in well A the radial flow is matrix-dominated while in some other cases, the radial flow could occur within the PFC’s and the permeability is close to the arithmetic average of channel intervals. In the latter case, the negative skin cannot develop. In the well “A” example, the radial flow regime characterised by the middle time stabilizations follows
a downward trend in late times. This partly-developed downward trend is difficult to interpret due to insufficient supporting data. This roll-over could be either the indication of flow expansion or it could equally be due to fluid contrasts as evidence of aquifer support.

2.3 The relations between the analytical and geological interpretation of the layered reservoirs

Although the geoskin, the gechoke and the ramp effect have well-described geological backgrounds, they can also be assumed as other members of the bigger family of the layered reservoir with/without cross-flow. Therefore, there should be some generic relationships between the classical homogenous layer-cake responses and these geotype curves. Figure 2.9 shows a classification of the layered reservoirs considering the geological footprints on the pressure derivative response curve.

Figure 2.9: The layered reservoir classification and the associated well-test signatures.

2.3.1 Cross-flow systems

The geoskin and the gechoke are both related to the cross-flow conditions. However, a simplified geoskin model has more resemblance to the homogenous layer-cake double permeability reservoir. Therefore, the geoskin can be seen as a general extension of the double permeability reservoirs where the high permeable streaks that are straddling the wellbore have a limited vertical and spatial extension. Lateral broadening of these patches can provide with a “clear” V-shape signature on the pressure derivative curve that is expected in double permeability reservoirs. This occurs whenever high permeable
patch thickness is small compared to the low permeable layer thickness (say less than 40% of the producing interval). This relatively high thickness contrast requirement for the V-shape signature appearance is because the porosity contrast in the clastic reservoirs may not fulfill the requirement for the small storage contrast, \( \omega \), even if the transmissivity contrast, \( \kappa \), is close to unity.

Figure 2.10 represents an isotropic two-layer reservoir model with a dominant matrix permeability of 20 md where a thin high permeable streak (with permeability of 2000 md) straddles the wellbore. The lateral extension of the model is reasonably large (\( r_e = 10000 \) ft) and the porosity and the coefficient of isothermal compressibility are kept invariant within the system (\( \varphi = 0.18, c = 3 \times 10^{-6} \) 1/psi). A single draw-down of 100 STBO/day is simulated to generate a series of type-curves showing the effect of lateral broadening of the high permeable streak.

![Figure 2.10: A simplified two-layer reservoir representing the double permeability model. The thin high permeable layer can have a variable lateral and vertical extension.](image)

Figure 2.11 presents a set of draw-down derivative responses of the two-layer system where the lateral extension of the thin high permeable streak changes from 25 ft to 6400 ft. The figure shows that the geoskin effect at the shorter length changes to a classical double permeability response with a V-shape signature in the larger lengths. The small hump after the early linear flow is inherited from the double permeability signature and will convert to a perfect valley in larger patch lengths. This hump can eventually disappear from the derivative curves when the layer thickness contrast reduces. Figure 2.12 shows another set of the derivative responses where the high permeable streak thickness increases to 50 ft (half of the total system thickness). Obviously, the hump has
been disappeared from the derivative responses and the V-shape signature of derivative response in the larger streak lengths has also been significantly alleviated.

![Figure 2.11: A set of draw-down derivative response curves of a two-layer reservoir model (Figure 2.10) where the lateral extension of the “thin” high permeable layer (i.e. 10ft) is varied from 25 ft to 6400 ft.](image1)

![Figure 2.12: A set draw-down derivative response curves of a two-layer reservoir model (Figure 2.10) where the thickness of the high permeable layer has increased to the half of total interval thickness (i.e. 50 ft) and its lateral extension is varied from 25 ft to 6400 ft.](image2)

**2.3.2 Commingled systems**

The cross-flow and commingled behaviours (Figure 2.13) of the described two-layer model are remarkably different. Figure 2.13 shows the comparison between the commingled and the cross-flow responses of the model. The comparison has been made for different lengths of the high permeable streak. As it is expected, the commingled
responses are associated with an approximate unit slope trend in the early/middle times where the high permeable patch is depleting faster than the lower permeability layer. Moreover, the V-shape signature of the cross-flow systems is replaced with an equivalent single-layer response in the commingled system.

Figure 2.13: The commingled and the cross-flow comparison of the draw-down response curves of a simplified two-layer model (Figure 2.10) where the lateral extension of the thin high permeable layer is varied from 50ft to 6400ft.

The ramp effect is also a commingled response of the layered systems where the derivative curve deviates from the equivalent single-layer response of the homogenous layer-cake models. Although this deviation (that can reach a ½ slope trend in the middle times) is well described in a geostatistical manner, where the local and the spatial statistics affect the ramp shape, however some simplified analytical modes (e.g. multi-layered region-composite models) might sometimes be representative for its interpretation.

2.4 Chapter summary
This chapter contains an introduction to the multi-layered cross-flow and commingled reservoir. The basic transient well-test responses of these reservoirs under the isotropic and homogeneous layers are reviewed. However, these classical responses may be complicated in presence of complex geology particularly in fluvial environments. The three pressure transient extrema variants that are potentially present in fluvial systems are presented. These well-test family members include the geoskin, geochrome and a newly recognized “ramp effect” response. Thus, the layered reservoir classification is
extended to include these geological members and the genetic relations between them are examined through a set of generated geotype curve.
Chapter 3

Ramp Effect Characterisation

The ramp effect that was defined in Chapter 2 is a monotonic increase of pressure derivative curve over at least one logarithmic cycle in a commingled system. It is usually associated with an early and a late time plateau. The duration of the early time stabilization, the slope and the shape of the transient ramp are controlled by the reservoir geostatistical and statistical properties such as the variance and the correlation lengths. The existence of the pressure derivative stabilizations benefit the reservoir engineering practice as the average permeabilities can be estimated at each plateau, while the geology footprint is reflected in the transient ramp effect. The ramp effect is expected to be seen in fluvial environments (braided, meandering, and possible anastomosing) with zero (or extremely low) effective vertical permeability while lateral $k \times h$ is continuously reducing. The extremely low vertical permeability gives rise to very low vertical communication in which the pressure distribution is encapsulated within each layer and creates commingled-type behaviour at the larger scale. Therefore, the loss of lateral connectivity, a very low $k_v/k_H$ and the position of the wellbore within the reservoir provide the necessary conditions for the existence of the ramp in the pressure derivative responses.
Based on the numerical simulation results, the decrease rate of connectivity has a substantial effect on the slope of ramp, which can reach $\frac{1}{2}$ slope trend when there is enough contrast in permeability of the facies. Although the unit slopes are not common in the build-up derivative responses, these might also occur where the high permeable patches are effectively isolated within the pervasive and the very low-quality reservoir facies (e.g. low energy anastomosing environments). When the pressure disturbance diffuses out of the wellbore towards the reservoir, the low permeability patches are by-passed because the pressure drop across the low permeable facies is not enough to induce their depletion. However, when the pressure inside the surrounding high permeable facies is low enough a secondary pressure drop (micro cross-flow) will happen and the low permeable patches affects the pressure derivative response to rise up. Accordingly, the by-passed facies with low permeability and high porosity can give rise to ramp effect. This phenomenon reminds the same process in double permeability layered reservoir (macro cross-flow). The macro cross-flow effect takes place when the vertical communication increases and alleviates the ramp effect. The vertical flow create the path ways for the flow to find the easiest way to move.

### 3.1 Ramp-like phenomena

In the literature, there are many ramp-like examples detected in heterogeneous reservoirs and these are sometimes interpreted as linear flow (because of parallel faults) or in some cases unexplained. Elarouci (1994) illustrate a ramp that is interpreted as a single effective channel (Figure 3.1). Herweijer and Dubrule (1995) interpreted the ramp effect by a meandering channel bordered by two parallel low-permeability levees. Sagar et al. (1995) simulated a draw-down response of a 2-D permeability field for the optimization purposes with a signature of ramp effect in the well-test response. Thomas et al. (2005) studied the reservoir upscaling with the help of well-testing, and marked a rapid rise in pressure derivative response due to connectivity reduction between the well and the reservoir as fluid was producing. However, they could not characterise the phenomenon.
Although all of examples mentioned here have a geological rationale, there are also some ramp-like phenomena with different origins. The hydraulic fracturing, for example, creates a \( \frac{1}{2} \) slope trend in pressure derivative data at early time region (Alvarado, 1994). Reduction of mobility due to fluid heterogeneity can also cause an increase of pressure derivative level (Dahroug et al., 2005). In addition, the differential depletion of layers in the commingled reservoirs during the well-test can create a phenomenon resembling ramp effect. An example of this is the study carried out by Chu (1996) in which the sharp rise of pressure derivative in the draw-down response was attributed to differential depletion of the layers that reach the pseudo-steady state regime successively during the “draw-down” period. However, this could be distinguished from the so-called ramp effect by performing a build-up test, in which the pressure derivative response during pseudo-steady state regime follows a downward trend. Although it rarely happens, the layer backflow, which is of the same origin as the differential depletion during the draw-down response, might also provide a sudden rise in the “build-up” response. However, this requires the other sources of information as PLT to be diagnosed.

### 3.2 How to study the ramp effect

The spatially distribution of reservoir properties and/or complex channel networks can give rise to the general ramp response. In the former case, the system mobility (permeability in particular) can be considered as a random process. The ensemble average of an infinite number of realizations in the unconditional field under negligible
vertical flow could end up showing a ramp response. Furthermore, the ramp response can be studied in a single realization of a more complex random process. In this context, the first stabilization is indeed a function of the well intersecting cells. However, the final stabilization, if the investigation radius is greater than the underlying geostatistical correlation length, is a function of the system structure (Noetinger and Haas, 1996). Meanwhile, the ramp response can also be due to complex geological processes that create complex channel networks. These might be reflected in the geological modelling where the geological stacking and lateral combination of the high net:gross and low vertical permeability cases (in braided fluvial and meandering environments) push toward having a ramp effect. The ramp effect can also be modelled in the anatomising and low energy environments where the pervasive silty and/or shaly deposits reduce the lateral and vertical connectivity of the producing sand bars.

Taken together, the ramp effect interpretation can be carried out by different approaches as listed below

- Statistical interpretation
- Analytical-numerical interpretation
- Numerical and/or geological interpretation

### 3.3 Statistical Interpretation of the ramp effect

To interpret the ramp, a statistical approach might be useful to compute the effective hydraulic conductivity (or permeability) as applied in hydrology. This is usually suitable for fairly undemanding geological scenarios (without complex heterogeneities like internal no-flow or leaky boundaries) with a single well and a single reservoir facies. This helps make some direct analytical conclusions which relate the average permeability of the investigated medium to the geostatistical distribution parameters through which the ramp effect is explained.

In the statistical approach, the diffusivity equation is treated as a stochastic differential equation and the corresponding parameters have been assumed to be stationary random functions. Then taking the ensemble average of the flow equation and its boundary conditions one can use the concepts of Green functions (Ozkan, 2006) to obtain the effective permeability of the medium under certain conditions. The effective
permeability is the permeability which represents the statistically homogeneous medium at large scale.

In the absence of the extreme sampled permeability values (i.e. zero or infinity), the harmonic average is the lower bound and the arithmetic average is the upper bound of the effective permeability (Cardwell and Parsons, 1945; Dagan, 1989; Renard and de Marsily, 1997; Deutsch, 2002). The single-phase effective permeability of a porous medium is a general function of the flow regimes, geology and the geostatistical parameters (e.g. variance and the ratios between the length scales characterising the covariance function $C_Y$ (Dagan, 1989)) and is therefore difficult to represent by simple mathematical form. However, based on the stochastic approach and assuming an infinite-domain heterogeneous permeability field, several authors (Dagan, 1993; Paleologos et al., 1996; Tartakovsky et al., 2000; Jankovic et al., 2003; Gluzman and Sornette, 2008) attempted to obtain the effective permeability of the medium under steady state and uniform flow conditions. In all of these studies the isotropic permeability was assumed to be a stationary random function with a lognormal distribution and finite correlation range. For 1-D and 2-D flow conditions, the generalization of a perturbation approach, the so called Landau-Matheron or Landau-Lifshitz conjecture (Dagan, 1993; Paleologos et al., 1996), was found to be exact. The conjecture gives the effective permeability of the medium and is written as follows

$$
\frac{k_{\text{eff}}}{k_G} = \exp \left[ \left( \frac{1}{2} - \frac{1}{D} \right) \sigma_Y^2 \right]
$$

(3.1)

$$
k_G = \exp(m_Y)
$$

(3.2)

in which $k_G$ is the geometric average of permeability, $k_{\text{eff}}$ is the effective permeability at distances larger than the correlation length, $D$ is the dimension of the space in which flow takes place and $m_Y$ and $\sigma_Y^2$ are the mean and variance of $Y=\ln(k)$. The equation states that the effective permeability is equal to harmonic average or geometric average under 1-D or 2-D flow condition respectively. Noetinger (1994) stated that when the number of the flow dimensions increases (i.e. $D \to \infty$) the arithmetic average is obtained. In these situations, the correlation between the pressure gradient and the permeability
can be completely disregarded since the flow will preferentially avoid the low permeable regions.

For the 3-D flow condition, the conjecture failed to represent the exact effective permeability and it was demonstrated that there is a dependency on the shape of the covariance function (De Wit, 1995). However, Noetinger (1994) showed that the conjecture holds (but as an approximation) when there is no effect of correlations between different points in the field. He showed that the effective permeability of an uncorrelated isotopic lognormal permeability field in 3-D can be expressed as a power average of the model permeabilities: \( k_{\text{eff}} = \langle k^{1/3} \rangle^3 \) (the bracket “\( \langle \ldots \rangle \)” represents the ensemble averaging operator). This is the same result that obtained by Desbratas (1992).

Based on the Self-Consistent Approximation, Dagan (1993) defined the effective permeability of an isotropic lognormal permeability field in 3-D and 2-D flow condition without any restriction on the variance (equation (3.3)).

\[
 k_{\text{eff}} = \frac{1}{D} \int_0^\infty \frac{f(k)dk}{k(D-1)+k_{\text{eff}}} 
\]  
(3.3)

in which, \( f(k) \) is probability distribution function of the permeability and \( D \) is the flow dimension. Later Dykaar and Kitandis (1992) indicated that the underlying assumptions for equation (3.3) are not appropriate for the porous medium.

In transient states, on the other hand, the published solutions are limited to the case of mildly heterogeneous permeability field in an infinite reservoir in which the lognormal permeability distribution has a small variance compared to unity. This restriction is a stumbling block on the road to applicability of numerous theoretical analyses to real-world problems (Gluzman and Sornette, 2008). The transient effective permeability obtained in this manner, is a complex function of time and space and is dependent on the covariance function of the permeability. For the case of infinite domain, isotropic and exponential covariance function, the effective permeability can be formulated in a useful mathematical form that can be obtained by the following equation (Dagan, 1982)
Chapter 3: Ramp Effect Characterisation

\[ \frac{k_{\text{eff}}}{k_G} = 1 + \sigma_y^2 \left[ \frac{1}{2} - \frac{1}{D} + \beta(t) \right] \]  
(3.4)

where,

\[ \beta(t) = \frac{1}{\sigma_y^2} \int C_y(|x-x'|)G(x,x',t)dx' \]  
(3.5)

\[ G(x, x', t-\tau) = \frac{1}{8\pi \sqrt{\eta_x \eta_y \eta_z} (t-\tau)^{3/2}} \times \nonumber \]
\[ \exp \left[ -\frac{(x-x')^2}{\eta_x} + \frac{(y-y')^2}{\eta_y} + \frac{(z-z')^2}{\eta_z} \right] \frac{1}{4(t-\tau)} \]  
(3.6)

\[ C_y(|x-x'|) = \sigma_y^2 \exp \left[ -\frac{(x-x)^2}{l_x^2} + \frac{(y-y)^2}{l_y^2} + \frac{(z-z)^2}{l_z^2} \right] \]  
(3.7)

in which, \( C_y(|x-x'|) \) is the covariance function, \( t \) is time, \( l_x, l_y \) and \( l_z \) are the integral correlation lengths, \( k_G \) is geometric average of permeability, \( D \) is dimension of flow and \( G(x, x', t) \) is the transient Green function. The Green function is defined for a differential equation with specific boundary conditions and corresponds to instantaneous point source solution (Ozkan, 2006). In physical terms the point source solution corresponds to pressure drop at point \( x \) and time “\( t \)” where a point source of unit strength, located at point \( x' \), is used to inject (or withdraw) a finite amount of fluid instantaneously at time “\( \tau \)”. For the finite domain, the Green function takes a more complex form including infinite series that might avoid easily formulating the effective permeability of the medium. However, having used an isotropic covariance function in an infinite domain, two useful conclusions can be made: firstly, if \( t \rightarrow 0 \) then \( \beta(t) \rightarrow 1 \) and secondly if \( t \rightarrow \infty \) then \( \beta(t) \rightarrow 0 \) (equations (3.8) and (3.9)). Therefore,

\[ \frac{k_{\text{eff}}}{k_G} = 1 + \sigma_y^2 \left[ \frac{1}{2} \right] = \exp \left( \frac{\sigma_y^2}{2} \right) \equiv \text{arithmetic average} \ (t \rightarrow 0) \]  
(3.8)

\[ \frac{k_{\text{eff}}}{k_G} = 1 + \sigma_y^2 \left[ \frac{1}{2} - \frac{1}{D} \right] = 1 \equiv \text{geometric average} \ (t \rightarrow \infty, D = 2) \]  
(3.9)
These equations indicate that the ensemble pressure derivative response could “plateau-off” at the early stages indicating the arithmetic average of permeability and finally should stabilize over a lower level indicating the steady-state effective permeability value. However, for any particular spatial realization of an infinite 2-D lognormal permeability distribution, the initial value of the stabilization is dependent upon the local permeability value intersecting the well and the final plateau gives the geometric mean (equation (3.8)) whenever the pressure investigation is beyond the correlation length.

In order to investigate the transient effective permeability in a wider range for the cases without any restriction on the input permeability variances, we can use the numerical simulation. In this case the geostatistical and petrophysical parameters (other than permeability) are kept constant. The standard deviation of permeability in “Ln-space” is solely allowed to vary from smaller values (e.g. 0.01) to larger values (e.g. 3). The model has a permeability distribution with a constant geometric average of 50 md, an isotropic correlation length of 100 ft and a porosity of 0.2. Figure 3.2 shows the effect of standard deviation on the pressure derivative response of a 2-D heterogeneous permeability field. This mimics the ramp effect on the transient pressure derivative response. One can observe that as the standard deviation of the permeability field increases, the shape of the pressure derivative response deviates from that of a homogenous system with a flat signature and a small standard deviation. The increase of the standard deviation causes the value of the early stabilization to reduce, which in turn changes the ramp slope. The ramp slope can reach a half slope trend when the standard deviation is greater than unity. Larger standard deviations (greater than 3) might create the internal no-flow boundaries effectively. Therefore, the early pseudo-steady state regime, distinguished by a unit slope trend on the pressure derivative response, is not unlikely (Figure 3.2 and Figure 3.3). This middle time unit-slope trend can also be formed in multi-layered commingled reservoirs where one or few of the layers are limited in extend.

It is worth noting that well-test response is divided in to three main regions; early, middle and late time regions. These regions are typically interpreted from the Log-Log diagnostic plots. The middle time region represents the infinite acting radial flow regime. The derivative curve follows a zero slope trend on a Log-Log plot from which the permeability (or more generally permeability-thickness product) can be estimated.
The early time indicates the wellbore and near wellbore phenomenon (e.g. hydraulic fracturing and wellbore storage). In particular, the wellbore storage is associated with a unit-slope trend in the derivative and pressure drop curves. Finally, the late time region represents the effect of the external boundaries. A late transient flow period (e.g. due to a linear no-flow boundary) or a pseudo steady state flow regime (reservoir depletion) are characteristics of late time region. In particular, the linear no-flow boundary effect is associated with a zero-slope derivative trend while the pseudo steady state (in drawdown period) follows a unit slope trend in the derivative curve.

The effect of standard deviation can also be reflected on the spatial pressure distribution. Figure 3.4 shows the pressure drop maps for two systems with small and large standard deviations. The spatial distribution parameters (i.e. covariance function and correlation lengths) of the underlying permeability field are kept invariant between the two systems. The figure reveals that for the small standard deviation the pressure deviation is essentially radial and leads to having a model with homogeneous-like behaviour. However, for the large standard deviations the pressure distribution is no longer radial and the spatial heterogeneities are illuminated.

![Figure 3.2: Effect of standard deviation (SD) of lnk_H on the pressure derivative response in 2-D domain. As the SD of lnk_H increases, the slope of the derivative curve increases.](image-url)
Figure 3.3: The early and late pseudo-steady state regime shown on the draw-down pressure response. The early pseudo-steady state can develop when the facies contrast are large enough for the pressure disturbance front to be confined within the high permeable facies and then to be released toward the low permeable facies. The upper curve is the pressure drop and the lower curve is the pressure derivative response.

Figure 3.4: The spatial pressure drop maps after 5hr of production for two permeability fields with small (left: $\sigma_{lnk} << 1$) and large (right: $\sigma_{lnk} >> 1$) standard deviations.
3.3.1 Ramp effect in 3-D: Effect of vertical permeability

The numerical methods can also be applied in the 3-D domain to investigate the statistical pressure derivative responses. In this sense, the simulated pressure derivative responses of infinite number of model realizations should be averaged. Due to practical issues, only 500 realizations of any single scenario have been used in here. To do so, a synthetic single-facies 15-layer reservoir model was constructed. Apart from the permeability, all of the other petrophysical properties of the reservoir were kept constant. The Direct Gaussian Simulation was applied to create the isotropic horizontal permeability realizations with different correlation lengths and different covariance functions. The horizontal permeability was assigned to have a lognormal probability distribution function with a mean and variance of $m_Y$ and $σ_Y^2$ respectively (the mean and variance are measured in logarithmic space: $Y=\ln k$). Moreover, the reservoir was allowed to either have the same value of vertical permeability as horizontal permeability ($k_V=k_H$) or have a commingled production ($k_V=0$). The model consists of 200×200×15 coarse cells in x, y and z directions respectively and each cell measures 20 ft×20 ft×10 ft of the reservoir volume. A single well ($r_w=0.31$ ft) was placed in the middle of the model to simulate a single draw-down test of 2000 STBO/day from the total thickness of the reservoir (150 ft).

Scenario 1: Random permeability field: Figure 3.5 shows the average draw-down test response taken over 500 realizations of a model that is constructed with an exponential covariance function, a small horizontal correlation length (i.e. $l_H=5$ ft $< Δx=10$ ft) and an isotropic 3-D permeability distribution (i.e. $k_X=k_Y=k_Z$). The continuous dark-coloured dotted curves show the average test response (red curve: average pressure derivative; black curve: average pressure drop) and the vertical bars present the range of variability for the values obtained from the simulations. This model theoretically represents a random medium and as it expected, the extracted permeabilities from the well-test responses of realizations converge to the geometric mean of the permeability distribution (~20 md). The early unit-slope trend of the derivative curve in Figure 3.5 results from the pseudo (or fake) wellbore storage effect (FWBS), which is a numerical artefact and arises from the use of pseudo steady-state (or Peacemann’s) well index in transient well-test simulations. Using a transient well index (Archer and Yildiz, 2001; Archer, 2010) or applying highly refined cells near the wellbore area minimizes this
Chapter 3: Ramp Effect Characterisation

phenomenon. The time at which the FWBS ends can be estimated from Archer and Yildiz’s equation (2001)

\[ t \geq 1600\Delta L^2 \frac{\varphi \mu c}{k} \]  \hspace{1cm} (3.10)

in which, \( t \) is the time in “hr”, \( \Delta L \) is the dimension of the square block in “ft”, \( \varphi \) is porosity, \( c_t \) is the total compressibility in “1/psi”, \( k \) is the permeability in “md” and \( \mu \) is the viscosity “cp”.

For the commingled system \((k_v=0)\) when the layers do not communicate in the reservoir, the statistical response of the model is different from that of the cross-flow system. Figure 3.6 shows the average test response and its associated variation range for 500 simulations of the model under commingled flow conditions. The average derivative response results in well-test permeability of 10 md, which is half of the geometric average of the permeability field. Furthermore, in comparison to the cross-flow case (Figure 3.5), there is a wider range of variability at the late time where the pressure derivative stabilizes. This shows that the vertical communication can affect both of the shape of the derivative response and the estimated permeability values.

Figure 3.5: Statistical test response of a cross-flow model with random permeability distribution. The continuous curves show the average test response taken over 500 simulation models and the vertical bars show the corresponding range of variation. The late time plateau gives the geometric mean of the permeability.
Although, in a random field the ramp effect does not appear (as the derivative starts to quickly stabilize over the average well-test permeability), this helps scrutinize the effect of vertical flow in the systems where the analytical solution are available. For instance, the effective permeability of a random field is the geometric mean of the permeability distribution.

**Figure 3.6:** Statistical test response of a commingled model with random permeability distribution. The continues curves show the average test response taken over 500 simulation models and the vertical bars show the corresponding range of variation.

**Scenario 2: Correlated permeability field:** In this scenario the effect of correlation length and vertical permeability on the ramp effect were studied. The variance of the log-permeability field was assigned to be unity ($\sigma_Y^2=1$) and the mean was kept unchanged ($m_Y=3$). In addition, an exponential covariance function with an increased horizontal correlation length of 300 ft was implemented to create 500 different realizations for this scenario. The variance effect of the 3-D permeability field on the statistical test response will be discussed later.

Figure 3.7 displays the statistical draw-down response for the cross-flow case where $k_V = k_H$. The pressure derivative response stabilizes over 23md which is slightly higher than the geometric mean ($k_G=20$ md). The variation range of the derivative response decreases by the time and finally converges at the pseudo steady state flow regime. This is characterised by the late time unit-slope trend.
Chapter 3: Ramp Effect Characterisation

Figure 3.7: Statistical test response of a cross-flow model with a correlated permeability distribution and the correlation length of 300ft $\sigma^2 = 1$. The continuous curves show the average test response taken over 500 simulation models and the vertical bars show the corresponding range of variation.

The vertical communication can affect the test response. If the vertical permeability of the system switches to zero to simulate the no-cross-flow behaviour, the ramp effect emerges (Figure 3.8). That is where the pressure derivative monotonically increases and creates a second plateau. This stabilization that is shown in Figure 3.8, provides an effective permeability of around 11 md that is almost half of the geometric average and this is comparable to the test permeability of the commingled random system (Figure 3.6).

**Scenario 3: Correlated permeability field: Variance effect:** The variance is a measure how the data are spread out from each other. Therefore having a higher variance (or standard deviation) of the permeability distribution implies that there are higher contrasts between the model permeability values. Figure 3.9 shows the statistical well-test response of the same cross-flow model as described in scenario 2 with a higher permeability variance (i.e. 6). The areal coverage of the geological model has been extended to postpone the emergence of unit-slope derivative signature, which is due to pseudo-steady sate flow regime. Starting from a wide range of variability for the pressure derivative curves at the early time, the average derivative responses converges to a stabilization that leads to a permeability value of 55 md that is far greater than the geometric average of the permeability distribution. As it is expected for the cross-flow system, the ramp effect does not emerge. However, the commingled behaviour of this
system (Figure 3.10) provides an extended ramp effect with the late time stabilization indicating the average well-test permeability of 7 md that is biased towards the harmonic average.

Comparing the statistical derivative responses of the commingled systems with different permeability variances (Figure 3.8 and Figure 3.10) reveals that the higher the variance
(or the standard deviation) in the probability distribution function, the higher the ramp slope. This is essentially due to the higher contrast of the distributed permeability values in the system with the higher variance. This higher contrast also affects the system cross-flow and changes the level of the derivative stabilization from which the effective permeability is estimated.

Figure 3.10: Statistical test response of a commingled model with a correlated permeability distribution and the correlation length of 300ft and \( \sigma_I^2 = 6 \). The continuous curves show the average test response taken over 500 simulation models and the vertical bars show the corresponding range of variation.

**Scenario 4: Correlated permeability field: Effect of the covariance function:** The geostatistical modelling approaches provide the possibility to implement different covariance functions to simulate the perophysical property distributions. This will affect the continuity and the smoothness of the resulting distributions. Figure 3.11 elucidates the outcome of statistical draw-down responses for two commingled systems where only the covariance function was allowed to change. The mean and the variance (in Ln space) and the correlation length were kept unchanged between the scenarios (i.e. \( m_I = 3 \), \( \sigma_I^2 = 4 \), \( l_H = 300 \text{ft} \)). As above, a total of 500 realizations were used for each scenario. The red curve is average pressure derivative response for the cases where the exponential covariance function is used and the blue curve is for the equivalent Gaussian covariance function. The figure (Figure 3.11) shows that the covariance function does not change the position and value of the stabilizations. However, the shape of the ramp is somewhat affected by the choice of the function. The Gaussian ramp curve (i.e. blue curve) appears to be slightly lower than the exponential ramp (i.e. red curve). This is mainly
due to the nature of the Gaussian function which smooths out the variations and biased the response towards having a homogenous-like behaviour.

**Figure 3.11:** Statistical test response of two commingled models with different covariance functions and the same correlation lengths of 300ft and $\sigma^2 = 4$. The blue curve shows the average derivative response taken over 500 simulation models of the case with exponential covariance function and the red curve is that of the Gaussian covariance function.

**Scenario 5: Correlated permeability field: Effect of correlation length:** The correlation length can affect the ramp signature on the pressure derivative response. Figure 3.12 shows the statistical test response for a commingled system implementing different correlation lengths. The other geostatistical properties remain unchanged amongst the models (i.e. $m_Y=3$, $\sigma^2 = 4$ and exponential covariance function). The figure reveals the correlation length in a large (i.e. an unbounded) system does not change the stabilization level. However, the ramp shape is affected by the correlation length. The shorter the correlation length (i.e. pure nugget effect in extreme cases), the earlier the pressure derivative stabilization.
Figure 3.12: The correlation length effect on the statistical derivative response of a commingled system. The red curve shows the average derivative of the random model, while the green and blue curves represent the average response for longer correlation lengths (150 ft and 300 ft respectively).

**Scenario 6: Correlated permeability field: Effect of correlation lengths ratio:**

Increasing the vertical correlation length, $l_V$, (while the horizontal correlation length remains invariant) can have the same effect as decreasing the horizontal correlation length, $l_H$, (while the vertical correlation length remains invariant). This is shown in Figure 3.13 and Figure 3.14 where the $l_V/l_H$ increases from 0.03 to unity. Figure 3.13, that is the statistical well-test response of a commingled system, indicates that the final stabilization of the derivative curve is independent of the lengths ratio for a commingled system. On the other hand, Figure 3.14 reveals that for a full cross-flow system where $k_V = k_H$, increasing the vertical correlation length of the permeability leads to an overall upward shift of the derivative curve toward the random field response. This is because with increasing the vertical length (to infinity), the flow domain eventually reduces to a 2-D field (Dagan, 1989) where according to Landau-Lifshitz conjecture the exact geometric mean is achieved.
3.4 Analytical-numerical interpretation of ramp

This second approach provides a further insight into the permeability averaging process of well-testing and is based on perturbation solution of the radial diffusivity equation with a smooth permeability variation in radial direction (Oliver, 1990). This relates the instantaneous permeability concept to a weighted-harmonic average of several “rings” of...
the permeability within an investigated region. The rings are obtained by superimposing several concentric circles of geometric or logarithmic progression on the original 2-D permeability field (Sagar et al., 1995; Gautier and Noetinger, 2004). The kernel function, \( K(r_D, t_D) \), is used as a time-dependent weighting function and implies that 98% of the total contribution of the permeability to pressure derivative response comes from the annulus between two radii \( r_{D,min} = 0.12 \sqrt{t_D} \) and \( r_{D,max} = 2.34 \sqrt{t_D} \) (Sagar et al., 1995). Here \( r_D \) and \( t_D \) are the dimensionless radius and dimensionless time respectively and will be defined in the following section. Although the Kernel function has originally derived for the cases when the permeability is varying slowly in space, Sagar et al. (1995) and Feitosa et al. (1994) extended the approach for higher permeability contrast. The approach was found to be essentially robust in 2-D permeability field. It is emphasized that the algorithm proposed by Feitosa et al. (1994) and Sagar et al. (1995) assumes that there is no effect of wellbore storage and leaky or no-flow boundaries on the well-test response. The algorithm presumes that any variation in the pressure (or pseudo pressure) derivative response is due to permeability variation. This means that the only recognized variable is the permeability variation in the system. Moreover, the model should be essentially infinite in size. As long as these boundaries affect the well-test response, the algorithm will not provide with the promising results.

Following Sagar’s approach (1995), the analytical instantaneous permeability, \( k_{ins}(md) \) which is extracted from the clean draw-down pressure derivative response (without wellbore dynamic and external boundaries effects), is defined by

\[
k_{ins} = \frac{1}{2} \times \frac{141.2 q \mu B}{h \frac{\partial P}{\partial (Int)}}
\]

in which, \( q \) is the production rate in “STBO/day”, \( \mu \) is viscosity in “cp”, \( B \) is oil formation volume factor in “STBO/STB” and \( h \) is the effective thickness in “ft”. This is numerically approximated by

\[
\frac{1}{k_{ins}(t)} \approx \sum_{j=1} w_j(t) \frac{1}{k_j}
\]
where, \( k_j \) and \( w_j \) are the ring permeability and the averaging weight respectively, and they are defined as follows

\[
w_j(t) = \frac{\int_{r_{D_{min}}}^{r_{D_{max}}} K(r_D, t) \, dr_D}{\int_{r_{D_{min}}}^{r_{D_{max}}} K(r_D, t) \, dr_D}
\]  
(3.13)

\[
\sqrt{t_{D_i}} K(r_D, t_{D_i}) = 0.5 \sqrt{\frac{\pi t_{D_i}^2}{t_{D_i}}} \exp\left(\frac{-r_D^2}{2t_{D_i}}\right) W_{1/2,1/2}\left(\frac{r_D}{t_{D_i}}\right)
\]  
(3.14)

in which,

\[
r_D = \frac{r}{r_w}
\]  
(3.15)

\[
t_{D_i} = \frac{2.637 \times 10^{-4} k_{ms} (t) t}{\varphi \mu c r_w^2}
\]  
(3.16)

and

\[
A_j \times \ln(k_j) = \sum_i A_i \times \ln(\Delta k_i)
\]  
(3.17)

where, \( r \) is the radius “ft”, \( t \) is time in “hr”, \( \varphi \) is porosity, \( A_i \) is a portion of the area of a gridblock i in “ft^2” which falls inside the ring j with total area of \( A_j \), and “\( \Delta k_i \)” is the gridblock permeability in “md”. \( W_{1/2,1/2} \) is Whittaker’s function (a confluent hypergeometric function (Gradshteyn et al., 2007)). The procedure is visually shown in Figure 3.15.

\[\text{Figure 3.15: The proportion area, } A_i, \text{ of each cell “i” with permeability } \Delta k_i \text{ that falls within the ring “2” (defined by } r_1, r_2) \text{ with area of } A_2 \text{ and permeability of } k_2.\]
Chapter 3: Ramp Effect Characterisation

The algorithm is based on the fact that the analytical instantaneous permeability (equation 3.11) is a weighted harmonic average of the equivalent radial permeability distribution (Feitosa et al., 1994) (equation 3.12). An equivalent radial grid is defined by assuming a series of concentric circles where their radii follow a geometric (or logarithmic) progression. The user can control the number of the circles to be used. In this application 100 circles were defined. The outermost circle should have a radius equal or less than the nearest boundary in the model. The equivalent radial grids are superimposed on the 2-D Cartesian grids with the well in centre. The equivalent permeability of each annulus is defined as an area-weighted geometric average of the Cartesian cell values that are confined in each annulus (equation 3.17 and Figure 3.15). Eventually the kernel function is used to define the time-dependent weights for each annulus (equation 3.13) and the numerical instantaneous permeability profile is constructed by implementing equation (3.12). The analytical-numerical approach is not a specific solution to the ramp effect. It provides a moving average window, which numerically investigates the spatial permeability field and constructs the transition state from the well-location permeability toward the effective permeability of the system. In some cases, where the wellbore permeability is close to the maximum permeability, then the transient ramp effect appears. Consequently, the transition state is strongly dependent upon the wellbore position within the unconditional permeability field.

Figure 3.16 (lower) is an example that shows the robustness of the analytical-numerical method in predicting the analytical instantaneous permeability (extracted from the flow simulation) in 2-D domain. The red curve is the constructed analytical instantaneous permeability curve from a simulated draw-down well-test response (equation 3.11) in a specified permeability distribution (Figure 3.16: upper right and upper left). The downward trend of the red curve after a middle time stabilization is caused by the pseudo steady state regime (PSSR), where $\frac{\partial p}{\partial \ln(t)}$ is steadily increasing (unit slope) which leads to the reduction of the instantaneous permeability. This means that the boundaries effect has been attributed to the permeability variation which is not correct. The PSSR has reached after one hour, as the reservoir size is finite and the permeability is relatively high. This part of the curve is not of particular interest. The blue curve, on the other hand, is the estimated well-test permeability from the numerical-analytical approach (equation 3.12). There is an excellent agreement between the curves in the transient state.
Chapter 3: Ramp Effect Characterisation

Figure 3.16: The instantaneous permeability curves from the draw-down flow simulation of the well-test (analytical $k_{\text{inst}}$: red curve) and the analytical-numerical approach (Numerical $k_{\text{inst}}$: the blue curve). This is the case for a 2-D permeability field (top: right) with an arbitrary permeability distribution (top: left) and the variogram.

Figure 3.17: Instantaneous permeability profile from the simulation (analytical $K_{\text{inst}}$) and from the numerical-analytical approach (Numerical $K_{\text{inst}}$). This is the case for a 2-D permeability field (top: right) with an arbitrary permeability distribution (top: left) and the variogram.
Figure 3.17 shows another example for a complex instantaneous permeability in a 2-D field. This has been tested for many other 2-D permeability fields, and the excellent agreement was achieved.

In the commingled 3-D permeability fields, the speed of the moving kernel function is different in each layer which avoids the method to predict the correct instantaneous permeability field. Figure 3.18 shows the position of the normalized kernel function for two different layers of different permeabilities. It emphasizes that at a specific time (0.01 hr) after imposing the pressure disturbance, the kernel function in the higher permeability layer (700 md), is moving faster toward far distances from the well, than the lower permeability case (7 md). This implies that when there is a contrast between the layer properties, the arithmetic average of numerically-extracted instantaneous permeabilities of the layers fails to reconstruct the analytical instantaneous permeability profile. In these cases a power average of the instantaneous permeabilities could match the analytical instantaneous permeability profile. However, the exponent factor in power average cannot be obtained by a trivial procedure and the validation procedure with a trial-and-error procedure is generally required.

To test this algorithm in 3-D, a two-layered commingled reservoir with different permeability distributions was constructed. The model has dimensions of 2000 ft×2000 ft×60 ft and was discretized into 200×200×2 cells, each cell measuring 10ft×10ft×25ft for the first layer and 10 ft×10 ft×35 ft for the second layer. Except for the horizontal permeability, the other rock properties were kept constant to ensure that any change in
pressure derivative response is only due to spatial changes of the permeability field. The unconditional Sequential Gaussian Simulation was then implemented to simulate the log-normal permeability field within each layer of the reservoir (Table 3.1).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Min (md)</th>
<th>Max (md)</th>
<th>Geometric Mean (md)</th>
<th>SD in Ln space (md)</th>
<th>Correlation Length(ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Layer</td>
<td>12</td>
<td>6404</td>
<td>976</td>
<td>0.5</td>
<td>220</td>
</tr>
<tr>
<td>Lower Layer</td>
<td>4</td>
<td>1000</td>
<td>375</td>
<td>0.5</td>
<td>220</td>
</tr>
</tbody>
</table>

A production well was placed near the middle of the model and a black oil simulator (E100) of single phase with slightly compressible fluid was applied to generate a single draw-down of 100 STBO/day for 500 hr over 60 ft of the reservoir thickness. The simulations were performed without any effect of the wellbore dynamics. Transforming the pressure derivative data into instantaneous permeability, the analytical instantaneous permeability profile was obtained (Figure 3.19: black curve). The rapid decrease of the instantaneous permeability at late time is due to the pseudo-steady state flow regime and is not considered as the reservoir response and matching procedure. To apply the algorithm, the reservoir was divided into 50 annular rings with geometric progression that is started from a number slightly greater than the wellbore radius (i.e. \( r_1 \) =0.39 ft) to a large radius (i.e. \( r_n \)=1000 ft). The area-weighted geometric average of the gridblock permeabilities inside each ring (i.e. confined between \( r_i \) and \( r_{i+1} \)) was calculated and the resulting value was assigned as the ring permeability. The kernel-weighted harmonic average of the ring permeabilities were then implemented to provide the instantaneous permeability values at each time for either of layers. Because, the kernel function is a function of the instantaneous permeabilities obtained from the well pressure data, it assumes the same weight to each specific ring at different layers.

In this example, the numerical instantaneous permeability profile of the upper layer (Figure 3.19) increases from the cell permeability penetrated by well (i.e. 460 md) to geometric mean of the layer (i.e. 970 md). Equally, the numerical instantaneous permeability for the lower layer (Figure 3.19) shows a gradual decrease from 700 md to a final geometric mean of 370 md. The latter profile is the typical signature of the ramp effect. Despite robustness in 2-D, the algorithm cannot be simply applied in 3-D model.
In this sense, the analytical instantaneous permeability profile was not found to be a simple thickness-weighted arithmetic average of the calculated layer’s numerical instantaneous curves (Figure 3.19: dotted purple curve). The difference is magnified at the late times. Therefore, a thickness-weighted power average was implemented to combine the layer’s numerical instantaneous permeability curves. A trial and error approach could provide with a negative exponent factor of -0.3 for achieving a reasonable match. This is far from the arithmetic average with the exponent factor of unity. Although, a general rule could not be found, from several numerical experiments, it was observed that the small values of the exponent factor were frequently required for the cases in which the opposite trends of the numerical instantaneous permeability profiles cross each other at some point.

![Figure 3.19: Instantaneous permeability profile of a commingled two-layer reservoir. The red curve is the numerical instantaneous permeability profile for the upper layer and the grey curve shows the numerical profile of the lower layer (ramp effect). The overall numerical profile (blue curve) is expressed as the power average of two layer profile and fits the analytical instantaneous permeability profile (black curve).](image)
It should be noted that the diffusion front (i.e. pressure disturbance front) might not always follow a perfect radial expanding shape. This happens for the heterogeneous reservoirs with a high variation in spatial permeability values and is a limitation to the Analytical-Numerical approach as well. Figure 3.20 (right) presents a 2-D reservoir mode with a heterogeneous permeability distribution. The model consists of $250 \times 250 \times 1$ fine cells in x, y and z directions respectively and each cell measures 2 ft $\times$ 2 ft $\times$ 150 ft of the reservoir volume. The porosity is kept constant ($\phi=0.18$). The model is flowed with a constant draw-down of 100 STBO/day. Figure 3.20 (left) shows the influence of permeability at each location on the well pressure according to the sensitivities in Appendix A (equation A.1). The sensitivity values are computed after 0.03 hr of production. The calculated sensitivity values are based on He’s procedure (1997) and a set of C# codes were developed to implement this method. The sensitivities provide few important conclusions. Firstly, the well-test averaging procedure does not provide the same weights to all permeability values within the investigated region (the more sensitive values are shown by dark colour in the Figure 3.20. Secondly, the heterogeneous and/or anisotropic pressure disturbance expansion is not unlikely. Finally and probably the most interestingly is the facies bypassing which has been explained earlier.

Figure 3.20: Left: the influence of permeability at each location on the well pressure. The sensitivity values are calculated at a specific time. Right: the corresponding permeability distribution map. The dark coloured cells (left) represent the most sensitive cells during the draw-down period.
3.5 Numerical interpretation of ramp

Although the statistical and numerical-analytical approaches provide some tools to characterise and interpret the ramp effect response, the detailed geological modelling of the reservoir along with associated non-linearities show that use of such methods violate some of their underlying assumptions. Apart from the limitations in 3-D applicability of the mentioned methods, the dependency on the single-facies and a perfect lognormal permeability distribution is rarely acquired in the realistic situations. However, the numerical simulation of the detailed geological model provides a unique opportunity to examine the model and explore the effect of different input parameters on the outputs of the numerical simulations. In other words, the advantage of numerical models is that they allow systematic understanding (i.e., mapping) of the effects of various geological assumptions on the pressure response. Nevertheless, we should be aware of the point that due to complexity of the model the resulting quantification might be challenging and any static model might not easily reproduce the dynamic behaviour of the real reservoir. This would be magnified in the multi-facies models with multi-modal probability distribution function where there are major petrophysical and stochastic anisotropies. Equally, many realisations might reproduce the real reservoir behaviour and in this way, any model scenario will not provide a unique solution. These multiple realizations are helpful to generalize a well-test phenomenon (e.g. ramp effect) and to ensure that it is not peculiar to a single model. If a phenomenon were realized to be universal to a scenario, then one approach would be to study a proper single realization possessing the phenomenon. This proper realization is commonly obtained after careful studies and preferably after matching with available observed data.

3.5.1 Model set-up

The model studied here for the numerical well-testing is based on a realistic pixel-based model of a multi-facies, high net:gross, braided fluvial reservoir (Zheng et al., 2007). The braided fluvial channels usually have the high width:depth ratio and are associated with higher channel slopes. The model includes four different facies (Figure 3.21) ranging from high quality sands (facies 3), moderate quality sands (facies 1), low quality sands (facies 2) and shale (facies 4). The “Conditional” Sequential Indicator Simulation and the “Conditional” Sequential Gaussian Simulation were implemented to distribute the reservoir facies and properties throughout the reservoir (Table 3.2). This was performed by availability of some real log and core data. The base horizontal correlation
lengths for all facies and properties were assumed to be isotropic and had a fixed value (250 m).

![Depositional environment of the braided fluvial Stream. The model includes 4 different facies from high quality sand to shale.](image)

**Figure 3.21:** Depositional environment of the braided fluvial Stream. The model includes 4 different facies from high quality sand to shale.

<table>
<thead>
<tr>
<th>K_H</th>
<th>Min (md)</th>
<th>Max (md)</th>
<th>Arithmetic Mean (md)</th>
<th>SD (md)</th>
<th>No of cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facies 1</td>
<td>0.35</td>
<td>915</td>
<td>161</td>
<td>204</td>
<td>79207</td>
</tr>
<tr>
<td>Facies 2</td>
<td>0.001</td>
<td>4</td>
<td>0.2</td>
<td>0.6</td>
<td>15745</td>
</tr>
<tr>
<td>Facies 3</td>
<td>473</td>
<td>660</td>
<td>585</td>
<td>77</td>
<td>2244</td>
</tr>
<tr>
<td>Facies 4</td>
<td>0.001</td>
<td>1.4</td>
<td>0.17</td>
<td>0.4</td>
<td>6004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>φ</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Facies 1</td>
<td>0.1</td>
<td>0.24</td>
<td>0.16</td>
<td>0.03</td>
<td>79207</td>
</tr>
<tr>
<td>Facies 2</td>
<td>0.04</td>
<td>0.11</td>
<td>0.07</td>
<td>0.02</td>
<td>15745</td>
</tr>
<tr>
<td>Facies 3</td>
<td>0.22</td>
<td>0.23</td>
<td>0.23</td>
<td>0.002</td>
<td>2244</td>
</tr>
<tr>
<td>Facies 4</td>
<td>0.03</td>
<td>0.1</td>
<td>0.04</td>
<td>0.02</td>
<td>6004</td>
</tr>
</tbody>
</table>

The extracted sector model for performing the test experiments covers the area of 2133 m (6997 ft) × 1158 m (3798 ft) over the average reservoir thickness of 50 m (162 ft). The model consists of 86×48×25 cells, each coarse cell has average dimensions of 25 m (82
ft)×25 m (82 ft)×1.9 m (6.2 ft) in x, y and z direction respectively (Figure 3.22). A black oil reservoir simulator of slightly compressible fluid was then applied to flow the model with 990 STBO/day over the total reservoir thickness. A very fine Cartesian local grid refinement (C-LGR) along with adopted logarithmic time-steps have also been used in the well-test simulations in order to reduce the associated numerical artefacts reflected in the subsequent pressure derivative responses. Although, the layer skin factor may significantly change the well-test behaviour of the multi-layered reservoirs, all the simulations here are performed in absence of the wellbore dynamics and the mechanical skin factor.

![Figure 3.22: A three-dimensional realization of a sector of the geological model in the braided fluvial environment. The models has a variety of different quality facies.](image)

### 3.5.2 Effect of vertical permeability

Vertical flow communication within the reservoir has a noticeable effect on the pressure derivative response. Even with small values of the vertical permeability a significant flow can transfer between the layers (Raghavan and Kuchuk, 2009). To study the effect of vertical permeability on the ramp effect, the vertical permeability of the model was reduced from a value close to the horizontal permeability (i.e., the isotropic case) to zero (i.e. strongly anisotropic). Figure 3.23 shows that the ramp slope increases with decreasing the vertical permeability. When the vertical permeability is equal to isotropic horizontal permeability, the ramp slope is around 0.1 while in the case of the complete commingled flow, the slope increases to 0.25 on the logarithmic plot. Although we have already showed the slope of the ramp is also dependent on the diversity of the permeability distribution (i.e. standard deviation) and could even reach ½ slope in some cases, in this study, the probability density function of permeability was kept intact and
solely the vertical permeability effect was studied. The existence of the slope in 3-D flow conditions shows that the ramp effect is limited to the 2-D flow condition. However, the slope generally remained small. This is indeed due to existence of low quality facies with some lateral extents where they limit the dynamic communication within the model layers. Reduction of the ramp slope with increasing the vertical permeability can also be explained by the stochastic approach in steady-state regime which explains that in the mathematical sense the effective permeability approaches to the arithmetic average in the infinite flow dimension. Another important point highlighted in Figure 3.23 is that in the commingled reservoirs the time to reach the total pseudo steady state could be much longer than the cross-flow system (Cobb et al., 1972; Earlougher et al., 1974; Raghavan et al., 1974; Tariq and Ramey, 1978).

![Effect of Vertical Permeability on Ramp](image)

**Figure 3.23:** The effect of vertical permeability on the pressure derivative response. The figure shows that the slope of the ramp increases when the vertical permeability decreases. The red curve ($k_H = k_V$) shows that a ramp, with gentle slope, could exist even though the vertical permeability is not nil (black curve).

### 3.5.3 Effect of correlation length

The rate of increase of variogram is an indicator of the rate at which the “influence” of a sample decreases with increasing distances from the sample site (Sarma, 2009). The
correlation length or the zone of influence is the distance beyond which the data are uncorrelated and at which the value of variogram corresponds to variance of the population. The correlation length in geological modelling is a concept that is applicable for both the facies and the petrophysical distribution. However, here the term “correlation length” refers to “facies” correlation length unless it is specifically mentioned. The correlation length has a significant impact on the ramp effect signature. An increase in the correlation length would lead to the simulation of a more smoothly changing function, which in turn would produce thicker facies units (Yarus and Chambers, 1994). A thicker facies produces a longer plateau in pressure derivative response. Figure 3.24 shows the effect of correlation length on the pressure derivative response of a commingled multi-facies stochastic model. The correlation length in this study corresponds to the range of an isotropic exponential variogram without nugget effect. There might be possible geological arguments for making use of anisotropic covariance function, but as the downstream direction can radiate over a wide range in braided fluvial reservoirs, the isotropic areal variogram will likely suffice (Corbett et al., 2005) for realistic models. As the correlation length of all facies increases, the early time stabilization increases as well. For instance, in the case with 750 m of correlation length the early plateau is longer than the other cases with shorter correlation length. A second stabilization appears when the pressure-investigated volume is not affected by the external no-flow boundaries and the diffusion front is, by far, areally larger than the correlation lengths. In this situation the second plateau should be stabilized over its steady-state value of the effective permeability. The model with correlation length 25 m, for example, has a fully developed ramp with a second plateau representing the effective permeability of total system. Although there is a short transition from the first stabilization toward the ramp effect, the slope of the fully-developed ramp does not depend upon the correlation length.
Chapter 3: Ramp Effect Characterisation

3.5.4 Effect of no-flow/leaky boundaries on the ramp effect

The existence of no-flow or leaky boundaries near the wellbore can affect the ramp response. Figure 3.25 shows a permeability distribution map that is associated with a ramp response on the Log-Log diagnostic plot. The map includes two non-sealing boundaries (i.e. boundaries 1 and 2) that are far from the well and will not affect the well-test response. However, the model includes another fault (i.e. boundary 3) which is close to the well (i.e. 300 ft away from the wellbore). The effect of size and transmissibility of this boundary will be considered here. The existence of this boundary can affect the original ramp effect response. Two cases with different fault lengths will be considered in here; one with a shorter length and another one with a much longer length. Figure 3.26 shows the constant rate draw-down response curves for the first case where the fault has a shorter length (i.e.1500 ft). Three scenarios are considered where the transmissibility (T) is set to zero (sealing fault), 0.1 (leaky fault) and 1 (non sealing or fully transmissible fault). The figure indicates that the reduction of fault transmissibility creates a higher pressure drop and a different ramp effect. In particular, level of the second plateau has been shifted upwards. Therefore, the analysis of the second plateau, while ignoring the fault effect, will provide the erroneous results in permeability estimation from the late transient stabilization. Figure 3.27 shows the draw-down well-test response for the second case, where the fault length has been
extended to 7500 ft. Clearly, in this case, decreasing the fault transmissibility has increased the ramp slope to approximately 0.75. More interestingly, the second stabilization has disappeared from the plot within the considered test period. Figure 3.28 shows the associated spatial pressure maps for the second case at the end of test, where different fault transmissibility values have been implemented. The figure indicates how fault transmissibility can affect the spatial distribution of pressure.

**Figure 3.25:** Permeability distribution map corresponding to the ramp effect. There are two fully transmissible faults (i.e. boundaries 1 and 2) and one no-flow/leaky boundary near the well location (boundary 3).

**Figure 3.26:** Effect of a short fault (boundary 3) with different transmissibilities (i.e. \(T=0, 0.1\) and \(1\)) on the ramp effect.
3.5.5 Statistical pressure response

The dynamic pressure response is a function of the spatial distribution of the properties inside the geological model. Any changes in these properties might lead to a different well-test response. On the other hand, the geostatistical techniques can generate multiple realizations of the reservoir properties. These realizations provide all possible and equiprobable outcomes of the reservoir model. Therefore, the geomodel test-response can be interpreted as an ensemble averaging of all well-test responses over all possible realizations. This ensures that the pressure derivative obtained is a general response for a family of models sharing the same statistics and geology. Figure 3.29 presents the ensemble averages of pressure derivative responses with associated standard deviations for correlation length of 75 m. The ensemble has been taken over 50 different realizations. The upper curve shows the commingled response of the reservoir when
$k_V/k_H = 0$ whilst the lower curve is the response when $k_V/k_H = 1$. Both curves started at the same near wellbore permeability with the same degree of variability shown by the vertical error bars diminishing with time. This confirms that the early-time response is independent of the vertical permeability (vertical or macro cross-flow) and is basically close to arithmetic average of the horizontal permeabilities of the penetrated cells. After 0.03 hr the commingled pressure derivative response deviates from that of the cross-flow system developing a ramp effect. It is worth mentioning that as the contrast in the average horizontal permeabilities of the layers is small, the V-shape response, the characteristic of the double permeability reservoir with macro cross-flow, does not appear.

Figure 3.29: Statistical pressure response of two different systems with and without vertical macro cross-flow. Each curve is the ensemble average of the pressure derivative response over 30 different realizations of the reservoir with associated vertical error bars (vertical bars).

3.5.6 Particular aspects of commingled and cross-flow systems from numerical experiments

Equivalent single-layer response

The “extracted” permeability from the early time well-test responses should always be close to the arithmetic average of local effective permeability values intersected by the
well. This is irrespective of the reservoir cross-flow condition. However, the extracted value is contingent upon how early the pressure data points can be recorded and how laterally extensive are the local permeability values near the wellbore regions (reflected in correlation lengths). In contrast to the early time, the vertical upscaling of permeability by arithmetic average could not exactly match the late time well-test response of the multi-layered model. This depends on the macro (or reservoir) cross-flow condition and the complexity of permeability distribution. For example, Figure 3.30 shows the comparison between the build-up response of the full cross-flow multi-layered system (i.e. the same model as Figure 3.22 with $k_V=k_H$) and those of the single layered upscaled models. Although, the arithmetic upscaling of the horizontal permeability provides a generally close well-test estimate to the multi-layered cross-flow system, a better match is obtained for the late times if the power average upscaling (with exponent of 0.55) is used. However, this is associated with the early-time mismatch.

It was also noted that the equivalent single-layer modelling using the power averaging did not seem to suggest the unequivocal results for the commingled systems with complex 3-D permeability distribution (i.e. having the ramp effect).

![Figure 3.30: The comparison of the build-up response of the multi-layered cross-flow model ($k_V=k_H$) and the vertically upscaled models (single-layer). The arithmetic averaging of horizontal permeability in vertical direction could be a good estimate of the layered response at the early time. However, a power average could predict the later time response.](image-url)
Partial depletion and well backflow

Figure 3.31 shows that the pressure derivative response is monotonically increasing over four logarithmic cycles which indicates a sustained ramp effect. However, the last cycle after about 30 hr is a response due to boundary effect and/or the partial depletion of the commingled layers. In the commingled reservoir the compressible zone ahead of the pressure disturbance will move faster in the high permeable zones and leads to earlier depletion of the layers. This phenomenon mimics a “false ramp” effect on the pressure derivative response and is due to external boundaries effect. This signature can be distinguished from the true ramp effect based on some lines of evidence. Firstly, the false-ramp slope “gradually” increases to unity when the whole reservoir layers are depleting (pseudo-steady state regime). This will take a very long time. Secondly, the build-up response of the true ramp matches the draw-down response whereas the false ramp has different signature on build-up and draw-down responses. The build-up response highlights the fact that the pressure derivative in pseudo-steady state regime (depletion) follows a downward negative trend toward infinity (Figure 3.31).

![Figure 3.31: The build-up response (black curve) deviates from the draw-down response curve (grey curve) which shows the boundary effect. The layered depletion starts after around 30 hr of production.](image)

Figure 3.32 shows the pressure-investigated volume after 32 hr of draw-down. This is a time immediately after one external boundary has been felt by the well-test response,
and emphasizes that the ramp effect is independent of any external boundaries. The investigated volume here is defined as a volume in which the pressure drop is greater than say 1% of maximum pressure drop at well location. This ensures that the results can be distinguished from numerical errors. The figure also illustrates the fact that a boundary might have reached by the pressure disturbance in one permeable layer, though the other layers are still in transient state. This shows the boundary effect should start after 30 hrs of production in this model, which is followed by the partial depletion of the layers at the late times.

The “well cross-flow” (or backflow) can occur in the build-up response of the commingled reservoirs where the highly depleted reservoir layer during the draw-down period is recharged by the low depleted layers in the build-up period. This flow occurs within the wellbore particularly when the well is shut in at the surface, and is detected by a negative layer rate using the PL tool. This can sometimes create a false ramp effect in the build-up derivative response. Although in a vast majority of the real well-test situations, the backflow is not a major impediment for the interpretation (Raghavan and Kuchuk, 2009), there might be cases where the total build-up is affected by this phenomenon. In this numerical study, the well backflow was not a major problem. The unique shape of the pressure derivative is sustained in the draw-down response and was persistence even in individual testing of many single layers separated from the reservoirs model.

Figure 3.32: Pressure distribution in the reservoir during the ramp response. The filtered pressure drop (P(0)-P(t)>0) indicating investigated region right after an external boundary is felt by the well-test response (32 hr) where the layers are not depleted at the same rate.
**Heterogeneity varying with volume of investigation**

The coefficient of variation, $C_v$, is a dimensionless statistical indicator showing the variability in a population and is defined as follows

$$C_v = \frac{\sqrt{\text{Var}(k)}}{E(k)}$$

(3.18)

where, $\sqrt{\text{Var}(k)}$ and $E(k)$ are the standard deviation and the expected value of the population (e.g. permeability in this case). The variance and expected value operate on the random field, therefore $C_v$ is also a random variable with any specified probability distribution function (pdf). The pdf function of the population is not usually known; hence, it is a common practice to use the sample variance and mean in the $C_v$ formula. This is usually valid when we have a sufficient number of samples for the population.

The coefficient of variation, $C_v$, is used to guide sampling density and can be used as a geological heterogeneity indicator (Jensen et al., 2000). Although, it is dependent on the distribution function, as a rule of thumb, the sample mean will be within ±20% of the parent mean for 95% of all of possible outcomes, where the minimum number of samples is greater than $I_0$ (Jensen et al., 2000)

$$I_0 = (10 \times C_v)^2$$

(3.19)

This comments on the representativeness of the sample statistics.

Figure 3.33 shows the $C_v$'s for the horizontal permeability in different geological environments. This induces that the $C_v$ can also be used as a heterogeneity-level identifier of a geological medium (Jensen et al., 2000) that is the medium is taken as either a homogeneous medium when $C_v<0.5$, or a heterogeneous medium when $0.5<C_v<1$ and or a very heterogeneous medium when $C_v>1$. 


Chapter 3: Ramp Effect Characterisation

Figure 3.33: Variation of $C_v$ with scale and depositional environment (Jensen et al., 2000). $C_v$ can be used as an indicator for quantification of the level of geological heterogeneity.

The effect of the macro cross-flow in the geological model can be observed on the $C_v$ if it is calculated over the effective reservoir volume. The “time-dependent” $C_v$ profile is constructed based on the numerical tracking of the pressure front at each time-step to calculate the arithmetic average and the standard deviation of the permeability values within the pressure investigated volumes. The time-dependent $C_v$ is defined as

$$C_v(t, x) = \frac{\sigma(\Omega(t, x)|\Delta P(t, x) > \epsilon)}{M(\Omega(t, x)|\Delta P(t, x) > \epsilon)}$$

(3.20)

where, $\Omega(t, x)|\Delta P(x,t)$ shows that the statistics (i.e. sample standard deviation ($\sigma$) and sample mean ($M$)) are taken from the samples (cell permeability values) within the domain $\Omega$ at time “$t$”, when the expanding pressure volume is defined by the pressure drop of the cells as they are slightly greater than zero. In practice, a uniform averaging of the permeability values is taken and the “$\epsilon$” is selected in a way to ensure that the numerical error propagation in the calculations is minimized (in this case $\epsilon \approx 0.001$ psi suffices). To capture the early time test response, the early time $C_v$’s are calculated from the local grid refinements and replace the coarse cell values. The time-dependent $C_v$’s for the cross-flow system can then be compared with those of the commingled system to probe the effect of dynamic communication of the layers on the pressure investigation.
Figure 3.34 shows that, at the early times, the $C_v$’s for the commingled and the cross-flow systems of the model shown in Figure 3.22 match. This matching can also be seen in the very late times where the pseudo-steady state (PSS) flow regime prevails and the whole model is affected by the pressure depletion. A careful look at Figure 3.34 reveals that there is an earlier flattening of the $C_v$ curve for the cross-flow system. This shows the higher tendency of the cross-flow model to reach PSS flow regime. Meanwhile, the $C_v$’s for the two systems deviate from each other in the middle times soon after the vertical flow commences. This results in different diffusion front expansions in two systems. Figure 3.34 also shows that although the $C_v$’s for the cross-flow system are greater than the commingled one (due to vertical communication of cells), the overall commingled well-test behaviour is more complex (i.e. the ramp response). This indicates that the vertical flow communication is a very important factor in altering the well-test response and development of the ramp effect.

![Figure 3.34: Time-dependent $C_v$’s for the commingled and cross-flow systems. The early and late time $C_v$’s of the two systems coincide. The separation of the curves shows the effect of the vertical communication in the middle times.](image)

### 3.5.7 Production rate of separate layers

It is well known that in commingled systems when the total production rate is constant, the individual layers contribute to production according to their specific petrophysical and engineering properties (e.g. skin factor, geometry, storativity and transmissibility). The production rate of each layer varies with time and this takes more time to stabilize comparing to the cross-flow systems. This is shown in Figure 3.35 where the layered
production of the commingled and the cross-flow systems are compared for different production lengths (i.e. after 1 day and after 9 days).

Although, the selective layer testing is a matter of more detailed studies (see Kuchuk et al. 2010), it is important to look at a particular aspect of the layered derivative response in this section. Figure 3.36 displays the superposition derivative response curves for each individual layer, where the layer-wise rates and pressures have been taken into account by the superposition theorem. The reservoir is under an overall constant rate draw-down of 980 STBO/day and this is equal to the summation of individual layer rates at any time during the test. The figure elucidates that a majority of the layers carry the ramp signature while few of them (particularly the low permeable layers) have different derivative responses after a long initial unit-slope curve (i.e. fake wellbore storage). This point becomes explicated when we note that those layers with unit slope trend belong to tighter parts of the formation where their contributions to the total production are negligible.

Figure 3.35: The layer flow rates of the commingled and the cross-flow systems after one day (left) and 9 days (right). The commingled system suffers from the long term variation in the layer rates while the cross-flow rates stabilize rapidly.
Figure 3.36: The layer-by-layer superposition pressure derivative response where the total system is under a single draw-down \(q=980\) STB/day). Each derivative curve is obtained by including the variable pressure and rate of each individual layer. The summation of all rates at any specific time is equivalent to the total draw-down rate.

These individual test responses might be combined to result in an overall test response with a single constant production rate. Having this objective in mind, different averaging schemes were performed on these layered derivatives. Figure 3.37 shows that although the simple thickness-weighted arithmetic average results in a derivative curve, which follows the overall ramp response (yet is biased toward the extreme derivative values), the rate-weighted averages (equation 3.21) could reasonably match the single-rate overall ramp response. Equation (3.21) was an empirical equation that I derived by evaluating many numerical simulations and turned out to work. This also matches the superposition derivative of an average pressure curve that is obtained by averaging the layer pressures with time. The rate-weighted averaging methods is defined as follows

\[
M(t) = \sum_j \frac{q_j(t)}{q_T} \frac{\partial \Delta P_j(t)}{\partial \left(\text{Sup}_j(t)\right)}
\]  

(3.21)

where, \(M(t)\) is the rate-weighted average of the layers pressure derivatives. This is found to be very close to \(\frac{\partial \Delta P(t)}{\partial \ln t}\) that is the draw-down derivative response of the multi-layered reservoir with a single total production rate of \(q_T, \frac{\partial \Delta P_j(t)}{\partial \left(\text{Sup}_j(t)\right)}\) is the pressure derivative of a single layer \(j\) with respect to the superposition time, \(\text{Sup}(t),\)
which is in turn a function of the layers variable rates. The scaled pressure drop, $\Delta \bar{P}$, and superposition time, $Sup(t)$, are defined as follows (Kappa, Pers. Comm.)

$$Sup(t) = \sum_{i=1}^{n} \frac{q_i - q_{i-1}}{q_n} \ln[t - t_{i-1}]$$  \hspace{1cm} (3.22)

$$\Delta \bar{P} = \Delta P \times \frac{q_n}{q(t)}$$  \hspace{1cm} (3.23)

where, $\Delta P$ is the wellbore pressure drop, $q(t)$ is the rate corresponding to the point with pressure $P$, $q_n$ is the last rate value and $q_i$ is the step rate for the interval $\Delta t = t_i - t_{i-1}$.

Figure 3.37: The single draw-down pressure derivative response of a commingled multi-layered system (continuous black curve) and the derivative responses that have been calculated by different averaging method of the of the individual layers derivative responses. The derivative response of the “average pressure of the layer pressures” (dotted curve) and the “rate-weighted averaging of the layer superposition derivatives” (dashed curve) matches the overall single draw-down derivative response.

3.6 Chapter summary

Different approaches were employed to study the ramp effect. These were including the statistical approach, the analytical-numerical approach and the numerical (or geological) approach. The statistical approach uses an ensemble of the petrophysical realizations and provides the statistical dynamic response of the model. This is mainly used to study the relationship between the well-test effective (or average) permeability and the geostatistical parameters of the petrophysical fields. This can be done analytically in the
2-D fields while a numerical ensemble averaging may be used for the general 3-D cases. The analytical-numerical approaches scrutinize the averaging process of the well-test in the 2-D cases and can provide the well-test response of any individual permeability field without running the fluid-flow simulation. This approach helps understand the lateral variation of the permeability in a geostatistical field which is directly linked to the ramp effect. The numerical approach is used to study the well-test response of any 3-D reservoir model with any degree of the embedded facies and heterogeneities. This is the static transient test geological model (Kuchuk et al., 2010) that uses a geoengineering approach to integrate all sources of information. In this approach the ramp effect can be effectively characterised in terms of the geostatistical parameters of the system.
Chapter 4

Ramp Effect Case Studies

The ramp effect, which is diagnostic of a reservoir scenario, is constrained by the geological environment and well data, and in this sense, provides a more-constrained geoengineering description. In this chapter, the focus is on the geological well-test interpretation and modelling of few case studies in the commingled fluvial systems where the spatial distribution of the petrophysical and facies properties are complex. This has been reflected through studying the real-life geological well-test examples of the high-energy (i.e. braided) and low-energy (i.e. meandering and anastomosing) fluvial systems. The examples are ordered according to the increasing level of complexity in interpretation and modelling.

4.1 Example 1: The well-testing ramp-like response

4.1.1 Background

The Wytch Farm field is the largest onshore oilfield of the Western Europe which is located in the Purbeck region of Dorset, south of England (Bowman et al., 1993; McClure et al., 1995). The Triassic Sherwood Sandstone reservoir is the main producing group in the field, which has been deposited by the braided streams and the sheet flood processes in a semi-arid environment (Dranfield et al., 1987). The reservoir succession follows a fining-upward trend and becomes mud-prone towards the top (Hogg et al., 1996; McKie et al., 1998). The Sherwood Sandstone is divided into two different zones: a lower zone with high net:gross fluvial deposits and an upper zone with more common lacustrine and floodplain mud rocks (McKie et al., 1998). Approximately 70% of the oil
in place is contained within the particularly complex upper Sherwood member (Dranfield et al., 1987). The thicker sandbodies at the base have an average porosity of 18% and the permeability values commonly greater than 1.5 Darcy while isolated channel deposits at the top have an average porosity of 15% with a permeability of about 150 md (Hogg et al., 1996; Hogg et al., 1999). This forms a strongly layered reservoir with a $k_v/k_h$ value of 0.01 in the lower part compared with $k_v/k_h$ value of 0.001 in the upper part (Bowman et al., 1993; Hogg et al., 1996; Hogg et al., 1999) at the simulation model scale. Moreover, the complex facies-related distribution of discontinuous shales and the existence of caliche within the shale and siltstone horizons create some barrier to vertical flow (Dranfield et al., 1987).

Figure 4.1, from Hogg et al. (1999), is a schematic stratigraphic layering and zonation cross-section of the Sherwood Sandstone where the reservoir zones 10, 30, 50, 70, 80 and 100 are enclosed by the vertical transmissibility barriers of zones 20, 40, 60 and 90. The figure also shows that the western flank has a lower quality sands with abundant discontinuous shales, which can form an effective commingled system.

![Figure 4.1: The Schematic reservoir layering for the Sherwood Sandstone (Hogg et al., 1999)](image)

Figure 4.2 (McKie et al., 1998; Hogg et al., 1999) shows the representative west-east cross-section of the upper Sherwood Sandstone, which maps the grain size and the facies distribution across this reservoir section. This figure also indicates that the western part is mainly comprised of the thinly bedded fine-grain sandstones with more mud-prone
deposits while the eastern part is dominated by the coarse grain and the thick channel fill-deposits.

Figure 4.2: Representative west-east transect across the field showing the facies architecture of the upper zone of the Sherwood Sandstone (McKie et al., 1998; Hogg et al., 1999).

### 4.1.2 Well-test interpretation

A well-test example with a ramp-like response is that of Well F in the western part of the Sherwood Sandstone. The Well F perforated through the upper part of the Sherwood Sandstone and produces from 29 m of the perforated interval. The gross reservoir interval was 142.5m. The DST had a short flow period of 2.38 hr at a rate of 250 STBO/day, which was followed by a draw-down of 550 STBO/day for 7.9 hr, and a longer build-up period of 12.24 hr (Figure 4.3).

Figure 4.3: The history plot Well-F in Wytch Farm field. A build-up period of 8 hours follows a two-rate draw-down period.
Any well-test interpretation in this reservoir carries with it a high degree of uncertainty because the well-test data were not supported with enough engineering, geological and geophysical information. A general top-structure map interpreted from surface seismic data (Figure 4.4) and some poor core data were the only supporting information available to this study.

![Figure 4.4: The top structure map of the Wytch Farm field indicating the seismic faults and well locations. The well F is located in the western part of the field.](image)

Because the depth measurements for the core and the perforation were different, the perforation intervals could not be checked against the core permeability and the porosity values. Table 4.1 summarize the basic statistics of the available Well F core data.

**Table 4.1: Well-F statistics extracted from the core measurements.**

<table>
<thead>
<tr>
<th>Well</th>
<th>Parameter</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_H, md</td>
<td>Arithmetic Average</td>
<td>560.65</td>
</tr>
<tr>
<td></td>
<td>Geometric Average</td>
<td>16.04</td>
</tr>
<tr>
<td></td>
<td>Harmonic Average</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>1207.83</td>
</tr>
<tr>
<td></td>
<td>Variation Coefficient C_v</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>27.2</td>
</tr>
<tr>
<td></td>
<td>Mode</td>
<td>200.1</td>
</tr>
<tr>
<td></td>
<td>Sample Sufficiency No.</td>
<td>462</td>
</tr>
<tr>
<td></td>
<td>Sample No.</td>
<td>301</td>
</tr>
</tbody>
</table>
The permeability Variation Coefficient ($C_v$) of 2.15 represents that the core data were taken from a highly heterogeneous reservoir environment. However, this should be used with caution, as the estimated sample efficiency number is greater than the available sample size. This indicates that the sample statistics might not fully represents the population statistics.

Figure 4.5 shows the Log-Log plot of the build-up period. The pressure derivative curve steadily increases over two log cycles with a slope less than 0.5. This increase can be related either to the existence of external boundaries or deterioration of the reservoir properties (e.g. mobility) away from the wellbore which resembles the ramp effect. Modelling the test response with parallel faults provides two boundaries with the distances of 25 ft and 11 ft away from the well. The available seismic map does not confirm this, as the closest mapped fault is 220 m away from the well. However, considering the possibility of sub-seismic faults present and the general low quality of the surface seismic data, the faulted model interpretation should not be thoroughly overlooked. Alternatively, the ramp effect interpretation can be adopted. In this sense, the early zero-slope derivative curve provides the permeability-thickness product, $k\times h$, of 571.2 md×ft while the faint late flattening of the derivative curve gives rise to a $k\times h$ of 181 md×ft. Considering 29 m of productive interval, therefore the permeability varies between 5.9 md to around 2 md that lies between the geometric and the harmonic mean of the core data. Recalling the ramp effect concept presented in Chapter 3, this might indicate that channel sands are sparsely distributed within a low quality matrix, in an effectively commingled system, leading the well-test permeability to be estimated close to the harmonic average. Moreover, the existence of thin sandbodies interbedded with low quality shale lamine reflected in the core data, favours the existence of the spatial ramp effect in which the final stabilization is biased toward the harmonic mean. This point is visually shown in Figure 4.6, which relates the approximate well-test permeability in various fluvial environments. Comparing the available core data from the other wells in the field support this geological interpretation that Well F has been deposited in a distal area of the fluvial system where there are thinner flow units. Although, the high permeability streaks found in the core data, the negative geoskin was not seen. This might be due to either very limited extent of the high permeability streaks, which could not be monitored by the available poor gauge resolution or high wellbore skin (damage skin) that compensates the negative geological skin value.
Figure 4.5: Well F normalized build-up response representing a gradual increase in the pressure derivative curve (ramp-like phenomenon).

Figure 4.6: The approximate relations between the core and the well-test average permeabilities in different geological environments. Well-test permeability is close to: 1. the “arithmetic average” of the core permeabilities in the layer-cake reservoirs with perfect horizontal flow (left), 2. the geometric average in the random channelised systems (middle) and 3. the harmonic average in the sparse laterally connected channelised systems (right).

This example illustrates the non-uniqueness nature of well-test modelling and clearly indicates that the integration of geosciences and well-testing can reduce the uncertainty in the modelling and can improve the reservoir description.

4.2 Example 2: The well-testing ramp response in a meandering environment

4.2.1 Background

This example is taken from a well in India which was also the subject of a more detailed study (Gurav, 2010). The vertical profile shows a perforated interval across several discrete fluvial sandbodies (Figure 4.7). The fining upward sequence of the gamma ray (GR) log is characteristic of the fluvial environment. The central clean GR was found to be correlated with increased resistivity log attributed to oil-bearing high permeability coarse sands (shown in yellow) while the increased GR at the top and the base including
the mud drape and calcrete minerals were characteristic of the point bar deposits (shown in orange).

Figure 4.7: Perforated interval in a well from India. Four point bar intervals are interpreted. A rock-typing approach is used to apply petrophysical properties to the sand facies intervals (yellow and orange boxes).

A very detailed geological analysis was not part of this screening study, rather the tested interval in the well was simply interpreted as an interval containing a series of point-bar deposits, an analogue chosen and a 3-D model built to capture aspects of the interpretation model (high proportion of lateral accreting point bars in discrete layers). The model is therefore illustrative rather than trying to exactly match log and core data. The latter was not available to this study.

4.2.2 Multi Point Statistics (MPS) modelling

The Indus River is used here in this project as a modern river analogue for this geological setting (Figure 4.8). This image was taken from Google Maps™ which detail the river Indus near Katiar a small town near Hyderabad in Pakistan. The satellite photo of the river was obtained from Google maps and the lateral dimensions of the point bar deposits were measured for estimating the ranges for the training image. The sand body dimensions were measured and a range of lengths varying from 100m to 2000m was obtained. These were used for the generation of the training image.
Derived in part from the modern analogue, a training image was used for multi-point facies simulation (Guardiano and Srivastava, 1993). Complex geological structures present in the reservoir may be represented by use of multiple-point statistics, which can express more complex patterns (than from two-point, variogram-based statistical techniques). It is a pixel-based algorithm that can create the facies models look like the object-based models (Schlumberger, 2009b). A training image does not need to carry any locally accurate information on the reservoir but only needs to reflect a prior stationary geological/structural concept. Thus, training images can be generated by object-based algorithms freed of the constraints of data conditioning (Strebelle and Journel, 2001).

Most current stochastic reservoir simulation algorithms aim at reproducing statistics inferred from the well data solely. Since these data are typically sparse they can give, at best, only an estimate of the two-point correlation within the reservoir. The modelling of specific meander-loop patterns calls for characterising the spatial continuity at three or more locations at a time. Reproduction of multiple-point continuity in the reservoir model is critical, not so much to produce geologically-realistic looking maps, but to provide more representative connectivity and more accurate flow performance predictions (Caers et al., 1999). The resulting meander-loop training image (Figure 4.9) was constructed with this objective in mind. The training image has 20×20×1 cells in x,
y and z direction and includes four isotropic facies: coarse sand \((k=3200 \text{ md and } \varphi=0.15)\), point bar sand \((k=114 \text{ md and } \varphi=0.10)\), abandoned channel mud and background deposits \((k=0.001 \text{ md and } \varphi=0.005)\). The juxtapositioning of different interpreted facies was the primary object in construction of this simplified training image.

Figure 4.9: A simplified Training Image constructed from analogues for multi-point facies simulation. The facies in orange is the coarse sand, the yellow is the point bar deposit and the red and brown are the abandoned channel and the background mud respectively (Gurav, 2010).

A layered model was built based on our interpretation of the outcrop and modern river analogue with the floodplain and abandoned channel mud facies acting as barriers to flow. The proportions for the individual facies were made to honour the proportions observed in the well log data (i.e. around 60% of point bar and coarse sand in 23m of the interval). The four facies that were classified on the training image were then distributed using multi-point facies simulation. The porosity and permeability values were assumed from analogues (Corbett and Potter, 2004), with separate rock types giving an order of magnitude difference in permeability for similar porosity values and assigned individually to the facies. A sector model along with the assigned properties was then exported for flow simulation. A single realization of the distributed facies is shown in Figure 4.10.
Figure 4.10: Facies distribution (top view) showing the juxta-positioning of the distributed facies in a high net:gross lateral system.

The use of multi-point statistical modelling gives rise to a partially, but nevertheless relatively well-connected reservoir, with discrete but communicating sand packages. Using a pixel modelling approach would possibly result in too much connectivity for such a high-net to gross system. Object modelling of point-bars is also a challenge with simple objects as it is difficult to capture the right degree of facies transition. The modelling technique used will potentially have a significant effect on the well-test responses seen. Use of pixel models without high-degree of variability in these simulations will not show the well-test responses which are essentially driven by facies-connectivity issues, rather than more transitional facies-trends.

4.2.3 Numerical well-test simulations

A static sector model was exported to simulate dynamic flow performance. A simulation model with 170×226×4 cells was used to evaluate the well-test response. The average coarse grid size resolution of 10m×10m×8.5 m in the x, y and z directions was used.

The well-tests were modelled using black oil, finite difference, reservoir simulation software. Average flow rates observed in the well-test were noted. The PVT data from the samples obtained during the well-test was provided by the operator and fluid properties were input in the model. No core analysis data was provided. Three facies were assigned saturation function tables and the relative permeabilities for the facies were calculated by using the Corey’s two-phase model. Initial water saturation was
estimated for the facies (0.35 for shale, 0.15 for coarse sand and 0.25 for fine sand). The numbers were suggested by the providing company.

Selecting the appropriate number of time steps is essential in order to capture the pressure changes and obtain sufficient number of pressure data to import into the analytical well-test analysis software. A logarithmic increment of time steps was required for this purpose and the appropriate size of time steps was selected to adequately capture the rapid pressure drop at the early time of the draw-down test and the rapid pressure build-up at early time of the build-up. This was also done in order to reduce any numerical dispersion effects.

4.2.4 Well-test simulation and validation of the ramp effect

The smaller size training image with average dimensions of 200 m×200 m was used in the modelling procedure (Gurav, 2010). The well was placed at a distance of 250 m from the boundary in the model to simulate the actual well-test response. To accurately reproduce the test response, the exact same flow intervals were implemented on the simulation model (Figure 4.11). The vertical permeability of the model was set to zero for the simulation run.

Local grid refinement was applied for the host cells adjoining the well to allow the model to capture the pressure gradient near the well and avoid false wellbore storage effect. Though radial grid refinement is more appropriate for modelling flow in a near well bore region, Cartesian grid refining was used after comparing the simulation results of the two techniques.

![Figure 4.11: Flow rate vs time plot - actual and simulated data. The simulation is controlled by production rate.](image-url)
The well was flowed for the exact time interval and the derivative of the resulting pressure response is as shown in Figure 4.12. As seen from the figure, we observe that the simulated pressure derivative matches the trend of the linear flow response observed in the actual well-test. Though the derivative and the pressure response from the simulation are shifted in time, the model has captured the linear flow response successfully. This result proves that an linear flow response can be obtained from a high net:gross laterally accreting reservoir model with vertical permeability equal to zero. This response is a clear example of the ramp effect and is postulated to be generated from the cross-flow caused between point bars in a high net:gross, laterally avulsing, meandering fluvial reservoir. Pressure support is offered from the laterally amalgamated point bars and as the pressure disturbance (i.e. diffusion front) moves outward through the reservoir, lateral cross-flow between the individual sandbodies takes place (Figure 4.13). The important point to note here is that the pressure perturbation and the ensuing flow is constrained to moving in the lateral direction only and there is no vertical communication or cross-flow between the four layers modelled. From the diagnostic, we observe that the simulated response displays a longer infinite acting radial flow regime.

![Figure 4.12: Comparison between actual and test simulation pressure derivative.](image-url)
The late time effect is unclear in the real well-test response and we can only speculate over whether or not the well-test has seen the effect of the reservoir boundaries. After shut-in, the pressure starts to build-up during the initial infinite regime and is observed here to stabilize and tend towards the average reservoir pressure. If we assume that the effect of boundaries has been seen by the well-test, one of the possibilities of the flattening of the derivative curve could be the onset of the pseudo steady state regime on the derivative which is followed by a rollover. A longer shut in duration would have helped in the elimination of this ambiguity. Another possibility and probably the most important one is the flattening could have happened due to lateral pressure communication between the sandbodies. This interpretation leads to selecting a proper analytical model for the well-test interpretation.

4.2.5 Analytical modelling
The idea of the lateral communicating sand patches can be furthered modelled analytically by a 2-D equivalent model (i.e. “2-cells compartmentalized system (Figure 4.14)” where two communicating compartments of a reservoir are separated by an effective transmissible boundary (Stewart, 2011).
The initial derivative stabilization (around 0.03 hr) in the first compartment, where the well is located, is followed by a linear flow (½ slope line) in the middle time that is eventually followed by a tendency toward the unit-slope in the later times (around 10 hours). This unit slope trend in the build-up represents the onset of the first compartment depletion. The analysis of this unit slope line, if it is well developed, could provide the information about the compartment’s volume. In this case, the unit slope is not fully developed. However, an upward trend is observed which is assigned to this phenomenon. Depending on the transmissibility of the leaky interface, the late time derivative is flattened or rolled over where the total pseudo steady state is triggered. That is when the second compartment starts to feed the primary reservoir volume. This naturally depends upon the supporting volume of the second compartment as well. If the supporting volume is small, the rollover is triggered earlier in time. Figure 4.15 shows a match obtained by application of this model. Although, the geology (i.e. the earlier performed geological well-test modelling) supports the idea, this should be checked against the other sources of information as the sandbody volumes interpreted from 3-D seismic data or preferably the time-lapse seismic data where possible.
Chapter 4: Ramp Effect case Studies

Figure 4.15: Analytical modelling of the ramp response based on the two-cell compartmentalized model. This option inferred from the geological modelling of the system and recognizing the lateral communicating patches of the sandbodies. The dotted curves are the actual well-test data and the solid curves are the fitted model.

4.3 Example 3: The complex ramp effect response in an anastomosing environment

4.3.1 Background

The sedimentary system of the real field X corresponds to a low energy anastomosing environment with expansive low quality overbank deposits. The main reservoir systems are those of the fluvial channel-fill deposits corresponds to elongated sidebars of the low sinuosity channels and the lobate point bars of the higher sinuosity channels. These individual sandbodies are small in size and are poorly connected. Furthermore, due to predominant overbanking and limited channel erosion the vertical connectivity of the sand patches remains extremely low.

4.3.2 Extended well-test program

An extended well-test program was carried out to investigate the furthered reservoir volume and to evaluate the reservoir properties at distant areas of the tested well (i.e. well X1). Therefore, six reservoir intervals with an overall thickness of 48m (and the net interval of 32m) within different reservoir units were selected for the perforation (Figure 4.16: left). The test history is composed of many short-term draw-downs, with variable rates, and a series of build-ups for a total duration of 11.1 days that are followed by a
very long build-up of 90 days. Figure 4.16 (right) shows the wellbore pressure and the imposed surface rate history of the extended well-test.

![Figure 4.16: The pressure-rate history of the extended well-test performed on well X1 and the associated perforated intervals within the main reservoir units (left).](image)

**4.3.3 Well-test interpretation**

Figure 4.17 shows the diagnostic Log-Log plot of the last build-up period. This is an interesting long “ramp response” example where the upward trend of the pressure derivative curves highlights the drastic lateral connectivity degradation away from the wellbore. In particular, the pressure drop and the derivative curves cross each other at the late times, which highlight the fact that the producing reservoir permeability disappears gradually but does not become zero (i.e. closed system). This can be modelled using a radial composite model where the mobility ratio is of an order of hundred.

![Figure 4.17: The diagnostic Log-Log plot (i.e. pressure drop and pressure derivative plot) for the main build-up in the real extended well-test.](image)
Chapter 4: Ramp Effect case Studies

Figure 4.18 shows the straight-line analysis approach for the well-test interpretation. The required input parameters for the analytical interpretation are listed in Table 4.2. By using this interpretation approach, the early time stabilization of the pressure derivative curve provides an effective permeability of 30md and a skin factor of -1.9. This negative skin factor can be an indication of geoskin where some high permeable streaks, with limited extend, crossing the wellbore. This early radial flow regime is followed by a half-slope trend and a secondary stabilization in the middle time region. The slope analysis of these flow regimes provides a channel width of 74 m (from the half-slope trend line) and an effective permeability of 4md with an associated skin factor of -5.4 (from the stabilization). It should be noted that although the “channel” interpretation looks valid in this fluvial environment, the half-slope derivative interpretation could be an immediate outcome of the composite regions with a mobility contrast of 7.5 (i.e. the outer region 4md to the inner region 30md). Moreover, the derivative response of the linear flow regime during “build-up” usually take a less than 0.5 slope trend on Log-Log plot (Stewart, 2011). Eventually, the derivative response follows a long unit-slope trend at the late times preceding a possible stabilization or turn-over after the times greater than 3000 hr which may provide an effective permeability of 0.03md. This unit slope trend can either be interpreted as an indication of a high mobility contrast or the reservoir compartmentalization in a sense that there is a poor communication between the high permeable spatial sandbodies.

Figure 4.18: The straight-line analysis for the main build-up period shown on the Log-Log plot. Three radial stabilizations, a half-slope linear flow regime and a late time unit-slope trend in the derivative curves are diagnosed.
Table 4.2: The formation and fluid properties used in the analytical well-test interpretation

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid volume factor</td>
<td>1.839 vol/vol</td>
</tr>
<tr>
<td>Fluid viscosity</td>
<td>0.248 cp</td>
</tr>
<tr>
<td>Fluid compressibility</td>
<td>$0.160 \times 10^{-3} \text{ l/bar}$</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>$0.680 \times 10^{-3} \text{ l/bar}$</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.18</td>
</tr>
<tr>
<td>Water saturation</td>
<td>10 %</td>
</tr>
<tr>
<td>Net thickness</td>
<td>32 m</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>$0.460 \times 10^{-3} \text{ l/bar}$</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>0.108 m</td>
</tr>
<tr>
<td>Total compressibility</td>
<td>$0.197 \times 10^{-3} \text{ l/bar}$</td>
</tr>
</tbody>
</table>

This 2-D analytical interpretation can be furthered improved assuming a “multilayered” reservoir scenario corresponds to the main productive perforation intervals. Although the layered interpretation looks more realistic, this is associated with more uncertainties in the modelling and in the interpretation itself. This is because the analytical models are the averaged and simplified dynamic models which may not honour the underlying reservoir geology. Besides, selecting the right analytical (and “average”) well-test model per each reservoir layer might be challenging. Therefore, an alternate modelling approach is employed to construct a detailed static (geological) model using the Multi-Point Statistics (MPS) approach which is then validated by the extend build-up data. This is an excellent example of integrating the multi-domain and multi-source information in the exploration phase.

4.3.4 MPS modelling

The reservoir model includes three main geological units. Based on the geological studies, a few modern analogue river systems were selected to construct different training images for each of these reservoir units. Figure 4.19 shows the satellite images of the Parana River System of Argentina as an analogue for the first two units and Figure 4.20 shows the Magdalena River System of Colombia as an analogue of the lowermost unit. The pictures have been taken from the Google Maps™.
Figure 4.19: Parana River- Argentina mouth delta: An analogue example for the first two reservoir units.

Figure 4.20: Magdalena River- Colombia: An analogue example for the third reservoir unit.

The extracted satellite images are then digitized to provide most representative training images. Figure 4.21 shows the constructed training images for each reservoir unit. The
training image for each layer has been constructed from different satellite images. The 3-D training images for each unit have 180×205×3 cells in x, y and z directions and each cell measures 10m×10m×1m of the reservoir volume. This implies that a 3-D search mask should be implemented within each reservoir unit to construct the local probability distribution functions. The training image size is large enough to reproduce the facies shapes and interactions. The training images for this fluvial environment include four facies ranging from high quality sandstones to low quality shales. Table 4.3 summarizes the colour codes and the various proportions of each individual facies within the training images. Moreover, Table 4.3 clarifies that, the unit 3 possesses the highest proportions of the high permeable facies (i.e. facies 3 and 4). However, the overall reservoir quality of the training image remains poor. This is then inherited by the MPS modelling which leads to having a full field facies model with a lower connectivity between the individual sand patches within dominant background shale.

The MPS modelling is performed on a fine-grid geological model (i.e. 290×320×106 cells in x, y and z directions and an average cell size of 11 m×11 m×2 m) to simulate five different facies realizations. Figure 4.22 shows the resulting facies images for the representative layers within the different reservoir units.
Chapter 4: Ramp Effect case Studies

Figure 4.21: The 3-D training images per reservoir units. There are three reservoir units and each training image includes three individual layers. These training images are used within the MPS algorithm to simulate different facies structures for each reservoir unit.

Table 4.3: The colour codes and proportions of different reservoir facies used in training image and the MPS modelling

<table>
<thead>
<tr>
<th>Code</th>
<th>Facies name</th>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alluvial shales</td>
<td>61%</td>
<td>57%</td>
<td>50%</td>
</tr>
<tr>
<td>2</td>
<td>Alluvial siltstones</td>
<td>30%</td>
<td>30%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>Low permeability sandstones</td>
<td>3%</td>
<td>4%</td>
<td>21%</td>
</tr>
<tr>
<td>4</td>
<td>High permeability sandstones</td>
<td>6%</td>
<td>9%</td>
<td>19%</td>
</tr>
</tbody>
</table>
Figure 4.22: The facies maps taken from the representative layers within each reservoir unit in five different MPS facies realizations.
To populate the petrophysical properties, the conditional Sequential Gaussian Simulation algorithm is implemented to distribute the porosity field within the geological model whereas the isotropic horizontal permeability values are estimated using a verified porosity-permeability cross-plot 1 (Figure 4.23). The facies statistical and geostatistical parameters required for porosity distribution are listed in Table 4.4. The vertical connectivity of the system, on the other hand, is extremely low. This is largely reflected in the $k_V/k_H$ ratio and the high proportion of the low quality facies in the system. The $k_V/k_H$ is equal to 0.0001 for facies 1 and 2 (F1 and F2) and is 0.01 for facies 3 and 4 (F3 and F4). Moreover, because of the high proportion of shale in the model, the sparse producing layers are effectively separated by low quality facies. This leads to production under commingled conditions which results in “ramp response” where the producing layers only communicate through the wellbore. This is confirmed from the PLT and the detailed geological studies.

![Figure 4.23: The porosity-permeability cross-plot for each reservoir facies. This cross-plot is used to estimate the permeability distribution after simulating the porosity field by Sequential Gaussian Simulation.](image)
Table 4.4: The parameters used in the geostatistical distribution of porosity.

<table>
<thead>
<tr>
<th>Facies</th>
<th>Spatial Distribution</th>
<th>Porosity Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_x (m) )</td>
<td>( \lambda_y (m) )</td>
</tr>
<tr>
<td>FG1</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG2</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG3</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG4</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

\( \lambda \) is correlation length in meter

4.3.5 Well-test simulations of different facies scenarios

A smaller sector of the fine model (i.e. 186\times 241\times 68 cells) is extracted to perform the numerical well-test simulation scenarios. Figure 4.24 shows the sector model and the main reservoir units embedded within the reservoir structural framework.

![Figure 4.24: A sector of the fine reservoir model and the associated reservoir units. This sector model is used in the well-test simulations. The first 12 vertical layers are within the unit 1, the next 19 layers within the unit 2 and the last 15 layers are within the unit 3.](image)

A black oil simulator is applied to perform the well-test simulations using the measured fluid and rock properties of the field X. The perforation interval and the rate history are taken to be the same as the real test operation.

Facies scenarios

Figure 4.25 shows the Log-Log diagnostic plot for five facies realizations. Although all of the realizations seem to follow the same upward trend, the overall qualities of matches are not acceptable.
However, a careful look at the diagnostic plots for all other realizations (Figure 4.25) reveals that two of those realizations (realizations 4 and 5) created a better trend than the other realizations (Figure 4.26). Despite a vertical shift, which indicates a much lower estimated lateral connectivity than the reality, the well-test response of realization 5 follows the same trend as the real build-up response at the early times. Clearly, there is a major issue in the late time trend, where the pressure derivative and pressure drop curves fail to reproduce the crossing feature seen on the real test response. Furthermore, Figure 4.26 also shows that the facies realization 4 reproduces the same late time signature as the real build-up while the early time trend is not correctly captured. In particular, the middle time stabilization does not appear in the simulated results and the duration of the linear flow regime (1/2 slope trend in the derivative curve) is overestimated.

Figure 4.25: The Log-Log diagnostic plots for the main build-up for different MPS facies realizations and the real well-test data.

Figure 4.26: The Log-Log diagnostic plot for the main build-up in the facies realizations 4 and 5 and the real well-test data.
Having found the proper facies realizations, few matching procedures were examined that are explained in details in the coming sections. These procedures are listed as follows

1. Uniform upgrading of the petrophysical properties
2. Petrophysical realizations
3. Facies hybridization and petrophysical realizations

Amongst these approaches, the “facies hybridization” could provide the most reasonable match satisfying the concerned geological constraints.

**Uniform upgrading of the system properties**

The vertical shift observed on the test responses of the facies realizations 5 and 4 suggests that the overall connectivity of the system needs an improvement. Therefore, the overall system permeability ($k_H$ and $k_V$) was improved by a factor of 2.3. This factor, which is the amount of pressure derivative shift upwards, is equal to the ratio of the pressure derivative for the facies realization 5 (or realization 4 but at the late times) to the real test derivative value. Figure 4.27 and Figure 4.28 (orange curves) show the well-test simulation results for this uniform upgrading of permeability in facies realizations 5 and 4. The response curves have shifted downwards and get closer to the build-up curves. However, there is still an obvious mismatch in the late times. It should be noted that by increasing the permeability values not only there is a vertical downward shift in the derivative response curve but also there is a shift toward the left. In other words, the spatial heterogeneities are felt earlier in time with permeability upgrading.

To overcome this shortcomings, the porosity values might also be improved by the same factor. Figure 4.27 and Figure 4.28 (purple curves) show the well-test response curves for the facies realizations 4 and 5 where the porosity distribution is “also” upgraded by the same factor of 2.3. As it is expected, the overall matching is fairly improved. However, there are still some clear mismatching features. The facies realization 5 (Figure 4.27: purple) cannot reproduce the expected late time feature and the overall well-test matching obtained from facies realization 4 (Figure 4.28: purple) is not convincing (in particular, the middle time stabilization does not match). This approach with porosity upgrading should remains as a sole sensitivity study and cannot be considered as right matching procedure. This is because porosity upgrading with a
factor 2.3 violates the confirmed range of porosity variations from the geological study of the field X.

![Figure 4.27](image1.png)

*Figure 4.27: The Log-Log diagnostic plot for the real well-test data and the facies realization 5 where the “permeability” or “permeability and porosity” are upgraded by a factor of 2.3.*

![Figure 4.28](image2.png)

*Figure 4.28: The Log-Log diagnostic plot for the real well-test data and the facies realization 4 where the “permeability” or “permeability and porosity” are upgraded by a factor of 2.3.*

**Petrophysical realizations**

In this section, the effects of different petrophysical realizations are exercised. This is to examine the petrophysical variations effect on the early and late time signature of the well-test response in facies realization 5 and to find out whether the these variations can lead to a reasonable well-test match. In this context, two hypotheses (Hyp1 and Hyp2) are considered and five different realizations per each hypothesis are simulated. The Hyp1 uses the same geostatistical parameters as previous simulation studies whereas in the Hyp2, the horizontal and the vertical correlation lengths of porosity (i.e. \(\lambda_x\), \(\lambda_y\) and \(\lambda_v\)) are changed. These changes are based on the geological understanding of the
porosity and permeability variations in field X and are within the accepted ranges of uncertainties. Table 4.5 summarizes the input parameters required for the Sequential Gaussian Simulation algorithm. Because the porosity and the permeability are correlated, this automatically changes the permeability field within the individual facies as well.

Table 4.5: The spatial distribution parameters required to generate ten petrophysical realizations under two different hypotheses.

<table>
<thead>
<tr>
<th>Facies</th>
<th>Hyp1: Realization 1 to 5</th>
<th>Hyp2: Realization 5 to 10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \lambda_x ) (m)</td>
<td>( \lambda_y ) (m)</td>
</tr>
<tr>
<td>FG1</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG2</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG3</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>FG4</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 4.29 shows the build-up diagnostic plot for simulated petrophysical realizations. The figure indicates that increasing the petrophysical correlation length tends toward flattening of the derivative curve. This, for instance, can be observed in realization 6 (of Hyp2) where the middle time stabilization is disappeared. It is also noted that the late time crossing feature is nearly reproduced (Figure 4.30). Meanwhile, the preceding upraise derivative (with a unit-slope trend) has been flattened and more importantly the overall connectivity increase is not remarkable. However, this is common for all of the simulated petrophysical realizations. Furthermore, a careful look at Figure 4.29 reveals that the pressure derivative curves for different realizations cross each other at some points. This is attributed to the different spatial permeability distribution that has been created by each petrophysical realization.

This exercise shows that overall connectivity of sand patches remains as an issue yet and highlights the fact that the facies distribution may be considered as a stronger controlling parameter than the geologically constrained petrophysical variations for this general sand-shale reservoir.
Figure 4.29: The Log-Log diagnostic plot for the real well-test data and the different “petrophysical” realizations for “facies” realization 5.

Figure 4.30: The Log-Log diagnostic plot for the real well-test data and a particular “petrophysical” realizations of “facies” realization 5 (Realization 5-6). The late-time crossing feature is somewhat reproduced.

4.3.6 Facies hybridization

Several facies realizations (that most of them are not documented in here) were generated. However, only two of those realizations (i.e. realizations 4 and 5) showed a closer trend to the real data than the other realizations. The facies realization 5 could closely match the early time while the facies realization 4 could preserve the late time crossing feature of the $\Delta P$ and $\Delta P’$. The hybridization was an engineering idea to take advantage of both models and to wisely combine parts of the spatial facies structures in the realizations 4 and 5 in order to achieve a reliable well-test signature. This is to ensure that the early time signature observed on the well-test response of facies realization 5 (in particular, the early time liner flow and the middle time stabilization) and also the late time signature seen on the test response of the facies realization 4 (the late time crossing feature), are preserved in a combined facies model. The hybridization
algorithm seems functionally similar to gradual deformation method (Roggero and Hu, 1998; Hu, 2000) where two realizations are geostatistically combined in Gaussian space and creates realizations that change smoothly while preserving the global statistical features of the model (Gallo and Ravalec-Dupin, 2000). However, in the hybridization algorithm two engineering approaches were implemented to combine the facies models.

1. Effective Volume Cropping
2. Box Cropping

1. Effective volume cropping: In order to define a proper effective volume around the wellbore in facies realization 5, a single draw-down with an average production rate is simulated. This is to have a more clear view on the pressure diffusion process and to reduce the unnecessary simulation time. With a cumulative production of 6375 m$^3$ during 11.18 day (just before the start of the main build-up period), an average oil rate of 570 m$^3$/day is obtained.

Figure 4.31 shows that the hybridization concept where the merging time of 35 hr is selected based on the derivative signatures. This time is used in defining the effective volume where the pressure drop in any location of the reservoir model is greater than a small value ($\varepsilon$). This is mathematically shown as follows

$$\mathbf{EV} = \Omega(x, t_m) \left| P(x, t=0) - P(x, t_m) > \varepsilon \right.$$ 

in which, $\mathbf{EV}$ is the effective volume and $\Omega(x,t_m)$ is the volume defined by the pressure drop after $t_m=35$ hr of production. In the presence of numerical artefacts, defining a meaningful “$\varepsilon$” may not be easy and needs a trial and error attempt. However, several numerical experiments (under commingled conditions) suggest that selecting different $\varepsilon$’s for the reservoir layers provide adequate results. In this context the volume is defined in such a way that the spatial pressure drop in each layer of the reservoir are greater than the 1 to 5% of the maximum pressure drop at each individual layer-connection to the wellbore. Figure 4.32 (left) shows the 3-D effective volume for the facies realization 5 obtained by this procedure. The figure also highlights the non-homogeneous nature of pressure diffusion in heterogeneous layered reservoirs. Obviously, the layers within the unit 3 (the lower layers of the model) have a higher
Chapter 4: Ramp Effect case Studies

petrophysical quality and therefore the pressure diffusion process is faster in those layers.

![Graph showing pressure vs. time for different facies realizations.](image)

**Figure 4.31:** Selecting the hybridization time of 32 hr. This time is used to define an effective volume within which the geological properties of the facies realization 4 are replaced by those in the facies realization 5.

2. **Box Cropping:** In this approach, a cuboid is defined around the tested well in facies realization 4 which is then replaced by the one cropped from facies realization 5. This cube can be defined by the effective draw-down procedure where the cuboid sides are defined by the maximum I, J, K indices of the affected cells in either sides of the wellbore. Figure 4.32 (right) shows a cuboid obtained by this approach. This volume, which is obtained from facies realization 5, replaces the equivalent portion of the model in facies realization 4.
Figure 4.32: The volume obtained from the Effective Volume Cropping using ~5% of maximum pressure drop in each layer (left) and the Box Cropping using 80 < I < 130 and 45 < J < 110 (right).

Figure 4.33 shows the build-up responses obtained by these two different hybridization approaches. Although both of the approaches preserved the required features of the facies realizations 4 and 5, the box cropping approach still requires a permeability upgrading. This is in contrast to effective volume cropping while gives satisfactory results. The resulting well-test signature not only shows the expected features of the facies realizations 4 and 5 but also shows an important connectivity upgrading. This is because the spatial feature of the realization 5, within the effective volume, has been spatially superimposed on the spatial facies structure of the facies realization 4 and the model gets richer in facies 4 (high permeable facies). This point is clearly illustrated in the Figure 4.34. This figure illustrates the facies hybridization workflow within one layer of the reservoir model (layer 33) and shows that the lateral connectivity has been improved in near wellbore areas. Therefore, the pressure derivative response shifts downward and results in a proper derivative matching. It is important to note that the model obtained by the effective volume cropping is geologically consistent and follows the common facies structures generated by the MPS facies modelling.
Chapter 4: Ramp Effect case Studies

Figure 4.33: The Log-Log diagnostic plot for the real well-test data and those obtained by well-test simulation of models obtained by hybridization algorithm: the Effective Volume Cropping and the Box Cropping approaches. Unlike the Box Cropping that only preserves the overall trends, the Effective Volume Cropping provides a lateral connectivity upgrading as well.

Figure 4.34: The Hybridization algorithm shown on a layer of the reservoir model (layer 33). An effective region of the facies realization 5 replaces that of the facies realization 4.
The facies realization obtained by hybridization algorithm provides the necessary framework for the conditional petrophysical distribution. Ten different petrophysical realizations under two hypotheses categories are simulated using the Sequential Gaussian Simulation. The geostatistical distribution parameters and the probability distribution function of porosity for the either of hypotheses follow the same values as presented in Table 4.4 and Table 4.5. The only difference in here is that, the first five realizations use the cross-plot 1 (Figure 4.35) to estimate the permeability field, while the cross-plot 2 (Figure 4.35) is used for the second set of five. The cross-plot 2 uses a different porosity-permeability relationship for facies 4 (F4: high permeable facies) where its estimated permeability values are twice the predicted values by the cross-plot 1 (Figure 4.23).

![Upscaled Poro-Perm transforms (2)](image)

Figure 4.35: The porosity-permeability cross-plot 2. This cross-plot is used to estimate the permeability distribution of the petrophysical realizations in the hybridization algorithm. F4(old) curve is used for the first five realizations and the F4(new) is used for the next second five.

Figure 4.36 shows the well-test response for all ten petrophysical realizations. As it is expected the derivative curves for the “second” five realizations are somewhat shifted down the real derivative curve and fail to match middle stabilization. The late time behaviour has also been slightly affected; hitherto, the crossing feature has been reproduced in all of the realizations.
Figure 4.36: The Log-Log diagnostic plot for the real well-test data and the well-test simulations of ten petrophysical realizations for the facies model obtained by hybridization algorithm. The first five realizations use the X-plot 1 for the permeability distribution and the second five realizations use the X-plot 2 (with F4 (new)). The former provides a better match for the middle time stabilizations.

Of all the simulated realizations, the realization 1 provides a better well-test match. Figure 4.37 shows the build-up response for this realization.

Figure 4.37: The Log-Log diagnostic plot for the real well-test data and the well-test simulation of a particular petrophysical realization of the hybridization algorithm. Except the early time region, which needs a further improvement, the overall quality of match is good.

Clearly, there is a mismatch in early time derivative stabilization and/or the skin factor which is manifested in the vertical movement of the pressure drop curve. However, this can be adequately overcome by upgrading the permeability of few cells in the immediate vicinity of well. These cells are defined by employing another Effective Volume Cropping after 0.1 hr of the effective production that is around 30 m away from
the well in most permeable layers. The permeability values within this volume are multiplied by a constant value of 1.8 that corresponds to the derivative mismatch. It should be noted that this volume is very sparse along the wellbore and only few cells in the high permeable layers are affected. Figure 4.38 shows the superimposition of the effective volumes captured in high permeable layer 33.

![Figure 4.38: The affected cells in the layer 33 of facies realization 5 obtained by the Effective Volume Cropping. The red coloured cells shows a secondary defined cells (using another Effective Volume Cropping after 0.1 hr of production) whose permeability values are upgraded by a factor of 1.8. This is to match the early time test response of the hybrid model.](image)

Defining a secondary effective volume and upgrading the permeability within that volume can significantly improve the well-test match. Figure 4.39 shows the main build-up response where the pressure drop and the pressure derivative curves are vertically displaced and could match the real test response.

![Figure 4.39: The final match obtained using the hybridization algorithm by upgrading the permeability within a secondary Cropped Effective Volume (final match).](image)
The quality of the match can also be checked against the Semi-Log plot (Figure 4.40) and the history plot (Figure 4.41) as well. These figures show that although there are very small mismatches in the early times, the overall quality of match is very good.

Figure 4.40: The Semi-Log superposition plot for the main build-up of the real well-test data and the well-test simulation of the final hybrid model (final match).

Figure 4.41: The history plot of the real well-test data and the simulated extended well-test response of the final hybrid model. The upper plot shows the full production history and the lower plot is the enlarged view of the early history before the main build-up period (final match).
4.4 Alternative interpretation scenarios

The existence of the no-flow boundaries can affect the ramp effect signature. Besides, the ramp effect can be confused with the fault signature on the well-test response. Therefore, it is crucial to identify the existence of the flow barrier. Without having a fair knowledge of the system structure, it is unlikely that one can make a reasonable well-test interpretation.

The main reservoir structures such as reservoir top and regional faults can be mapped efficiently by 3-D seismic interpretation. These can be used to constrain well-test interpretation (Zheng et al., 2003) where relatively discrete sandbodies are present. However, smaller scale faults, stratigraphic discontinuities and permeability baffles show their effects in the dynamic behaviour of the reservoir and often their average properties can be mapped via 4-D seismic signals (Chapter 6). 4-D seismic data provide strong indicators that direct imaging of the impact of structures on flow will eventually become an essential tool in optimizing production from many complex reservoirs (Jolley et al., 2007). Besides, time-lapse (4-D) seismic data provide the information from which it might be possible to detect directly spatial changes in the reservoir (Gluyas and Warbrick, 2003). The integration workflow with time-lapse seismic data is discussed in the next Chapters.

4.5 Implications of misidentification of the ramp model

Interpretations of well-tests in fluvial reservoir require careful integration between geology, core, well-test and geophysical data (Zheng et al., 2000; de Rooij et al., 2002; Zheng et al., 2003). The depositional setting is critical – fluvial systems are not simple layers but complex micro-cross-flow systems and this gives rise for misidentification of faults and fractures. This point is important as some reservoir internal boundaries (e.g. sub-seismic faults) can create some ramp-like signatures. Sub-seismic faults are those faults with small throw (e.g. 5-10m) that cannot be resolved using the conventional 3-D seismic interpretation with low frequencies (e.g. 30 HZ). These faults can have substantial effect on the dynamic behaviour of the reservoir. Ignoring the existence of these features could mislead the well-test interpretation and can affect the reservoir modelling. One of the interesting examples is the one presented by Stewart (2011) where a minor sub-seismic sealing fault with limited extent is located close to a major long fault. The well-test response of a well between these parallel faults is associated with an
initial infinite acting radial flow regime close to the well, a ½ slope linear flow within the channel which is followed by a later hump and finally a hemi-radial stabilization indicating the radial flow in wider reservoir section (Stewart 2011). Therefore, this well-test response might be mistakenly interpreted as the ramp effect where the lateral mobility variations within the reservoir layers have formed this signature. In contrast to this situation, there are fluvial reservoirs that are described with posited sub-seismic faults that turn out on production to be well-communicating and not faulted at all (Corbett, pers. comm.). The fault interpretations result from the misidentification of channel boundaries and/or interaction as sealing or partially-sealing faults in braided fluvial reservoirs. This contribution is aimed to provide help in sorting out these responses and drawing attention to the existence fault- or fracture- like well-test responses in fluvial reservoirs without faults.

As an example of the application of a linear flow regime and non-linear regression interpretation in a channel refer to an example from Algeria (documented in Elarouci (1994)) where 74 m is derived as the best-fit channel width. The section of the Lower Devonian is interpreted as beach sand bar, estuary or tidal estuary but there are clearly horizontal elements where higher, correlated, net: gross flow units could give rise to the areal, micro-cross-flow described in this chapter. The work described here has considered only single-phase flow simulations. Fluid effects (e.g. in gas condensate reservoirs) are considered separately in Chapter 7.

4.6 Chapter summary
The ramp effect is recognized in three real-life well-test examples in different geological environments under extremely low vertical flow conditions. These case studies range from a high-energy braided stream to low-energy meandering and anastomosing fluvial channel environments. The well-test interpretations are constrained to relatively sparse geological data. The single-model with approximate flow regimes and the more sophisticated equi-probable multi-point statistics models are utilized within a geoengineering workflow to validate the ramp effect.
Chapter 5

Addressing the Non-Uniqueness by Dual-Domain Seismic Imaging and Well-Testing

Although the well-testing and the seismic imaging utilize in different domains with different underlying physics, they are both the responses of the same porous medium and can be employed to improve the knowledge of the earth structure and dynamics. The 3-D seismic data has been traditionally used to constrain the well-test interpretation. However, the recent advances time lapse seismic technology provides a greater opportunity for integration purposes. In this chapter, there is a brief review on the fundamental comparison of the well-testing and seismic principles and finally 4-D seismic response is briefly introduced as an alternate integration approach which could lead toward better understanding of laterally occurring heterogeneities. This is done by tracking the changes in amplitude and time shift of the seismic traces that occur during production/injection. These changes can be linked to engineering domain properties (e.g. pressure) by implementing the right petro-elastic models. The applicability of the time-lapse modelling will also be discussed using a realistic outcrop modelling.

5.1 Reviewing the seismic wave propagation

The seismic wave propagation in porous media is governed by the wave equation that is a hyperbolic partial differential equation (Kreyszig, 2006). Hyperbolic equations are amongst the most challenging to solve because sharp features in their solutions will
Chapter 5: Addressing the Non-Uniqueness by Dual-Domain Seismic Imaging and Well-Testing

Persist and can reflect off boundaries (Shearer, 2009). For an isotropic homogeneous elastic medium the general wave equation may be written as follows (Sheriff and Geldart, 1995)

\[
\rho \frac{\partial^2 u}{\partial t^2} = f + (L + 2M)\nabla(\nabla \cdot u) - M \nabla \times (\nabla \times u)
\]

\[
(5.1)
\]

in which \( u = u(x,y,z,t) \) is the displacement, \( f \) is the source function, \( L \) and \( M \) are the Lame’s parameters that describe the elastic properties of the medium and \( \rho \) is the medium density. The first term in equation (5.1) corresponds to the compressional waves, where the displacement is in direction of the wave propagation, and the second term corresponds to shear waves, where the displacement is perpendicular to the wave propagation direction and in this case the propagation velocity is almost two times slower than the compressional wave. Equation (5.1) shows that the wave propagation is a function of the media properties which are reflected in density and Lame’s parameters of the medium.

The seismic waves are created by some energy sources (commonly considered as the point source) and are propagated by spherical fronts in the 3-D isotropic and homogeneous media. The velocity of the propagation is generally very fast and depends on the elastic properties of the media. The velocity can be expressed in terms of the p and s waves (\( v_p \) and \( v_s \)) and are the function of the elastic properties of the media (\( L \) and \( M \))

\[
v_p = \sqrt{\frac{L + 2M}{\rho}}
\]

\[
(5.2)
\]

\[
v_s = \sqrt{\frac{M}{\rho}}
\]

\[
(5.3)
\]

Moreover, the wave-front radius can be obtained as a linear function of time

\[
r = v_p t = \sqrt{\frac{L + 2M}{\rho}} t
\]

\[
(5.4)
\]
Figure 5.1, from Roth et al. (1998), shows the propagation of seismic waves in a layered reservoir at different times. The waves are spreading spherically and are reflected, and transmitted after reaching the boundary with different acoustic impedance, “AI” or “Z” (i.e. density×velocity).

When the acoustic wave hits a boundary with different acoustic impedances a part of the energy is transmitted and another part is reflected. In a normal incidence condition (Figure 5.2) where the ray path is perpendicular to the interface (i.e. the source and receiver are at the same position on the surface) the normalized reflection and transmission energy coefficients ($E_R$ and $E_T$) are defined as follows (Liner, 2004)

$$E_R = R_{0i}^2$$  \hspace{1cm} (5.6)
\[
E_r = \frac{Z_2}{Z_1} T_0^2
\]

where, the \(Z_2\) and \(Z_1\) are the acoustic impedances of medium 2 and 1, and \(R_0\) and \(T_0\) are the normal-incidence reflection and transmission coefficients which are defined as follows

\[
R_0 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{(\rho v_p)_2 - (\rho v_p)_1}{(\rho v_p)_2 + (\rho v_p)_1}
\]

\[
T_0 = \frac{2Z_1}{Z_2 + Z_1} = \frac{2(\rho v_p)_1}{(\rho v_p)_2 + (\rho v_p)_1}
\]

These equations (5.6 through 5.9) state that, if \(Z_1 = Z_2\) all the energy is transmitted through the interface or if the \(Z_2 \gg Z_1\) (i.e. very hard medium) then the all of the energy is reflected back from the surface.

After reflection, the heterogeneity information is transferred back to any receiver at the surface that will be processed for interpretation in terms of seismic traces. Besides the effect of different noise (e.g. surface waves) and the conventional geophone (receiver) resolution the interpretable seismic information is limited by the lateral and vertical resolution of the seismic wave. The lateral resolution is the ability to see the fine-scale features on the map or cross-section (Liner, 2004). This means that the seismic wave is not able to distinguish two separate objects that are closely positioned near each other. The first Fresnel zone defines an area from which the actual seismic information is
bounced back and therefore defines the lateral resolution of the seismic data (Figure 5.3). The Fresnel zone is a function of frequency and the depth of the object beneath the earth and is defined as follows (Badley, 1985)

\[ r_f = \frac{v}{2} \sqrt{\frac{T}{f}} \]  \hspace{1cm} (5.10)

in which, \( r_f \) is the first Fresnel zone in “m”, \( v \) is the average velocity of the media in “m/s”, \( T \) is the two-way travel time in “second” and \( f \) is the dominant frequency in “Hz” which is commonly between 8 to 80 Hz (Ferguson and Sen, 2004).

Although, equation (5.10) suggests using the higher seismic frequency reduces the Fresnel zone and can increase the lateral detectability of the seismic data. However, the high frequencies get absorbed in the earth and their penetration depth is limited (Veeken, 2007).

\[ h \]

\[ h + \frac{\lambda}{4} \]

\[ \frac{\lambda}{4} \]

\[ \text{First Fresnel zone} \]

\[ \lambda \]

\[ \text{reflection information comes from an area rather than a point. } \lambda \text{ is the wavelength.} \]

The vertical resolution on the other hand, is the ability to identify the individual peaks on the seismic trace with the top and base of a geologic unit (Liner, 2004). The clear pick is obtained where the unit thickness is greater than a particular value which is called the Tuning Thickness and is traditionally defined as

\[ z_{\text{tun}} = \frac{\lambda}{4} \]

\hspace{1cm} (5.11)
in which, $\lambda$ is the dominant wavelength of the seismic wave. Equation (5.11) states that the seismic wave is not able to distinguish the layers that have a thickness smaller than the tuning thickness.

Another important property of the seismic wave is that the wave energy and amplitude get weaker with distance of propagation. Apart from the resistant effect of the media, there are some other factors that result in the wave energy to reduce. For an impulsive point source in 3-D, as the wave field evolves the energy density and the amplitude decrease by $1/r^2$ and $1/r$ respectively (Liner, 2004; Veeken, 2007). The geometrical spreading along with the augmented effect of other mentioned and unmentioned phenomena with depth (e.g. the increase in medium velocity, the attenuation of higher frequencies, the subsequent increase of the dominant wavelength, and reduction of the contrast in acoustic impedance ($\rho \times v$) between the strata (Brown, 2004; Veeken, 2007)) result in an overall reduction of the seismic resolution.

5.2 Reviewing the pressure diffusion process in well-testing

The movement of the pressure disturbance front, on the other hand is governed by the diffusivity equation which is a parabolic differential equation (Kreyszig, 2006). For a single phase and slightly compressible fluid in an isotropic homogenous media the diffusivity equation can be written as follows

\[
\nabla^2 p = \frac{1}{\eta} \frac{\partial p}{\partial t} \tag{5.12}
\]

in which, $p$ is the fluid pressure, and $\eta = k/(\phi \mu c_t)$ is the hydraulic diffusivity. All parameters are in Darcy unit. Equation (5.12) states that the pressure diffusion process is function of the medium properties which are reflected in hydraulic diffusivity coefficient.

Although it is not physically exact, for the mere sake of simplicity and analogy with seismic wave propagation phenomenon, the diffusion process can be assumed as an expansion of the pressure head front (i.e. diffusion front). The pressure transients are usually created at the wellbore whenever any changes (e.g. production or injection) happen. As an analogy to the seismic waves, the pressure disturbance front is assumed to be reflected back whenever they reach any reservoir flow barriers (e.g. variation in the hydraulic diffusivity coefficient). This information will affect the measured wellbore
pressure and are translated to well-test diagnostic plots for interpretation. Figure 5.4 shows a set of situations where the mobility \((k/\mu)\) changes across a boundary near the wellbore which might result in a reflection or transmission of the pressure disturbance. If the mobility ratio (of the first region to the second region) is unity pure transmission occurs, while an infinite mobility ratio is equivalent to a pure reflection. The ratio of the storage coefficients of the regions, which is defined as \((\phi c h_t)_1/(\phi c h_t)_2\), is kept invariant (and equal to unity) amongst the considered cases. The storage coefficient is the amount of fluid released from a reservoir volume of height “h” per unit base area per unit change in pressure.

![Figure 5.4](image.png)

**Figure 5.4:** The contour maps of diffusion front in a porous medium in a specific time with a linear spatial discontinuity in mobility. The disturbance is, 1: completely transmitted without any variation in propagation speed where the mobility ratio is unity (top: left), 2: transmitted and reflected with considerable variation in propagation speed (top: right and bottom: left) and 3: completely reflected where the mobility of the second region is zero (bottom: right).

Although, there are different forms of pressure investigation geometries, the most common cases (i.e. fully penetrated vertical wells in an isotropic homogeneous
reservoir), the diffusion front moves cylindrically away from the wellbore (e.g. Figure 5.4). The radius of investigation is a measure of the distance that pressure disturbance has moved into the reservoir (Bourdet, 2002; Stewart, 2011). For a homogeneous and isotropic formation, the radius of investigation is defined as follows (Lee, 1982; Bourdet, 2002)

$$ r_i = 0.032 \sqrt{\frac{kt}{\phi \mu c_i}} $$  \hspace{1cm} (5.13) 

In which, \( k \) is permeability in “md”, \( \phi \) is porosity, \( \mu \) is viscosity in “cp”, \( c_i \) is the isothermal compressibility factor in “1/psi” and \( t \) is time in “hours”. The propagation speed of the diffusion front depends on the investigated medium properties. An expression for the speed of pressure disturbance might be obtained by differentiation of the radius of investigation (i.e. the leading edge of pressure head fronts) with respect to time

$$ v_d = \frac{dr}{dt} = \frac{1}{2} \sqrt{\frac{k}{948 \phi \mu c_i t}} $$  \hspace{1cm} (5.14) 

in which \( v_d \) is the velocity with which the pressure disturbance moves. The speed of the pressure disturbance front at the time greater than zero is slow, compared to the seismic wave propagation, and as it is indicated in equation (5.14), it only depends on the hydraulic diffusivity coefficient of the media.

It should be noted that the amplitude of pressure disturbance gets weaker with distance by \( 1/r^2 \) (compared with \( 1/r \) for the amplitude reduction of the acoustic wave propagation with distance) and eventually disappears in far distances. This is obtained by the impulse response of an ideal reservoir (isotropic, radial homogeneous, constant porosity and compressibility with finite thickness) by injecting some amount of fluid, \( q_{inj} \), in a very short time, \( \Delta t \rightarrow 0 \), so as to have \( q \times B \times \Delta t = 1 \) RB (reservoir barrel), and then to monitor the maximum pressure drop at a distance “\( r \)” away from the wellbore. In this way the arrival time of the impulse at any location within the reservoir is analyzed. The
pressure drop due to unit impulse can be written as follows (Lee, 1982; Sabet, 1991; Kuchuk, 2009)

\[
\Delta P(r,t) = \frac{-1694.4\mu}{kht} e^{-\frac{948\mu c_r r^2}{kt}}
\] (5.15)

Analysis of the impulse response indicates that the pressure disturbance created by the impulse has a diffusive nature. This means that the impulse response consists of different pressure components and some of these components travel faster than the others (Sabet, 1991). Differentiating the equation (5.15) and equating to zero gives the maximum pressure drop that occurs at \( t_{\text{max}} \)

\[
t_{\text{max}} = \frac{-948\mu c_r r^2}{k}
\] (5.16)

which leads to the following equation

\[
\Delta p_{\text{max}}(r,t_{\text{max}}) = \frac{1.787}{h\phi c} \frac{1}{r^2} \propto \frac{1}{r^2}
\] (5.17)

An immediate conclusion of such geometrical spreading is that although the pressure drop can affect very far distances; however, only the pressure drops that are greater than a threshold (i.e. gauge resolution) can be recognized. Therefore, the gauge resolution plays an important role on reflecting the heterogeneity effect on the well-test response. It should be noted that although the investigation radius does not seem to be directly dependent on the production (or injection) rate; however, the stronger the rate the stronger the pressure drop signal. This causes the far distance areas to have a pressure drop signal greater than the gauge resolution and to be detected. Furthermore, the size, contrast and the relative position of the heterogeneities with respect to each other can also affect the well-test response. In the case of a symmetrical linear sealing fault, for example, the numerical results showed that if the fault length is less than \( \frac{1}{2} \) of the well distance from the well to the fault plane, the pressure derivative will not reveals a sensible effect on the drawdown curve. In this case, however, if the fault position is -
### Table 5.1: Fundamental comparison between seismic wave propagation and pressure diffusion physics.

<table>
<thead>
<tr>
<th>Seismic wave</th>
<th>Well-test pressure disturbance front</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Governing Differential Equation</strong></td>
<td><strong>Parabolic differential equation</strong></td>
</tr>
<tr>
<td>Acoustic energy transmission by particle motions</td>
<td>Pressure drop diffusion</td>
</tr>
<tr>
<td>Obtained from,</td>
<td>Obtained from,</td>
</tr>
<tr>
<td>1. Newton’s law</td>
<td>1. Darcy law</td>
</tr>
<tr>
<td>2. Hooke’s law</td>
<td>2. Continuity equation</td>
</tr>
<tr>
<td><strong>Constitutional Laws</strong></td>
<td></td>
</tr>
<tr>
<td>Acoustic energy transmission by particle motions</td>
<td></td>
</tr>
<tr>
<td>Obtained from,</td>
<td></td>
</tr>
<tr>
<td>1. Newton’s law</td>
<td></td>
</tr>
<tr>
<td>2. Hooke’s law</td>
<td></td>
</tr>
<tr>
<td><strong>Propagation in homogeneous and isotropic media</strong></td>
<td><strong>Pressure drop diffusion</strong></td>
</tr>
<tr>
<td>Spherical propagation (3-D-body waves)</td>
<td>Obtained from,</td>
</tr>
<tr>
<td>Radial propagation (2-D-surface waves)</td>
<td>1. Darcy law</td>
</tr>
<tr>
<td></td>
<td>2. Continuity equation</td>
</tr>
<tr>
<td></td>
<td>3. Equation of states</td>
</tr>
<tr>
<td><strong>Investigated parameters</strong></td>
<td><strong>Investigated parameters</strong></td>
</tr>
<tr>
<td>Fluids and rock types parameters L, M, ρ</td>
<td>Fluids and rock types k, q, μ (viscosity). C_s</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Radius of investigation in ideal media</strong></td>
<td></td>
</tr>
<tr>
<td>$r_1 = v_s \times t = \sqrt{\frac{L + 2M}{\rho}}$</td>
<td></td>
</tr>
<tr>
<td>$r_2 = v_s \times t = \sqrt{\frac{M}{\rho}}$</td>
<td></td>
</tr>
<tr>
<td><strong>Velocity of propagation</strong></td>
<td></td>
</tr>
<tr>
<td>High velocity ~ 2000m/s</td>
<td>Low velocity</td>
</tr>
<tr>
<td>1. Body waves (P and S waves)</td>
<td></td>
</tr>
<tr>
<td>2. Surface wave</td>
<td></td>
</tr>
<tr>
<td><strong>Processing</strong></td>
<td></td>
</tr>
<tr>
<td>High processing for data preparation</td>
<td>Lower processing for data preparation</td>
</tr>
<tr>
<td><strong>Resolution</strong></td>
<td></td>
</tr>
<tr>
<td>$R_l = \frac{v}{2\sqrt{L}}$</td>
<td></td>
</tr>
<tr>
<td>$R_s = \frac{\lambda}{4}$</td>
<td></td>
</tr>
<tr>
<td><strong>Geometrical spreading</strong></td>
<td></td>
</tr>
<tr>
<td>Amplitude decrease with $1/r$ for the common case of spherical expansion</td>
<td>Amplitude decrease with $1/r^2$ for the common case of cylindrical diffusion</td>
</tr>
<tr>
<td><strong>Wave front propagation</strong></td>
<td></td>
</tr>
<tr>
<td>Reflection, transmission, diffraction</td>
<td>Analogy to reflection and transmission (superposition)</td>
</tr>
<tr>
<td><strong>Factors affecting the interpretation</strong></td>
<td></td>
</tr>
<tr>
<td>1. Tuning effect</td>
<td></td>
</tr>
<tr>
<td>2. Geometric spreading</td>
<td></td>
</tr>
<tr>
<td>3. Loss of high frequencies during propagation</td>
<td></td>
</tr>
<tr>
<td>4. Absorption and scattering</td>
<td></td>
</tr>
<tr>
<td>5. Noise from different sources</td>
<td></td>
</tr>
<tr>
<td>6. Time-depth conversion (velocity interpretation)</td>
<td></td>
</tr>
<tr>
<td>7. Effect of Ghost or “multipliers”</td>
<td></td>
</tr>
<tr>
<td>8. Dominant Frequency of acquisition</td>
<td></td>
</tr>
<tr>
<td>9. Acquisition geometries</td>
<td></td>
</tr>
<tr>
<td>10. Complex processing that may create artifacts (migrations)</td>
<td></td>
</tr>
<tr>
<td><strong>Anisotropic and heterogeneous media</strong></td>
<td></td>
</tr>
<tr>
<td>$= f $ (stiffness matrix, etc.)</td>
<td></td>
</tr>
<tr>
<td>1. Layering/layering induced anisotropy: VTI (Upadhyay, 2004)</td>
<td></td>
</tr>
<tr>
<td>3. In-situ stress</td>
<td></td>
</tr>
<tr>
<td>4. Orientation of microscopic mineral particle(especially in shale~ velocity</td>
<td></td>
</tr>
<tr>
<td>anisotropy)</td>
<td></td>
</tr>
<tr>
<td>5. etc.</td>
<td></td>
</tr>
<tr>
<td><strong>Domain conversion</strong></td>
<td></td>
</tr>
<tr>
<td>Time to depth conversion = f(medium velocity)</td>
<td>Time to distance conversion = f(medium hydraulic diffusivity)</td>
</tr>
</tbody>
</table>

126
changed by a horizontal translation to have an asymmetric positioning respect to the wellbore, the derivative response can show a weaker response (lower $\Delta P'$). Another example of the relative positioning effect on the well-test response would be that of a ramp effect where increasing of the correlation length, to create some patches of facies with correlated permeabilities, may create a perfect ramp effect with two derivative stabilizations.

5.3 Heterogeneity information from seismic and well-test interpretation

The pressure diffusion and the seismic wave equations have different physics behind them. For example, Table 5.1 summarizes the fundamental comparison between the seismic imaging and the well-test techniques and illustrates few main (amongst many) differences between these two reservoir illuminating techniques.

The pressure diffusion and the seismic wave propagation are affected by “different” properties of the “same” medium where the relationships between these multi-domain properties are not thoroughly known. The well-test and seismic signals provide a record of the medium’s response in different ways. The conventional seismic surveys provide with a 2-D or a 3-D images that use the energy transfer within the medium and offer an approximate structural framework of the reservoir.

Figure 5.5, from Brown (2004) present the applicability of the 3-D seismic imaging. Obviously, whenever the dynamic information is required (e.g. connectivity), the 3-D seismic data fails to provide a full support. On the other hand, the well-testing uses the mass transfer principles and convey the volumetric (or average) information about the investigated reservoir heterogeneities. Therefore, the majority of the integration workflows follow a jigsaw puzzle approach in a sense that the main structural discontinuities are taken from the 3-D seismic interpretations and the well-test is commonly used to fill in the structure with average hydraulic diffusivity values.
Slivensky (1990) introduced a novel but limited ray-tracing approach for structural integration of seismic and well-test data. In this approach any deviation from the radial homogeneous behaviour in the well-test derivative curve is attributed to the existence of some linear boundaries. Each pressure derivative point, which corresponds to an investigation radius, is converted to an arc with a specific circumstance. Therefore, numerous objects (i.e. arcs with different radii) are defined. On the other hand, the seismic provides with the structural map including the main recognized faults to constrain the workflow. The obtained objects are superimposed on this interpreted fault map and are rotated in order that one of their ends conform with the main faults while the other ends supply another sets of the boundaries that are linked to sub-seismic faults. This is shown in Figure 5.6 where the deviation from the homogenous behaviour shown by red arrows (Figure 5.6: left) has been translated to some arcs (right) where one end of the arcs shown in red are fixed on the S1 fault and the other ends, highlighted in green, show the other boundaries. Obviously this method may work whenever the reservoir properties are homogeneous.
Ayestaran et al. (1989) used the high resolution Vertical Seismic Profiling (VSP) to detect and measure the no-flow boundaries near the wellbore. They have then estimated the anisotropic horizontal permeabilities ($k_x$ and $k_y$) from the estimated well-test permeability and distances to the boundaries. Guerillot and Beydoun (1993) tried to use the synthetic seismic images and well-test data in order to cross validate porosity, permeability values and to enhance volume calculation. Sahni et al. (2007) used a seismic attribute map in meandering fluvial reservoir to match the well-test data by modifying the clay-plug channel permeability and improving the connectivity of the reservoir sands. The integration interface has also been expanded in the fracture reservoirs where Cardona et al. (2002) defines a relation between fracture compliance and the storativity coefficient of the fractured system. Cacas et al. (2001) used the well-testing as a dynamic validation tool to validate the fracture properties of a discrete fracture model. In their approach the seismic attributes (coherency, fault map, curvature analysis, shear wave polarization and etc) are used to obtain the fracture set, fracture orientation and fracture density to characterise fracture network.

Although all of these integration methods are very interesting, they are all limited to static detectability of the structural objects. However, there are many situations where
these objects are not revealed by the static seismic interpretation. For example, the permeability baffles or strike slip faults are commonly difficult to pick by 3-D seismic data. This is while the frequent time-lapse (or 4-D) seismic data may provide very useful lateral information about the dynamic behaviour of the system by tracking the pressure and saturation fronts and capture these phenomena. This integrated approach will be discussed in more detail in following chapters. The next introduces the time lapse seismic modelling through a turbiditic channel reservoir example. The synthetic time lapse modelling is frequently used in the other next Chapters where seismic data and well-test are jointly modelled and interpreted in presences of complex fluid and geological heterogeneities.

5.4 Time lapse (4-D) seismic technology

The 4-D seismic technology was introduced in 1980’s and commercialized in late 1990’s (Amudsen and Landro, 2007). The idea of the 4-D seismic survey is simple where several seismic surveys are taken over different times within the life of the reservoir. Each seismic survey can be considered as an image of the reservoir at a specific time during the production. The first survey is called the “Base” survey and the other surveys are the “Monitor” surveys. The difference between the base and the monitor is called time lapse or 4-D response and can be interpreted in 2-D or 3-D and in any seismic related property domain. For example, the impedance changes, the amplitude changes or the velocity changes can be reported as a measure of 4-D signal. These changes are related to some engineering interest parameters like spatial saturation and the pressure changes. In particular, the saturation and pressure changes during reservoir history affect the elastic properties of the saturated rock which is directly reflected in the seismic traces. Any changes in the amplitude or the travel time of the seismic wave can then be interpreted as a dynamic change in the reservoir. The rock physics (or petro-elastic) modelling is a key that link from the geophysical to engineering domain.

Nowadays, the time lapse seismic interpretation is a proven technology used in reservoir evaluation (e.g. boundary detection), reservoir monitoring and management (e.g. water flooding) and the reservoir developments strategies (e.g. new infill well locations) and the economic influence of 4-D seismic technology, in the North Sea in particular, has been noticeable (Amudsen and Landro, 2007).
Rock physics (petro-elastic) models describe how reservoir parameters relate to elastic properties. This enables us to predict and interpret the seismic effects related to lithology and fluid (Rock Physics Associates, 2007). The petro-elastic models can be used for 4-D feasibility study and also for the pressure and saturation inversion from the 4-D signatures and therefore is a strong tool to bridge between the Engineering and the Geophysics domains. In this chapter, the usefulness of the 4-D seismic data and the petro-elastic modelling has been demonstrated by the synthetic seismic modelling of a real outcrop, Ainsa II channelised reservoir (Pyrenees, Spain).

5.5 Synthetic 3-D and 4-D seismic modelling through an example: Ainsa II Channelised Reservoir

The Ainsa II turbidite sandbodies contains sediments that originate from fluvial and shallow marine systems located to the south east of the Ainsa Basin (Mutti, 1985; Mutti et al., 1988), Northern Spain. The Ainsa II turbidite system has been interpreted as laterally stacked channel systems bounded by erosional surfaces (Mutti et al., 1985; Clark, 1995). The Ainsa II turbidite system has an average thickness of about 40–50 m and contains at least five clastic sandbodies bounded by erosional surfaces (Bakke et al., 2007). Figure 5.7, from Clark et al.(2008), shows an outcrop section of the Ainsa II and the interpreted sandbodies. This outcrop is used in object based modelling of this reservoir.

5.5.1 Modelling and simulation set up

The object based technique was used for modelling of this channelised reservoir. This was based on the outcrop exposed in North of Ainsa. The 3-D model was built based on the longitudinal extrapolation of a lateral cross-section of the reservoir exposed on the outcrop over a background of non-reservoir rocks (Figure 5.8). The detailed modelling procedure is documented in Barreto (2008). The upshot of the modelling was a fine scale model of 25 million cells with a cell size dimension of 5m x 5m x 0.5m. The fine scale model was eventually upscaled to have 76×150×35 cells in x, y and z directions with a total number of 399000 uniform course cells (Figure 5.9). The deterministic rock properties of the facies in the upscaled model are shown in Table 5.2.
Figure 5.7: Outcrop panel (upper) and interpretation (lower) of turbidite sandstones near Ainsa in Northern Spain (Clark et al., 2008).

Figure 5.8: Outcrop modelling of Ainsa II. The longitudinal extrapolation of a lateral cross-section of the reservoir exposed on the outcrop over a background of non-reservoir rocks (Barreto, 2008).
Chapter 5: Addressing the Non-Uniqueness by Dual-Domain Seismic Imaging and Well-Testing

Figure 5.9: Ainsa II fine Cartesian geo-model has been upscaled into coarser grids for flow simulation purposes (Barreto, 2008).

Table 5.2: The deterministic rock properties of the Ainsa II model.

<table>
<thead>
<tr>
<th>Facies</th>
<th>$k_x$ (md)</th>
<th>$k_y$ (md)</th>
<th>$k_z$ (md)</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>0.3</td>
</tr>
<tr>
<td>Debris flow</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>0.15</td>
</tr>
<tr>
<td>Shale</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

A sector of the upscaled model that contains 74×63×35 cells in x, y and z directions was selected. A well was placed near the centre of the reservoir within the (purple) debris flow channel (Figure 5.10) and was allowed to flow over the debris flow channel thickness (i.e. 28 m from 2400m up to 2428m). A commercial black oil reservoir simulator (E100) was then used to simulate a single drawdown of 300m$^3$/day for around 700 hrs. In order to avoid the multiphase flow complexity the initial pressure of the reservoir was set far above the bubble point pressure and to be 185 bars measured at datum depth of 2400m.

Figure 5.10: The stacking channels of Ainsa II reservoir. The top purple channel is the low permeable debris flow channel.
5.5.2 Petro-Elastic Modelling (PEM)

The rock physics models relate the engineering domain to the geophysics domain where the pressure and saturation changes are linked to the elastic properties of the media. The petro-elastic modelling is mainly based on the widely used low-frequency Gassmann’s fluid substitution equation (Gassmann, 1951) that predicts the bulk and shear moduli of the saturated rocks. These moduli are used to define the $v_p$ and $v_s$ of the saturated rock and are eventually used in calculation of the acoustic impedance values. The Gassmann’s equation is based on the elastic properties of the fluids (Batzle and Wang, 1992), grain minerals and dry frame moduli (MacBeth, 2004). The dry frame moduli are a function of some stress sensitivity parameters (e.g. $K_\infty$, $P_k$, $S_k$, $\mu_\infty$, $P_\mu$, $S_\mu$) that are defined in MacBeth (2004). The petro-elastic modelling with underlaying equations and assumptions are discussed in great detail in Appendix B.

The above mentioned sets of equations are implemented to create the petro-elastic model of the Ainsa II reservoir model and predict the 4-D signature of the model. The required input parameters of the facies for PEM are listed in Table 5.3. Due to limited measurement data and for the sake of simplicity the same sets of stress sensitivity parameters were used for all facies. It should be noted that from the practical point of view, some of the rock minerals like the background facies take the same elastic parameters as the shale with the constant compression velocity of 3350 m/s and the shear velocity of 1525 m/s. At the same time, the reservoir simulation deals with background cells as inactive cells and are not considered as simulation flow cells.

The PVT properties of the fluid were borrowed from an analogue reservoir in the North Sea area with an oil API gravity of 38, the gas specific gravity of 0.64 and the water salinity of 250000 PPM. As the Gassmann’s equation assumes a single fluid phase, the Reuss’s averaging (Reuss, 1929), which is a saturation weighted harmonic average, method was used to estimate the effective bulk modulus of the fluid. Furthermore, the reservoir temperature required in the petro-elastic calculations was kept at $T = 107^\circ$C, the Biot’s coefficient (Biot and Willis, 1957) was selected to be unity and the overburden gradient of 1psi/ft was applied to the model.
Table 5.3: The required mineral input parameters for the PEM.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Sands</th>
<th>Debris flow</th>
<th>shale</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_m$ (GPa)</td>
<td>36.8</td>
<td>36.8</td>
<td>36.8</td>
</tr>
<tr>
<td>$\rho_m$ (Kg/m$^3$)</td>
<td>2270</td>
<td>2300</td>
<td>2360</td>
</tr>
<tr>
<td>$K_\infty$ (GPa)</td>
<td>12.85</td>
<td>12.85</td>
<td>12.85</td>
</tr>
<tr>
<td>$P_k$ (Mpa)</td>
<td>5.62</td>
<td>5.62</td>
<td>5.62</td>
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<tr>
<td>$S_k$</td>
<td>11.06</td>
<td>11.06</td>
<td>11.06</td>
</tr>
<tr>
<td>$\mu_\infty$ (GPa)</td>
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<tr>
<td>$P_\mu$ (Mpa)</td>
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<tr>
<td>$S_\mu$</td>
<td>1.08</td>
<td>1.08</td>
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</tr>
</tbody>
</table>

The input fluid and rock mineral parameters along with the grid structure and the simulations results (e.g. pressure, saturation, solution gas ratio, and formation volume factor at each time step) were taken to the Gassmann’s equation to predict the elastic moduli, velocities ($v_p$ and $v_s$), and the acoustic impedance of each grid cell in the simulation model. A Heriot-Watt University in-house modelling software, Sim2Seis (ETLP, 2009), was used to generated the petro-elastic models at each required time step. Figure 5.11 shows the simulation model along with the pressure changes and impedance changes (both measured from initial time) after some time of the production. The existence of a high permeable streak near the middle of the reservoir thickness causes the diffusion front to have a larger depth of investigation in those areas. Obviously, with increasing the production time the pressure- (and subsequently the impedance-) affected volume expends out.

A careful look at the Figure 5.11 also reveals that there is a positive correlation between the impedance and the pressure changes. This is because all of the changes in impedance are mainly due to changes of the pressure. This reservoir is therefore categorized as a “pressure dominant” reservoir in the 4-D concepts as all of the changes in impedance are controlled by the spatial pressure changes of the reservoir. Figure 5.12 shows the averaged impedance and permeability profiles within the investigated volume at different time steps of this example. The arithmetic mean is used for the permeability averaging while the elastic moduli averages are based on Voigt-Reuss-Hill (Hill, 1963) estimation which is the mean of harmonic and arithmetic averages. In this context, the impedance is not directly averaged while the mean values of the density and moduli of
the saturated rock are used to compute the average compressional velocity within the investigated volume. The P-impedance can then be calculated by multiplication of the average density and the compressional velocity. Figure 5.12 show that there is a clear negative correlation between the impedance and the permeability of the investigation region.

Figure 5.11: A visual comparison of the affected pressure region and affected P-impedance region at a time step of flow simulation and petro-elastic model.

Figure 5.12: The arithmetic mean of permeability and Voigt-Reuss-Hill mean of the P-Impedance within the investigated regions at different time step.
5.5.3 Synthetic seismic modelling

The outcome of the petro-elastic model can be used to create a set of seismic images. There are several methods developed to create a synthetic seismic cube from the impedance data (e.g. 1-D convolution (Sen, 2006; Amini et al., 2011), Ray tracing (Chander, 1977; Zhang et al., 2011) and finite differencing (Ilan et al., 1975; Hall and Wang, 2009)). Of those methods, the 1-D convolution was chosen to create the synthetic seismograms. This is the simplest and the most objective (Margrave and Manning, 2004) modelling approach through which the seismic traces are generated by convolving an appropriate wavelet of having a specified frequency (e.g. 30 HZ) with the reflectivity coefficient series. The 1-D convolution modelling approach assumes a normal wave incidence configuration, which means that the source and the receiver are located at the same location on the earth. In spite of the simplicity, Amini et al. (2011) showed that the 1-D convolution can be furthered modified to include a resolution function to produce a seismic image with high degree of conformance to the image produced by the costly finite differencing approach. Figure 5.13 is a cartoon showing the 1-D convolution approach where a wavelet is convolved with the reflectivity series and the outcome would be a seismic trace.

![Figure 5.13: A seismic trace is the result of the convolution (*) of a wavelet with the reflectivity series.](image)

**Time lapse seismic response**

The 1-D convolution method was implemented to create a set of synthetic 3-D seismic cubes at various drawdown times. Two similar wavelets of different frequencies were used in the convolution operation: the first one was a 30 Hz Ricker wavelet which is of the order of the surface seismic frequency, and is typically used for imaging of deep reservoir targets and the other one was a 90 Hz Ricker wavelet which is of the order of Vertical Seismic Profiling (VSP) frequency, and is typically used for imaging of higher level reservoir targets. Figure 5.14 is a cross section of the generated 3-D seismic cube.
Chapter 5: Addressing the Non-Uniqueness by Dual-Domain Seismic Imaging and Well-Testing

corresponding to cross-line 30 of Ainsa II sector model. The overall structure of the model can be followed by tracing the blue (trough) and the red (peak) lines.

![Synthetic seismic profile](image)

**Figure 5.14:** Synthetic seismic profile of Ainsa II (cross line30) obtained by convolving a wavelet having a frequency of 30 Hz. Two horizontal lines drawn on the cross-section are structure interpretation of the section.

Increasing the wavelet frequency leads the synthetic seismogram to show higher resolution and illuminates the finer scale structures. Despite that, the continuity of coherent traces somewhat reduces. Figure 5.15 is a cross section of the generated 3-D seismic cube corresponding to in-line 6 of Ainsa II sector model.

![Synthetic seismic profile](image)

**Figure 5.15:** Synthetic seismic profile of Ainsa II (in-line6) obtained by convolving a wavelet having a frequency of 90 Hz. The higher frequency used causes the higher resolution of the section.

The 4-D seismic analysis is normally based on the 2-D interpretations. The Root Mean Square (RMS) average is the common averaging technique that creates the 2-D attribute maps from the 3-D seismic data. To do so, the top and bottom reservoir horizons are
defined as an averaging window. The RMS operator averages out the amplitude of the traces within the confined window and the obtained values are then allocated to the areal position of the parent traces. In Ainsa II example, the averaging window is the amplitude region confined by the two-way travel times (TWT) of 20 ms and 60 ms. This is to ensure that the whole reservoir volume is covered by the averaging method. The RMSΔamplitude map for a 4-D seismic signature is defined as follows

\[
RMS\Delta amplitude = \frac{RMS(Initial \ amplitudvolume) - RMS(Final \ amplitudevolume)}{Current \ amplitudvolume}
\]  

(5.18)

It should be noted that sometimes the 4-D amplitude map is defined as \(RMS(Initial \ amplitudvolume) - RMS(Final \ amplitudvolume)\). However, when the time difference between the base and monitor is short, which is the case for the studied model, the amount of time shift is negligible and both definitions will provide almost the same result. Moreover, the 4-D map created by equation (5.18) will be always positive. However, this will not create any issue as we are considering only the one way pressure changes where the spatial reservoir pressure constantly decreases. As it is shown in Figure 5.16, at the early production times the pressure has not yet diffuse out far away from the well and the RMSΔamplitude shows the disturbance in P-impedance only in near well bore area. With increasing the production time, the pressure permeates deeper and leads to wider impedance changes. Eventually, at late time, when the diffusion front affected all of the external reservoir boundaries, the strongest geological heterogeneities (channel outlines, in this case) are illuminated. This structure can be used in numerical well-testing as an external source of the information which can limit the interpretation and help decrease the uncertainty.
Chapter 5: Addressing the Non-Uniqueness by Dual-Domain Seismic Imaging and Well-Testing

5.6 Chapter summary

The basic theory of seismic wave propagation and pressure diffusion process are reviewed and a fundamental comparison between these two domains is made. This helps understand how these two processes investigate the porous medium. The seismic data provide some useful information that can be integrated with the well-test data to reduce the non-uniqueness of well-test modelling. The integration workflow is conventionally based on the 3-D seismic data. However, the time lapse seismic data are introduced as another source of information that can elucidate the effect of lateral reservoir heterogeneities. The 4-D seismic information is linked to the reservoir engineering concepts through rock physics models. The overall integration process is illustrated through a real outcrop model example in a channelised environment.

Figure 5.16: RMS amplitude of volume generated with wavelet frequency of 90Hz at different time steps of numerical simulation.
Interpretation of 3-D seismic data has traditionally been used in reservoir modelling to pick the reservoir top structure and the major faults detection. Well-testing, on the other hand, been mainly used to illuminate the well/reservoir dynamic properties and the reservoir structures. Similar to seismic history matching (Stephen et al., 2005; Stephen and Macbeth, 2006; Edris et al., 2008; Villegas et al., 2009; Arwini and Stephen, 2011; Kazemi and Stephen, 2011) more benefits can be gained by integrating the time-lapse (or 4-D) seismic data and the well-testing both of which monitor the dynamic behaviour of the reservoir. The 4-D seismic images provide the snapshots of the reservoir at different time within the reservoir life and provide the spatial information about the fluid and reservoir heterogeneities. This will impose a new constraint to the well-test modelling and interpretation. The implementation of the time-lapse data in the reservoir history matching is well documented (Huang et al., 1997; Waggoner et al., 2002; Gosselin et al., 2003; Fahimuddin et al., 2010; Arwini and Stephen, 2011). However, approaches to the joint integration of well-testing and seismic data are not widely reported in the field of reservoir characterisation, and only the 3-D seismic data are commonly integrated (Zheng et al., 2003; Combe et al., 2005; Sahni et al., 2007). One of the reasons for that could be sought in the time difference between the conventional 4-D seismic and the well-test acquisitions. The conventional well-test data are usually acquired within few days up to, exceptionally, few weeks, while the conventional 4-D
seismic data are sparser in larger time and are usually acquired within few years (usually 3 monitors). This gives an impression that the integration might be impossible. However, recent advances in the permanent seismic monitoring could create substantial hope to reduce this issue. The permanent seismic monitoring is a time- (and cost-) effective technology in which the acquisition devices sets on the sea bed and frequent seismic volumes with higher quality can be acquired (e.g. Valhall Field (Lane et al., 2006)). The permanent seismic nodes enable measurement of the smallest detectable changes in the seismic signature emanating from production-induced changes in the reservoir. This is achieved by increasing the repeatability of the 3-D seismic data sets, more accurate acquisition geometry and survey orientation and increasing the shot-time interval (Landrø, 2010; Mondol, 2010).

By and large, the integration workflow can be seen as a mutual procedure. This means that the either the well-test can help 4-D seismic interpretation (e.g. transmissibility and connectivity estimation and infill well drilling) or time-lapse seismic signals can assist well-test interpretation (e.g. early detection of faults for the model recognition and new exploration well locations). The application and feasibility of the latter integration procedure is studied in this chapter.

6.1 Heterogeneous model response and interpretation

Model recognition is one of the main steps in well-test interpretation. Therefore, any non-unique model should be used cautiously to be consistant with the available data. Without having a fair knowledge of the system structure, it is unlikely that one can make a decent well-test interpretation. The main reservoir structures such as reservoir top and regional faults can be mapped efficiently by 3-D seismic interpretation. However, smaller-scale faults or stratigraphic discontinuities show their effect in the dynamic behaviour of the reservoir and often their average properties can be mapped via 4-D seismic signal. This can be interpreted using a forward and/or inverse method(s) that can effectively take into account the reservoir heterogeneities to illuminate the geological features allowing us to reject all but limited subset of possible well-test models.
Central to the implemented approach is to use the detailed 3-D geological models and create a set of characteristic well-test responses. Hence, a sector of the full field numerical model of a commingled braided fluvial reservoir was selected in which several well-test features such as mobility and diffusivity ratios are superimposed (Figure 6.1). In addition, the sector model has been modified to create a secondary model (Figure 6.2) which includes two parallel stratigraphic discontinuities near the well. Both models consist of 55×55×25 gridblocks in the x, y and z directions for a total number of 75625 cells. The average individual cell dimension is 50 m by 50 m and 1.9 m thick. A black oil simulator of single phase and incompressible oil was applied to generate a single draw-down of 980 STBO/day in 550 hr over 162.5 ft of the reservoir thickness. In all simulations, a very fine Cartesian local grid refinement (C-LGR) was implemented near the wellbore area.

Figure 6.1: A sector of the pixel-based heterogeneous braided fluvial model.

Figure 6.2: A sector of the pixel-based heterogeneous braided model with two simplified stratigraphic discontinuities (permeability baffles).
Figure 6.3 shows two sets of draw-down pressure-derivative responses of the sector models. The light-coloured curve is the test response for the unbounded sector mode, whereas the dark-coloured curve shows the response for the model where additional discontinuities positioned near the well bore area. Clearly, the derivative curves deviate from each other after a while (i.e. 1 hr of draw-down) due to the discontinuities.

The derivative response of the unbounded system (Figure 6.3: light-coloured curve) represents a pure ramp effect. This is recognized with a long derivative response with a slope close to $\frac{1}{4}$ on the Log-Log plot. However, the effect of the continuous parallel faults (i.e. stratigraphic discontinuities) results in reducing the test volume and the appearance of a middle time linear flow regime (Figure 6.3: dark-coloured curve). After a short stabilization time around 0.04 hr, the pressure derivative curve increases monotonically (with a slope of around 0.25). With increasing the draw-down time, a short half slope trend line appears after 3 hr (Figure 6.3: dark-coloured curve). Interpretation of this line provides information about the possible location of a well inside two parallel stratigraphic discontinuities. Eventually, at the late times (> 30 hr) the pressure derivative curve deviates from the half-slope trend with a slope less than unity. This is an indication of partial depletion in multi-layer commingled systems. The pressure disturbance (diffusion front) moves faster in the high permeable layers and leads to variable depletion times in different layers. Unsurprisingly, the well-test modelling of such a system could be non-unique as one might also model the system with a multi radial-composite model without considering any boundaries.
Figure 6.3: Draw-down well-test pressure response of a heterogeneous system (without well bore storage) in which no stratigraphic faults are present (upper curve), and for the case where two parallel stratigraphic faults exist near the well bore area (lower curve).

The numerical modelling of this response can be achieved with the aid of the deterministic (single-domain) or the inverse (dual-domain) methods. Deterministic approaches, implicitly assuming a radial composite model, use the instantaneous (or apparent) permeability concept (Feitosa 1994) in order to generate the permeability distribution away from the well bore. However, the inverse methods try to match the well-test response of the system with numerical methods by integrating the additional information about the architectural framework of the reservoir that could be determined from the time-lapse seismic interpretation.

6.2 Single-domain interpretation

There are different deterministic approaches for estimation of the radial permeability distribution from the draw-down response (e.g. Modified Yeh-Agarwal algorithm (Yeh and Agarwal, 1989; Feitosa et al., 1994) or Inverse Solution Algorithm (ISA) (Feitosa et al., 1994)). The modified Yeh-Agarwal (MYA) procedure assumes that the instantaneous permeability is the volume-weighted harmonic average of the permeabilities within the investigated region. On the other hand, the ISA presume that the instantaneous permeability (defined at each derivative point) is a weighted harmonic average of the true radial permeability distribution. The inverse solution algorithm
(ISA) (Feitosa et al., 1994) was used as a deterministic approach to the problem and is defined as follows

\[
\frac{1}{k_{\text{inst}}(t_i)} = \frac{h\Delta P'}{70.6q\mu B} = \int_{r_1}^{r_2} 2K(r_D, t_{iD}) \times \frac{1}{k(r_D)} dr_D
\]

Equation (6.1) shows that if the analytical \( k_{\text{inst}} \) is known (from pressure derivative curve, \( \Delta P' \) as it is defined in the left hand side of equation 6.1) the equivalent radial permeability distribution can be obtained by an iterative procedure as follows

\[
\frac{1}{k_{\text{av}}(t_n)} = \frac{1}{k_{\text{inst}}(t_n)} - \sum_{i=1}^{n-1} \int_{r_{nD}}^{r_{iD}} 2K(r_D, t_{nD}) \times \frac{1}{k(r_D)} dr_D,
\]

in which, \( k_{\text{inst}} \) is the instantaneous permeability, \( \Delta P' \) is the Bourdet’s derivative of the well-test data points, \( k(r_D) \) is the permeability distributions as a function of dimensionless radius \( r_D \) (this is the permeability distribution prior to the current sector which has already been determined by the pervious steps of ISA), \( k_{\text{av}}(t_n) \) is the unknown true permeability for the sector “n” that lies between radii of \( r_n \) and \( r_{n+1} \), \( t_{nD} \) is the dimensionless time at time \( t_n \) and \( K(r_D, t_{nD}) \) is the Oliver’s Kernel function, all of which have been defined in Chapter 3. This approach generates a radial composite model with “n” progressive sectors starting from the well bore radius \( (r_1=r_w) \) up to the reservoir outer radius \( (r_{n+1}=r_e) \). The ISA algorithm assumes that the overall test response is due to variation in permeability and ignores the variation in the other reservoir properties (e.g. porosity and thickness) or structural complexities (faulted reservoir). The estimated radial permeability distribution can then be used as a permeability field for the numerical simulation purposes to check whether the test response can be reproduced. It should be noted that the generated permeability field is actually the “equivalent” radial permeability of the reservoir. As it was discussed in Chapter 3, the actual 2-D permeability filed was converted to an equivalent radial model by superimposition a set of concentric circles and finding an area-weighted geometric average of the cell permeabilities that fall within each annulus. This is purely because the kernel function
operates only on the radially distributed permeability fields. Figure 6.4 shows the calculated permeability distribution of the draw-down response of the model with parallel no-flow boundaries (Figure 6.3: dark-coloured curve) calculated with different approaches. The ISA and the MYA are fairly close and both highlight the degradation of the permeability away from the wellbore (i.e. the relatively low permeability values of 0.01 md are obtained at the distance of 1600 ft). This is in agreement with the general trend of the instantaneous permeability as well. A uniform radial permeability in the immediate vicinity of the well bore is estimated that is corresponding to the short stabilization time at the early time. Figure 6.5 shows the relation between the well-test time and the investigation radius using the instantaneous permeability

\[ r_i = \alpha r_w \sqrt{t_{Di}} \]  

in which, \( t_{Di} \) is the dimensionless time and was defined in Chapter 3. The factor \( \alpha \) in equation (6.3) was chosen to be 2 that is based on the drainage area concept and ignores the pressure gauge resolution. This definition and choosing the constant \( \alpha \) is a matter of discussion and many authors defined different definitions for the investigation radius (Daungkaew et al., 2000; Kuchuk, 2009).

Figure 6.4: Radial permeability distribution calculated from different deterministic approaches. The ISA (black curve) shows a lower permeability than the MYA (light gray) and the \( k_{\text{inst}} \) (dark gray).
Figure 6.5: The estimated radius of investigation based on the instantaneous permeability and the constant $\alpha=2$.

The dynamic response of the generated composite field by ISA can be verified against the original pressure derivative response. To do so, the calculated radial-composite permeability distribution is fed to a numerical well-test simulation package. The simulation utilizes single-phase flow and the PEBI gridding technique to generate the well-test pressure response of the given history (i.e. extended draw-down with 980 STBO/day). Figure 6.6 shows the draw-down pressure derivatives of the 3-D commingled model and the equivalent radial composite model from the ISA algorithm. A perfect match was obtained for the whole draw-down response. The build-up responses (Figure 6.7) show a reasonable match as well. The only mismatch at the early time (around 0.3 hr) is attributed to some numerical artefacts that arise from the 3-D flow simulation of the system. This is an engaging example which highlights the non-uniqueness nature of the well-test matching through which the effect of external boundaries could be mistakenly modelled using the lateral permeability variations. This is in fact one of the drawbacks of ISA algorithm which tries to replace the boundary effect (e.g. leaky faults in a homogeneous reservoir) by a permeability variation.
Figure 6.6: The extended draw-down derivative response of a permeability field, obtained by a deterministic approach (ISA) vs the realistic response from the 3-D commingled model.

Figure 6.7: The extended buildup derivative response of a permeability field, obtained by a deterministic approach (ISA) vs the realistic response from the 3-D commingled model.

Figure 6.8 is the draw-down pressure distribution of the equivalent radial composite model after numerical simulation of the draw-down. The sink surface is a pseudo-3-D...
geometry and shows the spatial pressure distribution after 600 hrs of production. This is when the outer radial boundary starts to affect the test response.

Figure 6.8: A sink surface. This is the spatial pressure distribution obtained numerically during the draw-down simulation for the equivalent radial composite model. Hotter colours indicate higher pressures.

6.2.1 The utility of radial composite inversion

The existence of “natural” radial composite reservoirs might be unlikely (Kuchuk and Habashy, 1997) as it may not be common for a well to be drilled in the middle of a circular reservoir (Houzé et al., 2011). However, the equivalent radial composite models (i.e. radial composite inversion) provide a utility to evaluate the equivalent radial property distribution of the geostatistical model and compare the transient responses of the model. The equivalent radial composite model can be used to understand the permeability (or the mobility) averaging process of well-test (Sagar et al., 1995). Hence, the Kernel function (defined in Chapter 3) can be applied to provide the time-dependent weights for the annuli in the inverted equivalent multi-radial composite model. The inverted model can be implemented within an optimization algorithm to constrain the geostatistical models by the well-test data. In this way, the reproduced permeability realizations should honour the geometric average within defined annuli in order to honour the radial permeability distribution by the Inverse Solution Algorithm (Sagar et al., 1995). Gautier and Noetinger (2004), used the inverted radial composite models as a fast approach to evaluate the transient response of the geostatistical models with skipping the time-costly fluid flow simulations. More interestingly, Yadavalle (1994) and Yadavalli et al. (1995) used the inverted composite model to estimate the horizontal variogram of the permeability from the single and multiple well-test responses.
It is worth noting that the application of radial composite models could be particularly useful in the gas-condensate systems where the liquid drop-out forms a radial saturation profile near the wellbore and create an effective composite system. This will be studied in details in Chapter 7.

Clearly, the radial composite inversion should be applied carefully not to include the effect of no-flow or leaky boundary because the radial composite inversion gives no limit to our ability in matching any responses (Houzé et al., 2011).

6.3 Dual-domain interpretation
The numerical well-testing can be performed using the additional data. The repeated 4-D seismic can be designed in a way that provides information about the structural and stratigraphic framework of the reservoir. In this study, a set of high frequency (60 Hz) rapid time-lapsed seismic data at different time steps (i.e. one base before production and four monitors after production of: 2 hr, 12 hr, 35 hr and 180 hr) have been generated. The 4-D responses at different time steps have been calculated and the opacity filtering (or rendering) has been applied to extract the illuminated geobodies. It should be noted that the actual value of the amplitude is not of interest in the geobody extraction algorithm. The opacity filtering option has been recently added to Petrel modeling software (Schulmberger, 2009) controls what part of the seismic data we can see. This is a what-you-see-what-you-get algorithm that helps remove the dross and clarifying the hidden geobodies (Schlumberger, 2009). Figure 6.9 shows the 3-D geobodies extracted from the 4-D signals by the opacity filtering of the seismic amplitudes. The depth converted geobodied can be directly gridded and be used in 3-D reservoir models. This can also be interpreted as a step-by-step illumination of the reservoir by the repeated (or rapid) 4-D seismic acquisition, in which the permeability baffles are enlightened. Moreover, the figure conveys some information about the layering of the model: some layers have more extensive affected regions than the others. The corresponding pressure drop volumes for the first and the last 4-D responses are shown on Figure 6.10.

Because it is a common practice to condense the 4-D seismic information in terms of attribute maps, a large 4-D interval (180 hr from the beginning of the test) was selected to be integrated with numerical well-test simulation. Figure 6.11 shows the
RMS amplitude map of the fraction of the 4-D signal changes in two cases with and without stratigraphic boundaries. For the former case, the sub-seismic discontinuities are clearly mapped (Figure 6.11: left) and the geological structure hidden in the system has been illuminated. This structure map can then be used as a guide in numerical well-test modelling to limit the well-test interpretation. There are also two major faults that are captured by 3-D (and 4-D) seismic maps that are clearly beyond the test volume of investigation.

Figure 6.9: 3-D geobodies extracted from the 4-D signals after different production time. The top left figure is the 4-D response after 2hr of production, the top right is the response after 12 hr, bottom left is for that of 35 hr and the last one is the response after 180 hr of the production. This is a successive illumination of the reservoir structure extracted from the opacity filtering of the 3-D time-lapse seismic data.

Figure 6.10: Filtered pressure drop volumes. The figure on the left is pressure drop ($P_i - P(t)$) after 2hr, and the figure on the right is pressure drop after 180 hr of production.
Figure 6.11: The generated short time (after 180 hr) 4-D seismic amplitude map of two different systems (Amplitude change fraction). The system in which the faults are not sensed by the draw-down duration (left) and the one in which the faults are sensed by the pressure derivative response curve but are difficult to deconvolve (right). Large faults mapped by seismic can be seen beyond the well-test volume of investigation.

If the repeated seismic response amplitude is $A_r(x, y)$, it can be written in terms of the base line response $A_b(x, y)$ and the time-lapse changes (MacBeth et al., 2006)

$$A_r(x, y) = A_b(x, y) + \frac{\partial A}{\partial S} \Delta S + \frac{\partial A}{\partial P} \Delta P$$

in which, $P$, $S$ and $A$ are pressure, saturation and amplitude respectively. In the case of single phase black oils simulation of an oil reservoir above the bubble point pressure, the 4-D signal may be attributed to the pressure change in the reservoir. Therefore, the seismic data can be used to relate the anisotropy and heterogeneity of the pressure diffusion to the diffusivity-coefficient ($D=k/\mu c_t$) variations. Thence, the RMS amplitude map (Figure 6.11: left), can reasonably be sectorized to several regions of different mobility ($M_{b/i}$) and specific storage coefficient ($S_{b/i}$) or diffusivity ($D_{b/i}$) ratios.

$$M_{b/i} = \frac{(k / \mu)_b}{(k / \mu)_i}$$
Chapter 6: Using Dual-Domain Heterogeneity Illumination

\[ S_{bii} = \left( \frac{\phi c_i}{\phi c_i} \right)_b \]  
\[ D_{bii} = \left( \frac{k / \phi \mu c_i}{k / \phi \mu c_i} \right)_b \]  

\[ (6.6) \]

\[ (6.7) \]

in which, \( k \) is permeability, \( \phi \) is porosity, \( c_i \) is total compressibility and \( \mu \) is viscosity.

The subscript “\( i \)” is a particular region with specified properties while “\( b \)” shows a base (or reference) region. Figure 6.12 shows the workflow for integrating the numerical well-testing with the 4-D seismic responses. The workflow for the inverse method starts with the 3-D seismic interpretation to map the major faults. The repeated 4-D seismic can help map the permeability baffles hidden from the 3-D seismic interpretation. These information help construct a 2-D structural framework with different regions which are then filled with the different mobility and diffusivity values. These values can be estimated using a non-linear regression algorithm offered in any numerical well-testing software. The selection of these regions is somewhat arbitrary; however, the seismic amplitude map can help in this issue. For instance, the regions can be selected where there are higher contrasts in amplitude changes. Moreover, the numerical experiences show that in some cases, applying the Laplacian operator to the amplitude map can help delineate the patches of high and low diffusivity coefficients. A numerical well-test simulator along with the 2-D Pedi gridding technique is used for the fast simulation of the draw-down response. The results are compared with the realistic results of the 3-D model results. However, to avoid the complexity the minimum possible regions (i.e. one region: a homogeneous reservoir) are used, and to acquire a reasonable match, the other regions are successively added to the map where there is a higher lateral contrast in the amplitudes. Perhaps an automatic history matching technique can be implemented in the loop to facilitate the matching process.
Figure 6.12: The general workflow for the numerical well-testing by the inverse method using the seismic data. Identifying the structures from the seismic, a manual or automatic history matching algorithm can be used to match the initial well-test response through allocating of different regions of unknown mobility and diffusivity coefficients.

In this draw-down example, the sector-composite map was constructed based on 8 regions (as there were higher contrasts in amplitude changes) and a base region (shown with white colour in Figure 6.13). Each region has a set of mobility and diffusivity ratios (which are defined by the ratio of the base region property to the property of each region). The base region average porosity and total compressibility were assigned to be 0.1 and $1.1 \times 10^{-5} \, \text{"/psi"}$. The base region average permeability was obtained to be 160 md from analytical well-test interpretation. The estimated values of $M_{b/i}$ and $D_{b/i}$ are obtained by the non-linear regression algorithm within the numerical well-testing software. These values are within the accepted range of permeability and porosity distributions in the reservoir model (Figure 6.13). Although, this sector-composite map provides an acceptable match in the early times, at later times, where the 4-D information is not available, the well-test data are not adequately matched. This might be tackled by adding extra composite regions away from the well. However, further 4-D seismic data and regional geology might help estimate the lateral variation of the reservoir properties and possibly lower the uncertainty of interpretation and modelling. The workflow followed in this example is based on the equivalent 2-D representation of reservoir where the implemented level of heterogeneity and detailed geology is limited. Clearly, the more detailed geological and geophysical data integration can be achieved.
within a 3-D geological well-testing workflow similar to the third case study presented in Chapter 4. The usages of a single-layer composite model instead of a multi-layered commingled system can sometimes cause serious problems. This point will be discussed later, in the following sections.

![Figure 6.13: The gridded (2-D Pedi) sector-composite map (upper part) within parallel faults-taken from the RMSΔamplitude map of the 4-D signal after 180 hr- and its corresponding well-test response (lower plot-continuous red curve). The data points (lower plot-dotted green curve) are the realistic pressure response from the 3-D Pedi model.](image)

Although, the quality of the match is comparable with the perfect match acquired in the deterministic approach, the well-test model is much closer to the reality as at least it honours the existence of the lateral permeability baffles. This is important as the existence of no-flow boundaries can considerably affect the “long-term” behaviour of the well.

### 6.3.1 Facies Illumination

From the numerical point of view, it is more appealing to know the effect of particular facies on the well-test and the 4-D seismic responses. To this end, a stepwise heterogeneeization (or equally homogenization) is performed. A homogeneous model with an average permeability equal to geometrical average is constructed ($k_v=0$). The reservoir heterogeneities (i.e. facies) are progressively added to achieve the full heterogeneous model. The numerical well-test and seismic responses of the models are sequentially evaluated. This workflow provides the outstanding information about the spatial influence of the facies on the test response (i.e. ramp effect in this case).
Figure 6.14: The facies illumination using well-testing and 4-D seismic. Different facies are added successively to the homogeneous model to construct the fully heterogeneous model (left). At each step, the well-test response is evaluated (middle figure). The effect of individual facies on the 4-D seismic can be explored if a suitable time window is selected and the 4-D signal is estimated within that window at each step. The 4-D maps can be differenced at each two successive steps to find the facies effect on the 4-D seismic signal (right).
Figure 6.14 shows the application of the proposed workflow on the studied model. The figure shows that the homogeneous well-test response (i.e. dotted black curve) follows a dramatic change, as the facies 2 is added to the model (i.e. red curve). Adding the facies 3 has a small effect on the combined test response (i.e. blue curve) and finally the facies 1 can further increase the ramp level (i.e. green curve). These effects can also be observed on the short-term 4-D seismic responses. To do so, a time window is selected and the 4-D seismic signals of the models are evaluated for this time interval during the stepwise heterogenization. The differences between each of these 4-D maps provide the information about the dynamic influence of each appended facies on the pressure distribution. This can mathematically be shown as follows

\[ 4-D_f = \Delta_f \text{RMS}(A_{t2} - A_{t1}) \]  

(6.8)

in which, “4-Df” is the 4-D signal due to addition of facies f, \( \text{RMS}(A_{t1} - A_{t2}) \) is the 4-D signal of each model within the time domain \((t_1, t_2)\) and \(\Delta_f\) is the difference between two 4-D map with and without facies “f”. For example, Figure 6.14 reveals that addition of the facies 2 has a greater spatial effect while the facies 3 has a negligible effect. These maps also show that the spatial contribution of individual facies could be different. For instance, the facies 2 and 3 have different local control on the pressure distribution. If the facies are added other way round, the results will not change. That is mainly due to the fact that facies 2 has higher volume proportion in the heterogeneous model which makes this facies to be more connective in the system. In contrast to facies 2, facies 3 has lower proportion in the heterogeneous model and is less connective. Therefore, the effect of facies 3 will be minimal.

**6.4 Applicability of dual-domain integration**

**Well-test domain:** It is noted that the well-test modelling was performed using an equivalent 2-D composite model, though the characteristic well-test response (i.e. ramp effect) was taken from a 3-D commingled reservoir model. This automatically imposes some critical issues on the model delineation (or reservoir description) and long-term prediction of the reservoir. For example, the partial depletion might be sometimes confused with the ramp effect, though they have different derivative slopes. The partial depletion in draw-downs happens when different speeds of the diffusion front within the
layers of different hydraulic diffusivities, cause the stepwise depleting of the reservoir layers. Moreover, careful attention should be paid to the validity of 2-D (or single layer) simplification at predicting the “long-term” performance or deliverability of the reservoir. Jordan and Mattar (2000) discussed the similar issue by comparing a simplified commingled two-layer homogeneous model (with the upper layer having a limited areal extend compared to the lower layer) with an equivalent single layer two-zone radial composite model in which the thickness was responsible for the composite behaviour. They observed that the transient test response of two models could have similar flow capacity \((kh)\). However, the composite model showed a lower (negative) skin, higher depletion rate, and a lower late production rate.

These issues are important if we ignore the layering and vertical communication in the well-test interpretation. However, accessing the multi-domain and multi-source information (e.g. core, electrical log, production log, and repeated 4-D seismic) the vertical flow temperament of the reservoir could be inducted. Figure 6.9, for example, clearly shows the layering nature of the system with potential successive layer-depletions. Therefore, the continuous increase of the pressure derivative at the late times (with slope <1) could be directly explained. Moreover, as a further step, the depth converted geobodies extracted from 4-D seismic data could be exported to a representative reservoir grid for a “3-D” numerical well-test simulation to be performed. This needs specific modelling techniques and is subject of a more detailed study.

**Seismic domain:** How large should be the pressure changes to be detected on the time-lapse response? The response to this question is not straightforward in terms of defining a “definite number”. The detectability depends on many parameters as rock physics model, overburden pressure, laboratory measurements, characteristics of the seismic acquisition (repeatability) and also 4-D processing and interpretation. The pressure effect can be observed on a single monitor whenever a large change of compressibility happens (e.g. unconsolidated sandstones or chalk). That is because it is not the pore pressure change indeed, but instead it is the effective pressure change that plays the role. Generally speaking, having either a lower overburden pressure or a higher initial pore pressure or a Biot’s coefficients close to unity, results in higher changes in dry bulk modulus. Figure 6.15 shows the dry bulk modulus versus effective pressure for the petro-elastic model used in this study. This shows that depending on the position of the
initial effective pressure, the strength of the plausible 4-D signal on the time-lapse seismic responses can significantly change. For example, if the initial effective pressure is 10 MPa, then 5 MPa increase of the effective pressure brings about 7% of changes in dry bulk modulus. This can be compared with 1.4% of change where the initial effective pressure is 20 MPa for the same magnitude of the effective pressure changes. The existence of the shale and its response to the pressure changes can also complicate the situation. This is reflected in the dry bulk modules calculation and directly affects the magnitude of the simulated seismic amplitude in the synthetic seismic results.

![Graph of dry bulk modulus vs effective pressure](image.png)

*Figure 6.15: Dry bulk modulus vs effective pressure. The lower the initial effective pressure, the higher the dry bulk modulus changes with production.*

There are other real-life factors that reduce the repeatability of the 4-D seismic. For example, source signature, ambient noise and changes in the near-surface with time are amongst the those that rather reduce the reliability of the 4-D seismic response and can possibly hide the reservoir-driven signals (Bacon et al., 2007). However, the development of seismic nodes for permanent seismic monitoring can significantly improve the repeatability of the time-lapse data. This provides the possibility of ultra-frequent acquisition surveys to monitor the short-term reservoir changes and to improve the detectability of the 4-D signals by stacking the different available survey (Landro and Skopintseva, 2008). The permanent survey can be placed near a desired well for illuminating the short- and long-term changes around the well. Figure 6.16 shows an example of a permanent seismic array designed for monitoring of the time-lapse
changes during the production. The receivers have been permanently planted on the sea floor and record the dynamic changes within the reservoir.

![Figure 6.16: A permanent seismic array example for time-lapse seismic acquisition (Fusion Petroleum Technologies Inc.)](image)

If the minimum detectable pressure change is known, then the minimum required time for the time-lapse seismic signal can be acquired by the numerical simulation or analytical calculation. Figure 6.17 is a cartoon which illustrates the stepwise illumination of a heterogeneity in time (Figure 6.17: upper) and/or in space (Figure 6.17: lower). The diffusion front (i.e. pressure disturbance) reflection from the no-flow boundaries, which are mathematically implemented by the method of image, have been neglected in this simplified plot. If the minimum detectable pressure is 200 psi, then it takes at least t2 hours to detect an infinitesimal part of the heterogeneity that is located at the distance L from the well. Clearly as time increase, the longer section of the boundary is detected.
Figure 6.17: Illumination of a linear heterogeneity by the pressure response. The top curve is the wellbore pressure with time and the lower curve is the spatial distribution of the pressure. The longer the production time the longer the illuminated length of the heterogeneity. The minimum detectable pressure is set to be 200 psi.

The calculation of minimum detectable pressure, $\delta P$, depends on many factors as the reservoir geometry and the petrophysical properties. Figure 6.18 illustrates a simple example of a 2-D homogeneous reservoir that includes two parallel linear boundaries. The pressure drop along the linear boundaries can be obtained by the superposition theory (Duhamel, 1833) and is calculated by considering the cumulative pressure drops of an equivalent infinite reservoir with an infinite number of image wells located away from the boundaries (Figure 6.18). The linear boundaries are located at the distance $L (>20r_w)$. This condition helps to use the simplified form of the transient pressure equation using the exponential integral, $E_i$, function. Therefore, the required minimum detectable pressure to illuminate $2F$ length of the linear boundary can be computed as follows

$$
\delta P = \Delta P(F,t) = 2 \sum_{i=1}^{\infty} \frac{70.6q\mu B}{kh} E_i \left( \frac{-948\phi \mu c_i \left( F^2 + [(2i-1)L]^2 \right)}{kt} \right)
$$

(6.9)
in which, $\delta P$ is the minimum detectable pressure in “psi”, $F$ is the illumination factor in “ft”, $L$ is the well distance to the fault in “ft”.

Figure 6.18: A homogeneous reservoir with two parallel no-flow boundaries and an active well of an equal distance to the boundaries. The pressure drop at $F$ is calculated by using the superposition theory where the boundaries can be replaced by an infinite number of image wells in both sides of the linear boundaries. For the matter of the calculation, the illuminated length of the boundaries, $F$, can be expressed as a linear multiple of $L$ (i.e. $F=i\times L$).

This procedure can be followed for any type of the linear boundaries. Figure 6.19 presents a set of different heterogeneity configurations in a homogeneous reservoir ($k=45$ md) and provides the minimum required time for illumination of a specific length of the boundary. The minimum detectable pressure change, created by a single draw-down of 1500 STBO/day, was selected to be 200 psi. The other reservoir and fluid properties were kept constant and are shown on the plot. Figure 6.19 shows that, for example, at least 56 hr is needed for the threshold pressure drop to reach a flow boundary located at 100 ft away from the well bore. However, the existence of the no-flow boundaries increases the pressure drop in the reservoir and is more favourable for the time-lapse detection. As an example, the threshold detection time of a single no-flow boundary is 5.5 hr (Figure 6.19: top right) and this reduces to around 4 hr (Figure 6.19: bottom left) for the case of two parallel faults. Obviously, the timing does not linearly change as it takes about 140 hr for illuminating 1000 ft of a single fault while it takes only 26 hr for the system with two parallel faults.
Chapter 6: Using Dual-Domain Heterogeneity Illumination

Figure 6.19: The required time for illuminating the specific length of different heterogeneity structures. The existence of no-flow boundaries increases the pressure drop which consequently reduce the minimum detection time for a specific length of a linear heterogeneity.

It should be noted that the reservoir and fluid properties could change the detection time, as well. For example, Figure 6.20 shows that the higher the reservoir permeability, the longer the detection time. For the matter of illustration, if the reservoir permeability improves from 45 md to 700 md, the detection time for 800 ft of the length of the two no-flow boundaries will change from 40 hr to 160 hr. This is the time for the minimum detectable pressure of 400 psi.

Figure 6.20: The reservoir properties have direct impact on the pressure drop within the reservoir. The more permeable the reservoir the less pressure drop within the reservoir.
Table 6.1 presents the typical 4-D seismic acquisition technique which is needed for specific time scale. For instance, if the required monitor time-scale is less than 2 days, the intra-survey acquisition fulfils this requirement. For the larger time scales, the conventional surface seismic or the permanent survey (seismic nodes) is required. This table has been constructed based on various personal communications with practitioners.

**Table 6.1: The different time-lapse acquisition techniques that fulfil the required time-scales**

<table>
<thead>
<tr>
<th>Time Scale</th>
<th>Practicality</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1-2 Days</td>
<td>Inter-Survey</td>
</tr>
<tr>
<td>2 Days to 1 Month</td>
<td>Might not be currently practical due to cost/ Ultra-frequent permanent survey</td>
</tr>
<tr>
<td>1 to 3 Month</td>
<td>Permanent survey, surface seismic</td>
</tr>
<tr>
<td>1 to 3 Years</td>
<td>Surface seismic</td>
</tr>
</tbody>
</table>

It is worth noting that the extended well-testing (e.g. Field X example in Chapter 4) could fulfil the requirements for more conventional surface seismic surveys where the test duration is around 90 days and the pressure changes are of order of 4000-5000 psi. The reservoir monitoring using the permanent down-hole gauges (PDG) could offer an alternate integration scenario, as well.

**6.5 One step forward: permeability extraction possibility from the 4-D seismic responses**

The rigorous way to estimate the spatial reservoir property distribution is to use the production or seismic history matching (Stephen et al., 2005; Stephen and Macbeth, 2006; Edris et al., 2008; Villegas et al., 2009; Arwin and Stephen, 2011; Kazemi and Stephen, 2011). The initial property distributions from the static data and using the geostatistical techniques are modified to match the predicted pressure and rate of the wells against the real available history. The 4-D seismic data provides extra constraints on the history matching process. However, this process is usually time costly as the required minimization algorithm relies on multiple realization as an initial statistical population. The direct inversion of the 4-D seismic data could be an alternate to the solution. However, this needs the knowledge of relating the 4-D seismic changes to the
dynamic reservoir parameters. To date, there have been many studies in time-lapse seismic domain that relate the 4-D seismic signals to the pressure and saturation changes (Tura and Lumley, 1998; Landro, 2001; Lumley et al., 2003; Floricich et al., 2005; MacBeth and Al-Maskeri, 2006; MacBeth et al., 2006; Ribeiro and MacBeth, 2006). In the single phase well-test situation, in particular, the problem is relatively straightforward as the spatial saturation changes remains very small and the reservoir is truly a pressure-dominated reservoir. This avoids the complexities associated with decoupling of pressure and saturation and to directly make a bridge between the seismic amplitude changes and the reservoir pressure change. On the other hand, these changes of pressure (and saturation) are in turn controlled by the reservoir properties like spatial distribution of the permeability and the porosity. Vasco (2004) and MacBeth and Al-Maskeri (2006) made some efforts on the permeability extraction from the long term steady state data in their studies. Vasco (2004), for instance, assumes that the reservoir changes affect the source term of the diffusivity equation and therefore neglected the time variation term and MacBeth and Al-Maskeri (2006) did not include the spatial permeability variations in the diffusivity equation and assumes a locally homogeneous field. However, the idea here is to extract the reservoir properties (i.e. heterogeneous permeability field in particular) from the time-dependent pressure field during a single-phase well-testing. The mathematical framework of the problem is constructed by integrating of the diffusivity equation over the test interval and then to solve the integral partial diffusivity equation to estimate the permeability variations. This is acquired by adopting a proper finite difference method.

The diffusivity equation of a single-phase slightly compressible fluid for a 2-D homogeneous porosity and heterogeneous permeability field is written as follows

\[
\frac{\partial}{\partial x} \left( \frac{\beta_c A_x k(x,y)}{\mu} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left( \frac{\beta_c A_y k(x,y)}{\mu} \frac{\partial P}{\partial y} \right) \Delta y + q = \frac{V_j \phi c_i}{\alpha_c} \frac{\partial P}{\partial t} \quad (6.10)
\]

in which, \(\beta_c\) is 1.127×10^{-3}, \(\alpha_c\) is 5.615 ft^3/bbl, \(A_x\) and \(A_y\) are the area normal to flow in x and y directions, \(\Delta x\) and \(\Delta y\) are the gridblock dimensions in “ft^2”, \(k(x,y)\) is the heterogeneous permeability field “md”, \(\phi\) is porosity, \(c_i\) is the coefficient of the
Chapter 6: Using Dual-Domain Heterogeneity Illumination

isothermal compressibility in “1/psi”, \( \mu \) is the viscosity in “cp”, \( V_b \) is the gridblock volume in “ft\(^3\)” and \( q \) is source/sink term in “bbl/day”.

Having a uniform areal gridblock (\( \Delta x=\Delta y \)) and integrating the equation (6.10) over the lapsed time, equation (6.10) takes the following form

\[
\int_{t_0}^{t} \left[ \frac{\partial}{\partial x} \left( k(x, y) \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( k(x, y) \frac{\partial P}{\partial y} \right) \right] \, dt + \frac{1}{V_b} \int_{t_0}^{t} q \, dt = \int_{t_0}^{t} \frac{\mu \phi_c \varphi}{\beta \alpha} \frac{\partial P}{\partial t} \, dt = \frac{\mu \phi_c \varphi}{\beta \alpha} \Delta P \quad (6.11)
\]

The main issue here is the evaluation of the first term that should be calculated at the “cell interface” rather than the cell centre. Applying the standard reservoir simulation technique to discretize the \( k(x,y)\frac{\partial P}{\partial x} \) or \( k(x,y)\frac{\partial P}{\partial y} \) as a whole, leads to a non linear equation and cannot be easily solved. However, assuming a fairly smooth permeability field, the first term can be further simplified by applying the partial derivative operators

\[
\frac{\partial}{\partial x} \left( k(x, y) \frac{\partial P}{\partial x} \right) = k(x, y) \frac{\partial^2 P}{\partial x^2} + \frac{\partial P}{\partial x} \frac{\partial k(x, y)}{\partial x} \quad (6.12)
\]

\[
\frac{\partial}{\partial y} \left( k(x, y) \frac{\partial P}{\partial y} \right) = k(x, y) \frac{\partial^2 P}{\partial y^2} + \frac{\partial P}{\partial y} \frac{\partial k(x, y)}{\partial y} \quad (6.13)
\]

These result in

\[
k(x, y) \left[ \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right] \left( \frac{\partial P}{\partial x} \frac{\partial k(x, y)}{\partial x} + \frac{\partial P}{\partial y} \frac{\partial k(x, y)}{\partial y} \right) \right] \, dt + \frac{1}{V_b} \int_{t_0}^{t} q \, dt = \frac{\mu \phi_c \varphi}{\beta \alpha} \Delta P \quad (6.14)
\]

In this equation the spatial pressure, \( P(x,y) \), is assumed to be known and the only unknown variable is the spatial permeability field, \( k(x,y) \). The variability of the porosity can also be simply included; however for matter of illustration a constant porosity was used. The Laplacian of the pressure can be calculated at each time step using the
numerical differentiation techniques including the central difference, polynomial or spline fitting methods. The time integration can also be numerically evaluated by the Trapezoidal rule or Gaussian Quadrature technique (Press et al., 1992; Visual Numerics, 2007). For example, the integration of the pressure gradient in x direction using the Trapezoidal rule is as follows

\[
\int_{t_0}^{t_s} \frac{\partial P}{\partial x} \, dt = \sum_{s=1}^{n_t} \frac{t_s - t_{s-1}}{2} \left( \frac{\partial P(t_{s-1})}{\partial x} + \frac{\partial P(t_s)}{\partial x} \right)
\]  

(6.15)

To extract the permeability from the pressure data, equation (6.14) is discretized in terms of permeability values in each gridblock.

\[
k_{i,j} \Gamma_{i,j} + \frac{\beta_{i,j}}{2\Delta x_{i,j}} \left( k_{i+1,j} - k_{i-1,j} \right) + \frac{\gamma_{i,j}}{2\Delta y_{i,j}} \left( k_{i,j+1} - k_{i,j-1} \right) = \xi_{i,j}
\]  

(6.16)

Where,

\[
\Gamma_{i,j} = \left[ \int_{t_0}^{t_s} \left( \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} \right) \, dt \right]_{i,j}
\]  

(6.17)

\[
\beta_{i,j} = \left[ \int_{t_0}^{t_s} \frac{\partial P}{\partial x} \, dt \right]_{i,j}
\]  

(6.18)

\[
\gamma_{i,j} = \left[ \int_{t_0}^{t_s} \frac{\partial P}{\partial y} \, dt \right]_{i,j}
\]  

(6.19)

\[
\xi_{i,j} = \left[ \frac{\mu \varphi c_t}{\beta_c \alpha_c} \Delta P - \frac{1}{V_b} \int_{t_0}^{t_s} q \, dt \right]_{i,j}
\]  

(6.20)

Using that natural ordering by rows in the discretization procedure (Figure 6.20) and assuming the new variables
\[ \beta_{i,j} = \frac{\beta_{i,j}}{2\Delta x_{i,j}} \] 
(6.21)

\[ \gamma_{i,j} = \frac{\gamma_{i,j}}{2\Delta y_{i,j}} \] 
(6.22)

the resulting coefficients of the sparse matrix \((N_x \times N_y) \times (N_x \times N_y)\) is constructed as follows:

The set of obtained equations (presented in matrix form) are then solved directly or sequentially by LU decomposition or Jacobi’s method (Visual Numerics, 2007) and the resulting permeability field is estimated.

To exercise the algorithm, a 2-D Cartesian model was constructed with \(100 \times 100\) cells of individual size \(10 \text{ ft} \times 10 \text{ ft} \times 100 \text{ ft}\) in \(x, y,\) and \(z\) directions respectively. The
porosity was kept constant ($\varphi=0.18$) and the Sequential Gaussian Simulation was implemented to generate an unconditional permeability field with a Gaussian covariance function and a horizontal isotropic correlation length of 200 ft. There is no flow and no pressure support from the external boundaries of the model. A well was placed in the centre of the reservoir and the Eclipse reservoir simulator (E100) was used to simulate the single-phase flow production of 1000 STBO/day. Twenty uniform time-steps, every 0.24 hr, were used in the simulation to be used in the required calculations. It should be noted that no noise has been implemented in the algorithm and only pure differencing has been used. Figure 6.22 shows the comparison of the estimated (Figure 6.22: right) and the original permeability maps (Figure 6.22: left). The estimated permeability field is relatively good and captures the patches of low and high permeability values. However, it could not completely retrieve the original map. The main reasons for that could be sought in the number of time steps used in the simulation and also the numerical computation of derivative and integration of the pressure fields in each time step. The numerical experiences showed that Laplacian of the pressure field is a very sensitive parameter and has a direct effect on the solution of the problem. Any artefacts in the calculations, will bias the solution towards unrealistic values of the permeabilities. Furthermore, some negative values for the permeabilities could also be emerged. Besides the former reasons, this could also be related to the discretization of the problem that assumes a smooth permeability field which enables to perform the calculations presented in equations (6.12) and equation(6.13).

![Figure 6.22](image)

*Figure 6.22: The original permeability map (left) and the estimated map from the integration of pressure distribution within the well-test framework.*
Apart from the novelty, obviously this approach needs more numerical adjustments to reduce the numerical artefacts, yet the approach with its present form could be used as a permeability “attribute” estimator which provides the direct information about the location of the patches of high permeability values.

6.6 Chapter summary
The dual-domain information are used to reduce the uncertainty in well-test modelling and interpretation of a complex transient well-test response. Using the single-domain information (i.e. well-test), the test response is attributed to a multi-radial composite system. However, the 4-D seismic maps are implemented for the identification of the parallel permeability baffles in the model that were previously hidden in the system (dual-domain information). The seismic amplitude information is also used to generate a sector-composite map providing the necessary geological and geophysical constraints for the numerical well-test interpretation. This leads to somewhat reduce the non-uniqueness in well-test modelling by identifying the lateral heterogeneities from the 4-D seismic signals. As a further step forward, the possibility of the permeability extraction from the seismic inverted pressure data are scrutinized and despite the limitations, the promising results are acquired.
Chapter 7

Additional Complex Fluid Implications

In this Chapter the effect of the complex fluids (i.e. rich gas-condensate) on the ramp effect is studied. Implementing a compositional simulator, a series of sensitivities are performed to highlight the impact of pertinent parameters (e.g. production time and rate, vertical permeability and correlation length) on the well-test signatures. The results clarify that the fluid heterogeneity competes with the geological heterogeneities and production parameters can alter pressure distribution and condensate saturation, and mask the native model well-test signatures.

The effect of condensate drop-out on the time lapse seismic data is also studied. This is reflected through forward seismic modelling during the draw-down and the build-up response of the reservoir model showing the ramp effect. The modelling takes the advantage of compositional modelling of the reservoir fluid. However, implementing a compositional fluid flow simulation highlights the limitations of the current petro-elastic modelling in conjunction with fluid composition change. Therefore, an effort is also been made in order to take into account the compositional changes of the reservoir fluid with time in the forward seismic modelling.

7.1 Gas-condensate reservoirs

Gas-condensate reservoirs are those with their temperature lying between the critical temperature and cricondentherm of the system (Figure 7.1). In such systems, retrograde
condensation occurs when the pressure falls below the initial dew point pressure (Muskat, 1949). This causes the liquid phase to build up near the producing well, which in turn can dramatically reduce the well productivity index (possibly as high as 65% (Fan et al., 2005)) even in lean gas-condensate reservoirs in which the liquid dropout is as low as 1% (Afidick et al., 1994). Recognition of this situation is important in reservoir monitoring and management decision since the secondary operations such as gas cycling would need to be triggered to re-pressurize the reservoir.

It is now well documented that in the near wellbore area, simultaneous flow of gas and condensate is also affected by the combined effect of coupling (increase in \( k_r \) by an increase in velocity or decrease in interfacial tension, IFT,) and inertia (a decrease in \( k_r \) by an increase in velocity) (Danesh et al., 1994; Henderson et al., 1996; Whitson et al., 1999; Jamiolahmady et al., 2010). The positive coupling effect also known as the capillary number effect is attributed to simultaneous coupled flow of the gas and condensate phase with intermittent opening and closure of the gas passage by the condensate at the pore level (Jamiolahmady et al., 2000, 2003).

Pressure transient analysis has traditionally been used as a diagnostic tool in such reservoirs. The analytical interpretation process begins by finding a well-test model that matches the real pressure data. The model parameters are tuned to obtain an acceptable match. The well-test models for analytical interpretation are often different from reality because they are the simplified average models, which mimic the dynamic behaviour of the reservoir. That is, some geological heterogeneities cannot be modelled with a simple predefined model. Furthermore, there could be many realisations with the same signature. In addition, the superposition of a complex geological model and a fluid model can have different effects on this interpretation. That is, the pressure distribution is a function of the spatial heterogeneities that are reflected in the distribution of the hydraulic diffusivity coefficient (\( D \)). In the particular case of gas-condensate reservoirs, the liquid drop out can also interfere with the expansion of pressure disturbance (diffusion front) These spatial and fluid heterogeneities are reflected differently in the draw-down and build-up transient well-test data hence, complicating the well-test interpretation even further. In this work it is shown that geology is a controlling parameter on the non-uniform formation and distribution of the condensate drop-out.
Moreover, the condensate formation interferes with the native geological test response of the reservoir.

Figure 7.1: A typical P-T diagram a gas-condensate reservoir. The initial reservoir state lies between the region confined by the cricondenbar and cricondentherm. As the reservoir pressure decreases, the retrograde condensation happens and the oil phase condenses out of the gas phase.

7.2 Well-test analysis approach in gas-condensate reservoirs

The fluid properties of the gas reservoirs (e.g. isothermal compressibility and viscosity) are strongly pressure dependent. This causes the subsequent diffusion equation to be strongly non-linear. However, the non-linearity of the diffusion equation can be reduced by employing the phase pseudo-pressure function. Al-Hussainy et al. (1966) introduced the single-phase pseudo-pressure function (equation 7.1) that is widely used in the well-test analysis of the dry-gas reservoirs

\[
m(p) = 2 \int_{p_0}^{p} \frac{p}{\mu(p)z(p)} \, dp
\]

(7.1)

in which, \(\mu\) and \(z\) are the viscosity and gas deviation factor respectively. This is a mathematical transformation which results in the governing diffusivity equation for the gas reservoirs to take the following form

\[
\nabla^2 m(p) = \frac{\phi \mu c_i}{k} \frac{\partial m(p)}{\partial t}
\]

(7.2)

in which, all parameters are in darcy units. This equation is not fully linearized because \(\mu\) and \(c_i\) are still function of pressure. However, Al-Hussainy et al. (1966) showed that if the fluid properties are calculated at a mean pressure the equation can be approximated
as a linear equation. Therefore, for practical purposes in the gas well-test analysis, the pressure is converted to the pseudo-pressure function and the liquid well-test analogy is applied for the well-test interpretations. However, in the case of a large change of gas compressibility during the test, this transformation does not exactly reproduce an equivalent liquid behaviour (Bourdet, 2002). Therefore, Agarwal (1979) introduced the pseudo-time function to further decrease the non-linearity of the differential equation that is applied in low-pressure gas wells and/or during the reservoir limit tests.

Jones et al. (1989) introduced a multi-phase pseudo-pressure function (equation 7.3 and equation 7.4), which can be applied to gas-condensate reservoirs

\[
m(p) = 2 \int_{p_g}^{p_o} \left( \frac{\rho_o k_{ro}}{\mu_o} + \frac{\rho_g k_{rg}}{\mu_g} \right) dp
\]

(7.3)

\[
m(p) = 2 \int_{p_g}^{p_o} \left( \frac{k_{ro}}{\mu_o z_o} + \frac{k_{rg}}{\mu_g z_g} \right) p dp
\]

(7.4)

in which, \( \rho_o \) and \( \rho_g \) are oil and gas molar densities and \( k_{ro} \) and \( k_{rg} \) are the oil and gas relative permeabilities. The value of incorporating relative-permeability effects in the pseudo-pressure function is that skin due to condensate banking is already being included within the two-phase pseudo-pressure integral (Osorio et al., 2005). The authors suggested to evaluate the pseudo-pressure function based on a 2-region fluid model proposed by O'Dell and Miller (1967) and Fussel (1973). These authors used the steady state concept and described the liquid saturation profile inside the reservoir as a two-region model: a single-phase region away from the wellbore in which the reservoir pressure is above the dew point pressure, and a two-phase region near the wellbore area in which the liquid and gas are both flowing. Fevang and Whitson (1996) proposed the existence of another intermediate region within which the liquid dropout is immobile and only gas is flowing. Gringarten et al. (2000) introduced the rate dependent relative permeability curves in simulation of gas-condensate well-tests by assuming the existence of a fourth region in immediate vicinity of the wellbore. Borges and Jamiolahmady (2009) showed that the dependency of relative permeability to IFT and velocity due to coupling and inertial effects, which cannot be taken into account in the two-phase pseudo-pressure calculation (equation (7.3) and equation (7.4)) makes such
WT interpretation unreliable. It has been shown that the multi-phase pseudo-pressure function is very sensitive to the input relative permeability (Roussennac, 2001). Roussennac (2001) showed that any inaccuracy in relative permeability measurements imposes higher error than using the single-phase pseudo-pressure measurement. Moreover, Xu and Lee (1999) acknowledged that the single phase analogy is simple and adequate to analyse transient pressure data if the condensate bank is deep and shut-in times are long.

In the light of the above findings, in this study the single-phase pseudo-pressure function was employed to analyze the well-test draw-down and build-up responses. As mentioned previously only the interfering effects of geology and condensate blockage and not those of coupling and inertia were scrutinized. In other words, due to the non-linear nature of flow, if velocity effects were included (Borges and Jamiolahmady, 2009), the simulations were conducted in absence of such effects.

In homogenous systems, using the single phase pseudo-pressure function yields a (radial-) composite behaviour on the Log-Log plot of the derivative response (Xu and Lee, 1999). This can be analysed using a two-zone radial composite model. If the test is long enough, two Semi-Log straight lines appear on a Horner or superposition plot. The analysis of the first and second straight lines slopes provides the mobility ratio ($M_r$), which is the ratio of the inner to outer region mobility. The total skin factor can be obtained by analyzing the second straight-line (single phase region) during build-up period using equation (7.5)

$$S = 1.151 \left[ \frac{m(p_{w, @ 1 hr}) - m(p_{w, @ tp})}{m} - \log \frac{k_{out}}{\varphi \mu_{out} c_{t-out} r_w^2} + 3.23 \right] \quad (7.5)$$

in which, $m$ is the Semi-Log straight-line’s slope on the Horner plot, $k_{out}$ is the outer region permeability in “md”, $\varphi$ is porosity, $c_{t-out}$ is the total compressibility in outer region in “1/psi”, $\mu$ is the viscosity in “cp” and $r_w$ is the wellbore radius in “ft”.

This skin factor is a combined effect of the mechanical and two-phase skin factor. The two-phase skin factor is assumed to be an apparent skin, which describes the influence
of the inner zone during the late time homogeneous behaviour. This skin reduces the connectivity of the wellbore to the reservoir. In this approach the condensate phase is assumed immobile.

Assuming a constant oil saturation in the condensate bank, Xu and Lee (1999) suggested that the total skin factor can be formulated as follows (equation (7.6))

\[ S_t = \frac{S_m}{k_{rg}} + S_{2p} \]  

(7.6)

in which \( k_{rg} \) is the gas relative permeability, \( S_m \) and \( S_{2p} \) are the mechanical and 2-phase skin factors respectively. This might be expressed in a more general form (equation (7.7)) using a radial composite system (Bourdet, 2002)

\[ S_t = \frac{1}{M_r} S_m + \left( \frac{1}{M_r} - 1 \right) \ln \frac{R}{r_w} \]  

(7.7)

where \( R \) is condensate depth of investigation in”ft”. As the mobility ratio \( (M_r) \) is less than unity, the apparent radial composite skin factor (second term of the above equation) is positive.

7.3 Geological model and simulation set up

This model is the same as the one discussed in Chapter 3 and is composed of 86×48×25 cells, each cell having a dimensions of 25 m×25 m×1.9 m in x, y and z directions, respectively. The facies and petrophysical correlation lengths were considered to be the same and were kept to be 250 m for the base case. This average correlation length, has been calculated from the analysis of the available core-log data for maximum six wells in this field.

It should be noted that this model was not originally related to a gas-condensate reservoir. However, the same gas-condensate reservoirs are presents in the North Sea (e.g. Heron field (Mckie et al., 2010), Frankin field (Suiter et al., 2005) and Marnock field (Fisher and Mudge, 1998)). Therefore, this model is used as an analogy in this study.
A single, fully penetrated vertical well has been placed near the middle of the selected sector model and different rate histories along with different geological scenarios have been simulated. The base rate history is 8 days of build-up with preceding 8 days of draw-down with production rate of 20 MMscf/day. Logarithmic time stepping along with an intensive and extensive Cartesian local grid refinement (C-LGR) around the wellbore were implemented to accurately capture the near well bore liquid formation and eventually reduce the associated numerical artefacts in the subsequent well-test derivative responses. This could be even more important in the case of the gas-condensate fluid systems in which the complex fluid behaviour such as condensation and re-vaporization takes place in the near wellbore region.

The fluid used in this study was a real case ten-component rich gas-condensate fluid (Table 7.1) with a maximum liquid dropout of 30%. The Peng-Robinson (PR) equation of state (EOS), which has been tuned against the Constant Composition Experiment (CCE), was implemented in order to model the in-situ fluid behaviour during the transient draw-downs and build-ups. The reservoir temperature was 250 °F and the initial reservoir pressure was selected to be slightly above the initial dew-point pressure (5341 psia). In this work, different sets of experimentally measured gas-oil relative permeability data for each facies have been used. Because the relative permeability data was measured in the absence of the interstitial water saturation, the initial water was not considered in the simulations. The well-test simulations were performed using adaptive implicit method (AIM) in E300 compositional reservoir simulator.

Table 7.1: The rich gas-condensate composition and properties used in this study

<table>
<thead>
<tr>
<th>Components</th>
<th>Composition %</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO2</td>
<td>2.97</td>
</tr>
<tr>
<td>C1N2</td>
<td>66.67</td>
</tr>
<tr>
<td>C2</td>
<td>9.68</td>
</tr>
<tr>
<td>C3-4</td>
<td>8.39</td>
</tr>
<tr>
<td>C5-6</td>
<td>3.12</td>
</tr>
<tr>
<td>C7-10</td>
<td>4.83</td>
</tr>
<tr>
<td>C11-14</td>
<td>1.77</td>
</tr>
<tr>
<td>C15-20</td>
<td>1.5</td>
</tr>
<tr>
<td>C21-29</td>
<td>0.87</td>
</tr>
<tr>
<td>C30+</td>
<td>0.2</td>
</tr>
<tr>
<td>Dew Point Pressure (psia)</td>
<td>5341</td>
</tr>
<tr>
<td>Maximum liquid drop-out (%PV)</td>
<td>30</td>
</tr>
</tbody>
</table>
Chapter 7: Additional Complex Fluid Implications

The viscosity and compressibility values needed to estimate the pseudo-pressure function were obtained using a commercialized PVT package (PVTi). The Peng-Robinson equation of state was used to generate the flash tables by simulating a Constant Composition Experiment (CCE). Figure 7.2 shows the resulting calculated single phase pseudo-pressure function as a function of pressure.

![Figure 7.2: Single phase pseudo pressure as a function of pressure.](image)

### 7.4 Native pseudo-pressure derivative response: heterogeneous model single-phase fluid

The native geological well-test response of the heterogeneous geological reservoir model when the pressure is above dew point was first studied. Figure 7.3 shows that the draw-down and build-up and the corresponding derivative responses of the base case model. The data shows a ramp effect phenomenon, which is a monotonic increase of (pseudo-) pressure derivative response over at least one log cycle. This is a characteristic response of a reservoir with laterally decreasing of the mobility that is usually observed in heterogeneous carbonate reservoirs and/or high net:gross commingled reservoirs in which the diffusion front is encapsulated within the reservoir layers.

The early time unit-slope trend of the pressure derivative curve is a numerical error associated with the grid-cell size penetrated by the wellbore (fake wellbore storage (FWBS)). The fake wellbore-storage is also repeated in the build-up response with the same strength and duration as long as the reservoir and fluid properties are not changed.
Therefore, the separation of the derivative curves during the draw-down and the build-up periods is an indication of the reservoir and/or fluid changes in near wellbore areas. Besides, the “layered (or partial) depletion” of the commingled system results in the late-time derivative response to rise in the draw-down period while the build-up derivative response is flattened and could finally be rolled over. This is a boundary dominated region and is not considered in this study.

### 7.5 Native pseudo-pressure derivative response: homogeneous model, two-phase fluid system

Figure 7.3 shows the single-phase pseudo-pressure derivative curves of a homogeneous system \((k=172 \text{ md})\) during 4 days of draw-down (with rate of 20 MMscf/day) and 8 days of build-up. In this system the initial reservoir pressure was above dew point but dropped below dew point during the draw-down period. It is noted that the condensate drop-out affects the build-up and draw-down well responses in different ways. That is, the condensate effect is magnified on the build-up response at the early time while draw-down response is sensitive to the condensate drop out at the late times. During the build-up, the saturation profile is that which has already been set up during the production phase and is assumed to be invariant afterwards. This profile may be approximated by the one at the end of the draw-down period.

![Figure 7.3: The build-up and draw-down well-test responses of a homogeneous gas-condensate system](image.png)
This behaviour can further be explained by looking at the propagation of the pressure disturbance front within the reservoir. Unlike the porosity averaging, the well-test permeability averaging is not a uniform average within the investigated volume. This can be explained by Oliver’s Kernel function (1990), $K(r_D, t_D)$, where 98% of the contribution of the weighting function coming from the annulus area encompassed by the dimensionless radii of $r_D = 0.12 \sqrt{\frac{t_D}{D}}$ and $2.34 \sqrt{\frac{t_D}{D}}$. The position of weighting function changes with time and can be interpreted as a moving average window. During the early time of build-up the weighting function passes over the accumulated condensate regions during the proceeding draw-down, whilst the derivative response at the late time reflects the virgin zones in which the pressure is above the dew point pressure. Figure 7.4 shows the spatial position of the normalized weighting function at different time ($t_1 > t_2 > t_3$) during the build-up period.

![Figure 7.4: The location of the permeability weighting function (Kernel function) away from the wellbore at different times during the build-up(upper). The lower curve is a cartoon shows the typical pseudo-pressure derivative response during the build-up.](image)

This point can be further explained for a build-up response in a radial homogeneous model with permeability of 20 md, porosity of 0.1 and the thickness of 165 ft. The reservoir is put into production to flow 30 MMscf/day for 4 days, which is then followed by 4 days of build-up. The original concept of the Kernel function was introduced for the draw-down response of oil reservoirs. Therefore, some modifications should be applied in order to implement the averaging procedure to the build-up response of a gas reservoir. In particular, these include using of the equivalent time and the definition of the dimensionless time (equations 7.8 through 7.11).
in which, $\Delta t_{eq}$ is the equivalent shut in time, $t_p$ is the production time in “hr”, $m(p)$ is the single phase pseudo pressure function in “psi$^2$/cp”, $c_g$ is the gas compressibility in “1/psi”, $\mu$ is viscosity in “cp”, $T$ is the reservoir temperature in Rankine (that is . 710 R in this case) and subscript $i$ in $(\mu c_g)_i$ represents the initial state. The condensate effect is assumed to be incorporated into the calculations by reducing the gas effective permeability. The Kernel function then operates over the radial gas effective permeability profile.

Figure 7.5 shows the gas effective permeability profiles at the shut in time, which is superimposed on the Kernel function positions at different build-up times (0.003 hr, 0.02 hr and 1 hr). This explains the averaging process of permeability that in turn affects the build-up derivative shape. The radial “gas effective permeability” profile has been obtained from the results of compositional simulation.

The Kernel function can be integrated over the effective permeability profile and estimate the instantaneous permeability profile that can reproduce the build-up derivative curve. Figure 7.6 shows the realistic build-up response, obtained from the compositional numerical well-test simulation (white dotted curve), and the one calculated by the averaging of the gas effective permeability using the Kernel weights at all time steps (dark dotted curve). There is an excellent match for the transition zone and some mismatch at the early and late times. Ignoring the early time regions where the numerical error is likely, the Kernel well-test response converges to the absolute
permeability of the 20 md while the characteristic test response flattens over 17 md. Apart from the assumptions for derivation of the Kernel function that are mainly for the single phase flow condition, this might also be sought in the input parameters of the Kernel function as $t_{DK}$, which assumes a constant $\mu c_g$ over the build-up period.

![Figure 7.5](image1.png)

**Figure 7.5:** The gas effective permeability at the shut-in time (green) and the moving Kernel function at different shut-in times: 0.003 hr, 0.02 hr and 1 hr (bell-shape curves). The red highlighting on the effective permeability profile shows that after 1 hr, the well-test derivative response comes from a non-uniform average of the harmonic effective permeabilities defined by the third Kernel function (blue curve).

![Figure 7.6](image2.png)

**Figure 7.6:** The build-up derivative response of the gas-condensate fluid in a radial homogeneous reservoir from the compositional numerical well-testing (white circles), and the estimated derivative response calculated from the Kernel-weighted harmonic gas effective permeability average (black circles). The gas effective permeability profile is generated at the shut-in time.
This permeability weighting function, works fine in situations, where the variations of the reservoir properties are negligible with time. This is the condition, which is verified in the transient build-up period. However, unlike the build-up period, the saturation profile can dramatically change during the draw-down period and this is eventually reflected in the late time increase of the derivative response. This phenomenon can be explained by the time-dependent sensitivity coefficients (TDSCs) concepts. The TDSCs show the sensitivity of the wellbore pressure with respect to the potential changes of any property (e.g. permeability) at any time and location within the reservoir (Appendix A, equation A.1). Figure 7.7 presents the calculated sensitivity coefficients for the homogenous reservoir following the method developed by He (1997). The figure shows that that at the early time (0.06 hr) the TDSCs are centred around the wellbore (Figure 7.7: left) and they will spread over a more extensive area of the reservoir at the later times (0.4 hr) (Figure 7.7: right). The figure highlights the fact that the wellbore pressure always remains sensitive to any potential permeability changes in near wellbore area.

Figure 7.7: The logarithm of the calculated well pressure sensitivity with respect to the cell permeability values at the early time (left) and at the late time (right).The units are in psi/md. This is the results for a homogenous permeability distribution. Note that the co-ordinate is the location of the cell whose permeability is being used in the sensitivity calculation.

Figure 7.8 present the fact that the native draw-down test response is remained intact at the early time when the condensate saturation is nil. However, this will be affected at the late time when the condensate saturation starts to build up and perturbs the effective permeability near the wellbore area.
Chapter 7: Additional Complex Fluid Implications

Figure 7.8: The draw-down well-test behaviour of a gas-condensate system. The early time behaviour is explained by the Kernel function averaging as there is not enough condensate dropout (left). However, the late time behaviour is explained by the combination of the Kernel function and the sensitivity coefficients (right). The Kernel function does not averages out the near well bore permeability while the well pressure changes due to the continuous reduction of gas effective permeability at the late times.

7.6 Interfering effect of the geological and the production parameters combined
gEOLOGY VS. FLUID SIGNATURES

7.6.1 Effect of rate and production time

From this section onward we will refer back to the heterogeneous model as it described in section 7.3 and 7.4. To investigate the effect of draw-down duration and production rate several sensitivities were performed. Figure 7.9 shows the effect of the production rate on the subsequent build-up response. Based on the materials presented in the previous two sections, we can assume that the upward trend arrow shown in this figure (Figure 7.9) reflects the multi-phase fluid effect whilst the diagonal arrow demonstrates the direction of the ramp effect. In this figure (Figure 7.9) the single phase pseudo-pressure function has been normalized by the production rate. It is noted that increasing the production rate, causes a very high pressure drop near the wellbore region, which in turn results in higher liquid accumulation in this area. The shaded area shows the effect of fluid on the response imposed by the higher production rate. At the early times (e.g. $t<10^{-3}$ hr), the multi-phase fluid effect dominates the response. The middle time region (e.g. $10^{-3}<t<10$ hr) in the build-up response has been affected by the interference of the geology and the fluid. In the early middle-times (e.g. $10^{-3}<t<2\times10^{-2}$ hr) the geology effect causes a rapid decrease in the derivative response followed by the ramp signature.
This interfering effect causes two stabilizations dominated by the fluid and geology effects. After $2 \times 10^{-2}$ hrs both the ramp and fluid effects dominate. This leads to a steepened short ramp but in a higher level than the case with low production rate. In the late time region (e.g. $t>10$ hr) the build-up derivative curve has been flattened in the case with higher rate production rate. The higher the production rate the longer the late stabilization and the shorter the ramp effect. Another important observation is the increase of skin factor by the increase of the production rate. This is shown by the separation amount of the pseudo pressure drop and the pseudo derivative curves (Figure 7.9). This difference is relatively higher for the case with higher production rate. It should be noted that as the liquid condensation can affect very large area near the well, the overall derivative curve can remarkably be shifted upward when the production rate has been increased. This is shown in Figure 7.9 by the shaded area within the two pseudo-pressure derivative curves.

The draw-down response can also be affected by increasing the production rate. Figure 7.10 shows that the ramp slope can be further increased in presence of a higher liquid saturation induced by higher production rate. The higher the production rate, the higher the liquid saturation increase and the higher the ramp slope.

![Figure 7.9: The production rate effect on the normalized build-up pseudo-pressure drop and pseudo-pressure derivative responses. The shaded area shows the effect of fluid imposed by higher production rate. The early time unit-slope trend of the derivative curve is due to the effect of Fake Wellbore Storage (FWBS).](image-url)
As noted above, the rate change can significantly affect the whole build-up derivative response. Nonetheless, the early and middle time region seem to be less sensitive to the preceding production duration (Figure 7.11) while the production rate is kept constant (20 MMscf/day). This can be observed by a small separation of the build-up derivative curves for different production times. Figure 7.11 shows that if we decrease the initial draw-down time from 8 days to 4 days or increase it up to 12 days, the subsequent saturation profile and eventually the build-up response is almost unaffected in the early and middle time regions. However, in the late time region the flattening of the derivative curves starts earlier in the case with higher draw-down time. Figure 7.12 is another example that shows that doubling the rate (20 MMscf/day for 4 days) has more impact on the build-up response than doubling the production time (10 MMscf/day for 8 days) while the cumulative production is the same (80 MMscf/day). These examples highlight the interfering effect of the ramp and the composite model imposed by the in situ phase separation.

Figure 7.10: The production rate effect on the normalized draw-down pseudo-pressure drop and pseudo-pressure derivative responses.
7.6.2 Effect of correlation length

Figure 7.13 and Figure 7.14 show the effect of the correlation length on the test response of the base commingled gas-condensate reservoir model. The production rate and the test durations are the same in these two plots. In the early time region, there is a separation between the build-up and the draw-down response curves. This separation is due to the condensate drop-out effect described earlier and is higher for the case with shorter correlation length (Figure 7.14), i.e. it is more pronounced in more heterogeneous case. Moreover, in contrast to the longer correlation length case (Figure 7.13), in which the geological effect prevails during the middle time region, the whole build-up derivative curve of the shorter correlation length test response has been
affected by the liquid dropout (Figure 7.14). This is manifested in the upward shifting of the derivative curve while retaining the ramp signature.

Figure 7.13: The build-up pseudo-pressure drop and pseudo-pressure derivative in the commingled case with the longest correlation length of 750m.

Figure 7.14: The build-up pseudo-pressure drop and pseudo-pressure derivative in the commingled case with the shortest correlation length of 25m.

Figure 7.15 shows the layer-by-layer spatial distribution of the condensate distribution at the end of draw-down period (4 days). The saturation profile for the case with shorter correlation length (Figure 7.15: right) is larger in size and also most of the layers are
affected by the condensate drop-out. In the longer correlation length case (Figure 7.15: left), on the other hand, the liquid drop-out is smaller in size and is limited to few layers of the reservoir due to lower required pressure drop to deliver the same rate.

Figure 7.15: The layer-by-layer saturation profile at the end of the draw-down response (after 4 days of production with the rate of 20 MMscf/day) for the case with the longer correlation length (left) and the case with the shorter correlation length (right).

7.6.3 Effect of $k_V$

Figure 7.16 shows the well-test response of the model with a short correlation length (i.e. 25 m) and an improved vertical permeability ($k_V=k_H$). Comparing this plot with Figure 7.14 reveals that increasing the vertical flow communication (or vertical permeability) between the reservoir layers, causes the ramp effect to disappear from the “native geological” well-test response of the model. More importantly, the liquid drop-out effect on the build-up response has been dramatically reduced, which is reflected in the reduced area between the draw-down and build-up pseudo-pressure derivative curves. This shows that, the fluid heterogeneity effect is more highlighted in the commingled response of the model where the diffusion front is encapsulated within individual layers. Besides, the vertical permeability alleviates the effect of the correlation length in presence of condensate drop-out. This point is revealed by comparing the pseudo-pressure derivative curves of the layer cross-flow cases with a shorter (Figure 7.16: 25 m) and a longer correlation length (Figure 7.17: 750 m), where the condensate effect on the derivative curve has been minimized. In general, the vertical permeability can be seen as a dominant parameter, which can flatten the ramp slope and even reduce the liquid accumulation in near wellbore area.
Chapter 7: Additional Complex Fluid Implications

7.6.4 Stepwise Homogenization

In this exercise, it was attempted to decouple the effect of condensate fluid and geological signature on the transient pseudo-pressure derivative curves. The same heterogeneous model as it was described in section 7.3 will be considered. For this purpose, the model homogenization is implemented that starts with removing the reservoir facies (and their associated permeability and porosity distributions) in sequential steps and replacing the petrophysical properties of each particular facies with a set of constant property values ($k=77$ md and $\phi=0.18$). This will eventually lead to
having a homogeneous reservoir at the end of this process. Figure 7.18 shows a set of build-up pseudo-pressure derivative curves obtained by implementing this procedure.

Figure 7.18: A set of build-up derivative responses during stepwise homogenization of the heterogeneous reservoir model

Figure 7.18 indicates how the reservoir heterogeneities could mask the fluid signature. Curve 1, is the heterogeneous model response in presence of the liquid drop-out. This is the case when the both geology and fluid affect the test response. The ramp response is evident while the fluid effect uplifts the curve particularly at the early times. However, the effect of shale on the response curves seems to be negligible. This is verified by replacing the shale with a uniform equivalent permeability and porosity (curve 2) giving almost the same response as that of the heterogeneous case with shale. Yet, the fine sand facies has a considerable effect on the test response and causes the level of the middle-time derivative response to decrease (curve 3). The derivative response is changed but the ramp effect is still present albeit with a smaller slope. The ramp response disappears when the sand facies is replaced by the uniform properties (curve 4). Therefore, in this case, the geology effect is dramatically reduced and the native condensate behaviour in a homogeneous reservoir dominates the build-up derivative response. Eventually, the coarse sand facies is stripped off and a homogeneous reservoir with pure fluid effect remains (curve 5) that has a signature similar to that of previous
curve. This highlights the minimal impact of the coarse sand facies on the test response. This is an excellent example where the competing effect of the geological heterogeneities and the complex fluid on the well-test response is scrutinized. If the facies are added other way round, the results will not change. That is mainly due to the fact that some facies have higher volumetric proportions in the heterogeneous model which makes those facies to be more connective in the system.

7.7 Seismic modelling in presence of complex fluid system

This section addresses the monitoring of the gas-condensate reservoir model in the presence of the geological complexities (while induce the ramp effect) by time-lapse seismic data. For the seismic modelling study, a conventional well-test design including a single draw-down of 20 MMscf/day for 15 days, which is followed by a shut-in period of the 15 days has been simulated. This is to provide enough time required for the detectable changes to appear on the seismic response (Figure 7.19). The inertia and the positive coupling effects (Jamio lahmad, et al., 2010) have not been considered in this modelling study.

![Figure 7.19: Schematic representation of a synthetic well-test. The base seismic cube is taken before the test (t=0) and the other monitors are taken at the end of the draw-down (t=15 days) and build-up (t=30 days). The build-up 4-D response is calculated from the shut-in time.](image)

7.7.1 Petro-elastic modelling in gas-condensate reservoirs

Batzle and Wang (1992) have proposed some black oil correlations for calculating the compressional velocity and bulk moduli of conventional reservoir fluids (i.e. dead/live oil and dry gas). The application of these correlations to the gas-condensate reservoirs
requires some specific modifications, which mainly affect the related gas phase calculations. To do so, some workflows were created and different approaches were applied to fulfil the required modifications to acquire the proper fluid models. These are accomplished by developing some E300 plug-ins to implement the workflows.

**Black oil approach**

The first approach is based on black-oil modelling. In this approach, the gas-condensate fluid system is simplified to a binary system composed of “wet-gas” and “live-oil” components. The liquid content of the gas condenses at the surface facilities within multi-stage separators and also within the wellbore whenever the pressure falls below dew-point pressure. Through numerical modelling, it was noted that in this studied fluid system (i.e. rich gas-condensate fluid) using the specific gravity of the produced “dry gas”, $\gamma_{DG}$, underestimates the gas density by 40%. This eventually reduces the estimated gas and oil bulk modulus by almost 50% as well. The so-called “recombination method” (Gold et al., 1989) is applied in order to calculate the correct well-stream gas gravity. In this method, the specific gravity of the dry gas and the oil, at different separators are recombined with their associated gas-oil-ratios to reproduce the wet gas specific gravity.

\[
\gamma_w = \frac{\sum R_i \gamma_i + 4602 \gamma_{oil}}{\sum R_i + (133316 \gamma_{oil} / M_{oil})}
\]  

(7.12)

in which, $R_i$ is the gas-oil ratio in scf/STB for each separator, $\gamma_i$ and $\gamma_{oil}$ are the gas and the oil specific gravity at standard condition, and $M_{oil}$ is the condensate oil gravity that can be predicted by the correlation proposed by Gold et al. (1989)

\[
M_{oil} = \frac{5954}{\text{API} - 8.8}
\]  

(7.13)

The studied fluid has a $\gamma_g = 0.85$ and API gravity of 42.05 and the producing gas oil ratio of 6440 scf/STB. This results in a well-stream gravity, $\gamma_w$, of 1.3. The calculated gas gravity should also be corrected for effect of “non-hydrocarbon” contents (Sutton,
The pseudo-critical properties of the gas-condensate are therefore, obtained using Sutton’s method (2005)

\[ T_{pc} = 187 + 330\gamma_w - 71.5\gamma_w^2, R \]  
(7.14)

\[ P_{pc} = 706 - 51.7\gamma_w - 11.1\gamma_w^2, \text{Psi} \]  
(7.15)

These pseudo-critical properties are ultimately used in calculations of gas deviation factor (Dranchuk and Abou-Kassem, 1975) and bulk modulus (Batzle and Wang, 1992). It is noted that the dry gas pseudo-critical properties used in the original Batzle and Wang method underestimate the reduced pressure and temperature by 10% and lead to an extra 1% error in gas deviation factor calculation.

The live oil component, on the other hand contains a considerable amount of gas in solution. The original Batzle and Wang model accounts for solution gas. Therefore, the calculated velocity and bulk modulus of oil are reliable. Figure 7.20 presents the overall workflow for the modified black oil model application in the petro-elastic fluid modelling.

![Figure 7.20: Overall workflow for the modified black oil model application in the petro-elastic fluid modelling](image)

The workflow for the gas phase includes the following steps

- The surface gas and solution gas oil ratios are recombined along with the stock tank oil gravity to obtain the well stream gravity (equation 7.12) at each time step.
- The well stream gravity is used to obtain the pseudo-critical properties
(equations 7.14 and 7.15). This is then used to obtained the pseudo reduced properties ($P_{pr}$ and $T_{pr}$), the gas compressibility factor ($z$), and the gas density at any temperature and pressure for each gridcell.

- The Batzle and Wang procedure is then implemented to estimate the gas bulk modulus and the gas compressional velocity.

And for the oil phase

- The oil density is obtained at any temperature and pressure for each gridcell
- The Standing-Katz method (Standing and Katz, 1942) is then used to obtain the equivalent oil density at surface condition ($\rho_{o,eq}$) by proper pressure and temperature corrections. This density is crude oil density (with all the dissolved solution gas) at standard conditions, that is, 14.7 psia and 60°F.
- The Batzle and Wang procedure is then used to estimate the oil bulk modulus and the oil compressional velocity.

Figure 7.21 and Figure 7.22 are the calculated oil and gas phase bulk moduli and the compressional velocities with production time. The oil bulk modulus and the velocity reduces from about 470 MPa and 950 m/s to about 440 MPa and 910 m/s respectively, while the gas bulk modulus and the velocity reduces from about 155 MPa and 600 m/s to about 100 MPa and 540 m/s respectively.

![Figure 7.21: The changes of fluid bulk moduli with production calculated from the modified black oil method.](image-url)
Chapter 7: Additional Complex Fluid Implications

Figure 7.22: The changes of fluid compressional velocities with production calculated from the modified black oil method.

Compositional approach

This approach is based on the prediction of fluid properties when the fluid composition is known. Unlike the first approach, in which the gas gravity was used to reproduce the pseudo-critical properties, here they are directly calculated from the changing gas phase compositions according to the so-called “SBV” method (Stewart et al., 1959). The pseudo-critical properties are then used to calculate gas density and gas deviation factor. For the oil phase, the pseudo-liquid density at standard condition, \( \rho_{osc} \), requires a multi-stage approach following the modified Standing-Katz’s method (Pederesen et al., 1984). In this approach \( \rho_{osc} \) has been correlated with the density of the \( \text{H}_2\text{S} \) and propane-plus fraction, \( \rho(\text{H}_2\text{S}+\text{C}_3+) \), the weight percent of \( \text{C}_1 \) and \( \text{N}_2 \) in the entire system, \( (\text{m}_{\text{C}_1+\text{N}_2})\text{C}_1+ \), the density and weight percent of ethane in the \( (\text{H}_2\text{S}+\text{C}_2+) \) and the density of \( (\text{H}_2\text{S}+\text{CO}_2+\text{C}_2+) \) mixture. This pseudo-liquid density is then used in calculation of the velocity and predicting the oil bulk modulus (Batzle and Wang, 1992). Figure 7.23 presents the overall workflow for the compositional model application in the fluid modelling.

The workflow for the gas phase includes the following steps

- The composition of the each gridcell at each time step with its pressure and temperature is obtained from the compositional simulation. This is used to calculate the gas molecular weight and the pseudo critical properties from
SBV’s method. The gas density can also be calculated from the real gas law. The Wichert and Aziz’s correlations (1972) should be used to correct for the effect of the non-hydrocarbon components.

- The pseudo reduced properties are used to calculate the gas compressibility factor.
- The Batzle and Wang procedure is then implemented to estimate the gas bulk modulus and the gas compressional velocity.

And for the oil phase

- The composition of the oil phase for each gridcell at each time step with its pressure and temperature is obtained from the compositional simulation.
- The Pederson’s procedure (Pedersen et al., 1984) is applied to calculate the equivalent oil density of the reservoir oil at reservoir condition (obtained from the compositional simulator) at the surface condition.
- The Batzle and Wang procedure is then implemented to estimate the oil bulk modulus and the oil compressional velocity.

Figure 7.23: Overall workflow for the compositional model application in the petro-elastic fluid modelling.
Figure 7.24 and Figure 7.25 present the calculated oil and gas phase bulk moduli and the compressional velocities with production time from the compositional method. The oil bulk modulus and the velocity reduce from about 480 MPa and 952 m/s to about 454 MPa and 916 m/s while the gas bulk modulus and the velocity reduce from about 142 MPa and 582 m/s to about 92 MPa and 523 m/s respectively.

**Figure 7.24:** The changes of fluid bulk moduli with production calculated from the compositional method.

**Figure 7.25:** The changes of fluid compressional velocities with production calculated from the compositional method.
It is important to notice that the effect of composition change can sometimes be hidden by the pressure changes. In the gas-condensate fluid system, it is expected that as the oil gets richer with the heavier components during production therefore the compressional velocity and the bulk modulus should consequently increase with time. However, it was noticed that the pressure could have a dominant effect. Figure 7.26 shows the bulk modulus can attain different trend when the pressure effect is removed. The same comparison can be made on the compressional velocity as well (Figure 7.27).

Figure 7.26: The composition effect on the oil bulk modulus during the draw-down where the oil phase gets richer in heavy components. This causes the increase of the oil bulk modulus. However, this effect can be hidden by the stronger pressure effect.

Figure 7.27: The composition effect on the oil compressional velocity ($v_p$) during the draw-down where the oil phase gets richer in heavy components. This causes the increase of the $v_p$ modulus. However, this effect can be hidden by the stronger pressure effect.
The acoustic properties of fluids from different approaches are summarised in Table 7.2. This is the results for an extended draw-down of 30 days. The results show that the compositional and black-oil models are in a good agreement. However, using dry gas model as it is documented in Batzle and Wang (1992) will result in erroneous values. Specifically gas density and bulk modulus are under-predicted by 40%, and oil bulk modulus by 50%. This will be reflected in the elastic properties of saturated rock, which for our PEM setting can make a difference in impedances of 1%. Depending on the PEM settings and the reservoir production mechanism with a higher rate of fluid composition changes, these differences can be more pronounced.

### Table 7.2: The acoustic properties of fluids and saturated rock from different approaches

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Saturated Rock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ρ_gas (kg/cu.m)</td>
</tr>
<tr>
<td>Batzle-Wang</td>
<td>259.6</td>
</tr>
<tr>
<td>Modified Black-Oil</td>
<td>404.6</td>
</tr>
<tr>
<td>Compositional</td>
<td>401.9</td>
</tr>
</tbody>
</table>

7.7.2 Forward seismic modelling

Having selected the compositional fluid modelling approach and dry frame PEM rock parameters calibrated to a real field the Gassmann’s fluid substitution was implemented. Dry frame stress sensitivity for sands is calculated using MacBeth (2004) model. A 30 HZ Ricker wavelet was then used to generate synthetic seismic volumes at different draw-down and shut-in times.

Figure 7.28 and Figure 7.29 shows a cross-section of saturation changes and associated 4-D seismic signatures during draw-down and build-up periods. The draw-down response (Figure 7.28) indicates a stronger amplitude change due to higher rate of the saturation change (e.g. |ΔSo| = S_max - S_min =0.16) while the build-up response (Figure 7.29) has a smaller saturation change (e.g. |ΔSo| = S_max - S_min =0.07) and subsequently a weaker amplitude change. This is due to the fact that the rate of condensate vaporization during build-up is less than the condensate dropout during draw-down. The small positive oil saturation changes in the flank of the reservoir during the build-up are ascribed to the lateral pressure communication between the neighbouring cells having the extreme high and low quality properties. This is due to high isothermal compressibility and low
hydraulic diffusivity of the gas phase in the medium, which causes a delayed response to the imposed pressure changes. The presence of some numerical artefact is not unlikely as well. However, this does not change the most effectual negative saturation change trend near the wellbore area. A more detailed comparison of the 4-D seismic cross-sections also shows that the 4-D signature of the build-up response is associated with a polarity reversal in near wellbore areas. This is credited to the opposite trends of pressure (and saturation) changes during build-up and draw-down.

Figure 7.28: The vertical cross-section of condensate saturation change (upper) during 15 days of draw-down and its corresponding time lapse seismic section (lower).

Figure 7.29: The vertical cross-section of condensate saturation change (upper) during 15 days of build-up and its corresponding time lapse seismic section (lower).

Furthermore, the lateral change of condensate saturation during build-up and draw-down has also been shown in the RMSΔamplitude map of the 4-D seismic responses (Figure 7.30 and Figure 7.31). Comparing these figures reveals the fact that a higher saturation change during 15 days of the draw-down period (Figure 7.30) produces a stronger spatial 4-D seismic signature than the 15 days of build-up (Figure 7.32). Comparing Figure 7.30 and Figure 7.31 also indicates that there is a “polarity reversal”
near wellbore region where the positive changes of RMS amplitude replace by the negative changes.

Figure 7.32 shows the 4-D response map of build-up computed from the dry-gas method assumption in the original Batzle and Wang’s procedure. This map is remarkably different from the compositional 4-D signature. It shows a particular stronger positive amplitude change near the wellbore area and the polarity reversal is not visible on the map.

Figure 7.30: The 4-D seismic response for 15 days of draw-down. This is based on the compositional method.

Figure 7.31: The 4-D seismic response for 15 days of build-up. This is based on the compositional method.
Figure 7.32: The 4-D seismic response for 15 days of build-up (S30-S15). This is obtained by using the original Batzle and Wang’s procedure which is based on the dry gas assumption.

It is should be noted that the 4-D response is due not only to changes in the fluid but also the rock petro-elastic properties, and these changes are not necessarily linear with pressure. Falahat et al. (2011) showed that the linearity could be attained whenever the pressure and saturation changes are scaled by thickness (or pore volume) of the reservoir that these effects occupy. Therefore, although the fluid effect is clearly mapped near the wellbore areas, the simple differencing could not wholly eliminate the rock effect. This causes some faint footprints of the rock (that controlled by the net: gross) to remain in the 4-D maps as patches of turquoise and yellowish colour.

7.8 Chapter summary

A heterogeneous commingled multi-facies, high net: gross, braided fluvial reservoir was considered. A series of sensitivities were conducted using numerical well-testing approach to evaluate the combined effect of geology and condensate drop-out on the created well-test responses. The results clarify that some geological heterogeneities and production parameters can alter pressure distribution and condensate saturation, and mask the native model well-test signatures. The stepwise stripping of the reservoir heterogeneity was also studied to show the impact of each facies on the build-up and draw-down transient pressure response. The time-dependent sensitivity coefficients were calculated to show that the draw-down test is very sensitive to effective permeability in near wellbore areas, where condensate is prone to build up with time. In the build-up, on the other hand, the condensate saturation is almost invariant with time and affects the early time region. The application of compositional simulation and seismic modelling in gas-condensate reservoirs are also studied. In such reservoirs, the fluid composition is continuously changing. Therefore, the widely used Batzle and Wang approach should be
modified for the gas-condensate systems to properly take the composition changes into account in the petro-elastic modelling. This is achieved by developing some new workflows using a compositional or a modified black-oil techniques. The modified petro-elastic model is convolved with a wavelet and the time-lapse seismic amplitude responses during draw-down and build-up are generated. Eventually, the 4-D seismic maps from different petro-elastic fluid models are compared with each other and the effects of geology and condensate banking are highlighted.
Chapter 8

Conclusions and Future Work

The multi-domain data integration in well-test interpretation is the central endeavour of this thesis. The importance of the data integration is well-described by John Owens of Maersk in the 2011 SPE/EAGE Agora workshop on *Effective Reservoir Models Leading to Business Solutions*, who said “If data integration is expensive what is the price of its ignorance?!” More specifically, this thesis confers a particular well-test response recognized in commingled fluvial environments (i.e. the Ramp effect). Well-testing, geological data, and time-lapse seismic data are utilized to address the characterisation and non-uniqueness in modelling of such reservoirs. This leads toward a better understanding and monitoring of the complex fluvial reservoirs showing the ramp effect. The thesis objectives have been largely met, and some key conclusions are drawn which are presented in the following sections.

8.1 Conclusions

8.1.1 Illumination and characterisation of Ramp Effect

The ramp effect is a newly-realized geological signature that is reflected in the well-test responses of particular commingled fluvial reservoirs. The ramp effect is manifested in draw-down or build-up responses as a monotonic increase of pressure derivative over at least one logarithmic cycle which may be confused with fault effect. The ramp should eventually ‘plateau off’ at the effective $k \times h$ of the system. The numerical simulations were employed as robust tools to describe the ramp effect. This is the main theme of Chapter 2 and 3 from which the following conclusions can be made.
1. The generalised Ramp Effect model is illustrated fully for the first time in this thesis. The ramp effect is classified as a new member of a well-testing family in multi-layered reservoirs that include the Geoskin, Geochoke reservoirs. This family of well-testing responses can be described by a series of Geotype Curves which emphasise the ‘continuum’ nature of the responses.

2. The ramp effect is expected to be seen in multi-layered fluvial environments with zero (or extremely low) effective vertical permeability while lateral $k\times h$ (or connectivity) is continuously reducing.

3. Lateral connectivity of facies and the impact of vertical permeability (i.e., vertical connectivity) are of the fundamental considerations.

4. The vertical permeability has a significant effect on the short- and the long-term dynamic response of the system. This is where the diffusion front is encapsulated within each layer and propagates with different speeds within them. This can make a partial depletion phenomenon where the reservoir layers will be sequentially depleted.

5. Using numerical methods and employing two-point statistics (i.e. pixel based modelling), the ramp slope and the level of the pressure derivative stabilization (i.e. effective $k\times h$ of the system) are found to be dependent on several factors. These include well location, geostatistical and statistical distribution parameters, (i.e. probability distribution function, covariance function, correlation lengths) and flow dimension.

- The ramp effect should occur if the permeability in the well location is sampled from the high end values of the probability distribution function.

- Having a proper well location for emergence of ramp effect suits for situation where increasing the contrast in spatially distributed permeability values (i.e. standard deviation of the probability distribution) causes the derivative slope in Log-Log plot to increase from zero (i.e. very low standard deviations) to a $\frac{1}{2}$ slope in larger standard deviation (i.e. $\sigma_{ln(k)}>1$). This can even reach a unit slope in extremely large contrasts (e.g. $\sigma_{ln(k)}>3$).

- Reducing the vertical flow communication between the reservoir layers increases the ramp slope.
• Increase of $l_H/l_V$ (i.e. horizontal to vertical correlation lengths) results in a longer initial stabilization of the pressure derivative curve. This is then followed by the ramp effect and a secondary plateau. In random permeability distributions (i.e. pure nugget effect), the pressure derivative is quickly stabilized over the effective $k\times h$ of the system.

• The effective permeability of the system is the geometric mean of 2-D and uncorrelated (i.e. random) 3-D systems with layer cross-flow. A lower effective permeability than the geometric average, which is biased toward harmonic mean of permeability, is likely for the commingled flow conditions.

• The Gaussian covariance function provides a smoother facies distribution which results in a longer initial well-test derivative stabilization compared with the exponential covariance function.

8.1.2 Interpretation of Ramp Effect in real-case examples

The ramp effect is seen in many real-case examples. In Chapter 4, three well-test examples, from various fluvial environments are presented. These cases range from a high-energy braided fluvial stream to low energy meandering and anastomosing systems. The extremely low vertical communication and lateral reduction of permeability (or connectivity) are the major sharing interfaces between these fluvial environments. For the first case (i.e. high energy commingled braided fluvial stream), no geological model is built and the ramp is attributed to lateral reduction of the permeability away from wellbore. However, for the second two cases, the geoengineering workflow is used to integrate all available data and match the dynamic response of the model with real responses. A very limited conditioning data were available. In these situations, the application of two-point statistics is erroneous. Therefore, the Multi-Point Statistics (MPS) modelling is employed to better capture the lateral facies connectivity in the presence of sparse hard data. This is achieved by creating some complex training images from modern river analogues. The main conclusions of this part of the research are listed below:
1. The ramp effect is present in variety of commingled fluvial environment and appears whenever the lateral connectivity (or \( k \times h \)) reducing.

2. In the high-energy commingled braided fluvial case (i.e. Well F in Wytch Fram Field) the continuous increase of the pressure derivative curve is attributed to lateral reduction of the permeability in distal area of the braided stream where derivative curve is biased toward harmonic average of the permeability. No-geological well-testing was made.

3. In the meandering case (i.e. an Indian Field well-test example), the ramp effect is postulated to be generated from the cross-flow caused between point bars in a high net:gross, laterally avulsing, meandering fluvial reservoir. This is proved by average flow-regime captured by employing MPS modelling technique which also helps in selecting a proper analytical model where the elongated sand patches are communicating within a low (but not zero) permeability background.

4. In the anastomosing case (i.e. Field X) the ramp effect is ascribed to the drastic lateral (or volumetric) reduction of connected high permeable facies. The results showed that in this sand-shale environment the facies distribution is the main controlling parameter of connectivity and has a stronger effect on the test response rather than permeability distribution (within the accepted variation range). The accurate spatial connectivity is modelled by a sophisticated multi-point statistics utilizing a multilayered training image. The final match is obtained by generating multiple facies and petrophysical realizations and applying a hybridization algorithm to combine the models that matches particular portion of the well-test response.

5. The ramp effect require an integration of static and dynamic data following a geoengineering approach to aid identification.

8.1.3 Reducing the non-uniqueness in well-test interpretation using the illuminating power of the frequently acquired time-lapse seismic data

There are fluvial reservoirs that are described with posited sub-seismic faults that turn out on production to be well-communicating and not faulted at all. The main reservoir structures such as reservoir top and regional faults can be mapped efficiently by 3-D seismic interpretation. These can be used to constrain well-test interpretation (Zheng et al., 2003) where relatively discrete sandbodies are present. However, smaller scale faults, stratigraphic discontinuities and permeability baffles show their effects in the
dynamic behaviour of the reservoir and often their average properties can be mapped via 4-D seismic signals. This is illustrated through synthetic seismic modelling in two distinct fluvial channelised environments. This is the main subject of Chapter 5 and 6 from which the following conclusions are made

1. The expansion of diffusion front in well-testing and propagation of seismic waves have different physics behind them. They investigate the porous medium in different ways. However, the rock physics models, in particular, make a bridge between these domains. The rock physic models relate the pressure and saturation to elastic moduli of the media, which in turn affect the seismic amplitude of the recorded traces.

2. The ramp effect can be confused with other linear flow responses (e.g., parallel faults, natural or artificial fractures). This creates some uncertainty in well-test modelling and interpretation. This can occur where a multi radial composite model is mistakenly used to represent the linear flow signature on the test responses.

3. The 4-D seismic data are proved to have a significant effect in reducing the non-uniqueness modelling by illuminating the lateral heterogeneities that are previously hidden in the system and are not recognizable by 3-D seismic data. Moreover, the high permeable patches can sometimes be interpreted from the 4-D signals that help in numerical well-test modelling.

4. The extended well-testing provide a favourable condition for running the time-lapse seismic surveys and integrating the seismic interpretation in well-test modelling.

5. The time-lapse seismic data are usually expressed in terms of 2-D maps. In these cases, the signal maps should always be integrated with the other sources of data to detect the potential reservoir layering possibilities. Ignoring the layering nature of the reservoir (i.e. ignoring the vertical flow communication) affects the long-term performance of the reservoir model. The layering nature of the system may be detected by considering the volumetric representation of time-lapse signals.

6. A method is developed to extract a permeability attribute map from the pressure inverted time-lapse seismic amplitudes. This method needs further improvements.
7. The decisive issue with incorporation of time-lapse seismic data into well-test interpretation is the relatively short duration of typical well-tests. This may impose some practical challenges on acquiring the short-interval seismic data and recognizing the 4-D signals. However, the extended well-testing (e.g. well-test example of Field X when the pressure has significantly changed by 5000 psi) can remarkably reduce this issue. On the other hand, the permanent seismic technologies also provide a real hope for acquiring the repeated seismic surveys and interpretation.

8.1.4 Complex fluid effects on seismic and well-test responses of the modelled fluvial reservoir

The fluvial reservoirs are amongst the most common reservoirs in the North Sea where the gas condensate fluid system are not uncommon (e.g. Heron field (Mckie et al., 2010), Frankin filed (Suiter et al., 2005) and Marnock filed (Fisher and Mudge, 1998)). Therefore, the implications of such complex fluid systems on the well-test and time lapse seismic behaviour are studied in Chapter 7.

The gas condensate vs. geology - who wins?

A heterogeneous commingled multi-facies braided fluvial reservoir is considered. A series of sensitivities are conducted using numerical well-test approach to evaluate the combined effect of geology and condensate drop-out on the created well-test responses. The main conclusions of this study are as follows

1. The production rate has a pronounced impact on the subsequent build-up curve whilst duration of period has minimal impact on this response.
2. The vertical permeability is a significant controlling factor on the condensate formation and dramatically changes the signature of the subsequent derivative build-up response.
3. The ramp effect is affected by the condensate formation. This interfering phenomenon is reflected on the derivative curves and is magnified in the presence of the shorter correlation lengths, the lower vertical communications and the higher production rates.
4. The build-up response is strongly affected by the condensate drop-out. The drawdown response also shows a deflection from the native geological response...
due to condensate dropout at late time period. A moving average window function (Kernel function) can be used to explain the build-up behaviour of the gas condensate reservoir, while the time-dependent sensitivity coefficients can be used for the explanation of the drawdown behaviour. These findings highlight that the geological heterogeneity is an important factor and its effect on the pressure derivative response should not be underestimated.

**Time lapse seismic modelling and application in gas condensate reservoirs**

The application of compositional seismic modelling in a complex gas/condensate reservoir is studied. In such reservoirs the fluid composition is continuously changing. The complementary information extracted from interpretation of well-testing and 4-D seismic interpretation give useful information for reservoir monitoring purposes.

1. This work highlights the application of compositional simulation in reservoir geophysics. The implications of the phase composition changes are implemented for the first time in this thesis.
2. The widely used Batzle and Wang equations (Batzle and Wang, 1992) for predicting the fluid elastic moduli are modified for the gas-condensate systems. These are achieved by compositional or modified black-oil techniques which mainly affect the gas phase calculations.
3. For the short time draw-down well-test responses, the composition and pressure changes have opposite effect on velocity and bulk modulus of the oil phase. However, the stronger pressure effects can hide the composition effects.
4. The synthetic time lapse seismic modelling is used to recognize the condensate banking around the well. The 4-D signals monitored during the build-up show a polarity reversal compared with the signature recorded during draw-down period. Moreover, the effect of saturation changes during each particular flow period can be realized by implementing the frequent time-lapse seismic surveys.

**8.2 Future research and recommendations**

1. The hybridization algorithm (Chapter 3) can be extended in an automated manner. In this way, the multiple generated models can be mixed to provide the required matching. The geological representativeness of the final model is the main concern. Therefore, this
should be considered as an assisted short-time history matching procedure where the reservoir engineer and geologist can dynamically monitor the changes.

2. In the case of high resolution seismic data (e.g. vertical seismic profiling) it might be possible to extract the 4-D seismic signals as 3-D geobodies to be integrated with the other sources of data. This could help in reservoir modelling and more constrained geological well-testing.

3. The permeability extraction algorithm from time lapse pressure maps (Chapter 6) can be further improved to reduce the numerical instabilities in the algorithm. The algorithm can be extended to include the saturation changes as well. In this way the time lapse saturation and pressure changes can provide an indication of permeability.

4. The positive coupling and inertia should be considered in well-test and seismic responses of gas-condensate reservoir. These are two important phenomena that significantly affect the gas-condensate reservoir performance. They have different functional implications on altering the productivity of the gas-condensate well. The unfavourable inertia effect happens in the gas reservoir when the high velocity of gas near the wellbore causes an extra pressure drop or skin near the wellbore. However, the favourable positive coupling effect, which is also a high gas velocity phenomenon, is associated with a low interfacial tension between the liquid and gas. The positive coupling effect modifies the shape of the original relative permeability curves towards the miscible curves and can reduce the near wellbore condensate blockage.

5. The time-lapse log data could also be considered as important sources of time dependent information. Although, they have a small lateral coverage, they seem feasible in current Industry practice. In particular case of gas-condensate reservoir they might provide invaluable information about the condensate blockage in immediate vicinity of the well and reduce the non-uniqueness in well-test interpretation and engineering decisions.
Appendix A

Sensitivity Coefficients

The sensitivity coefficients relate the well bore pressure at an observation point to the reservoir properties (e.g. permeability or porosity) at a particular gridblock in the reservoir model. In other words, the sensitivity coefficients indicate how strongly a change in that property would affect the pressure at the observation point (Chu et al., 1995). There are several ways to find the sensitivity coefficients

1. Direct or Influence Coefficient Method (Jahns, 1966)
2. Gradient Simulator Method (Anterion et al., 1989)
3. Jacquard or Variational Method (Jacquard, 1965)
5. Modified Generalised Pulse-Spectrum Technique (Chu et al., 1995)
7. Modified Carter et al. or He’s method (1997)

A.1 He’s method of estimating the time-dependent sensitivity coefficient
The method used in here is the method modified Carter et al.’s method (1974) that has been presented by He (1997) in his thesis. A comprehensive set of C# codes were developed to implement this algorithm. According to this method, the sensitivity of a well pressure at a specific time $t_n$, respect to a property (e.g. $k_x$, at location $l, m, n$) can be written as follows
\[ \frac{\partial P_{nf}^n}{\partial \alpha} = \frac{\sum_{k=l_1}^{l_2} (WI)_k \frac{\partial P_{i,j,k}^n}{\partial \alpha} + \sum_{k=l_1}^{l_2} (P_{i,j,k}^n - P_{nf}^n) \frac{\partial}{\partial \alpha} (WI)_k}{\sum_{k=l_1}^{l_2} (WI)_k} \]  

(A.1)

which can be summarized as

\[ \frac{\partial P_{nf}^n}{\partial \alpha} = \text{term1 + term2} \]  

(A.2)

in which, \( P_{i,j,k}^n \) is the gridblock pressure of any cell intersected by well at time \( t_n \), \( P_{nf}^n \) is the corresponding well pressure at time \( t_n \), \( k=l_1 \) to \( k=l_n \) are the perforated gridblocks in \( z \) direction, \( (WI)_k \) is the well index of the cells perforated by the well, and \( \frac{\partial P_{i,j,k}^n}{\partial \alpha} \) is the gridblock pressure sensitivity of the well intersected cell respect to any reservoir property, \( \alpha \), in any location in the model at time \( t_n \).

**Term1**: Term 1 is related to gridblock pressure sensitivities. If we take \( \alpha \) to be \( k_{x,l,m,n}(x-) \) permeability at location \( l, m, n \) then the first term reduces to

\[ \sum_{k=l_1}^{l_2} (WI)_k \frac{\partial P_{i,j,k}^n}{\partial \alpha} = \frac{C_1 \Delta y_m \Delta z_n \int_0^{z_{n/2}} \int_0^{x_{l/2}} \left( \frac{\partial P_d (x,y,z,s)}{\partial x} \times \frac{\partial}{\partial x} \frac{\partial}{\partial (t_n-s)} \sum_{k=l_1}^{l_2} (WI)_k P_{rd} (x,y,z,n,t_n-s) \right) dx ds}{\sum_{k=l_1}^{l_2} (WI)_k} = \frac{C_1 \Delta y_m \Delta z_n \int_0^{z_{n/2}} \int_0^{x_{l/2}} \left( \frac{\partial P_d (x,y,z,s)}{\partial x} \times \frac{\partial}{\partial x} \frac{\partial \hat{P}_{rd} (x,y,z,n,t_n-s)}{\partial (t_n-s)} \right) dx ds}{\sum_{k=l_1}^{l_2} (WI)_k} \]  

(A.3)

where

\[ \hat{P}_{rd} (x,y,z,t) = \frac{\sum_{k=l_1}^{l_2} (WI)_k P_{rd} (x,y,z,t)}{\sum_{k=l_1}^{l_2} (WI)_k} \]  

(A.4)

215
Appendix A

\[
(WI)_k = \frac{2\pi C_1 \Delta z_k \sqrt{k_{x,j,j,k} k_{y,j,j,k}}}{\mu (\ln \left( \frac{r_{ok}}{r_w} \right) + S_k)}
\]

(A.5)

\[
r_{ok} = \frac{0.28073 \Delta x_i \sqrt{1 + (\Delta y_{j,j})^2 / k_{x,j,j,k} \Delta x_i / k_{y,j,j,k}}}{1 + \sqrt{k_{x,j,j,k} / k_{y,j,j,k}}}
\]

(A.6)

in which, \( C_1 = 1.127 \times 10^{-3} \), \( \Delta x, \Delta y \) and \( \Delta z \) are the cell dimensions in x, y and z in “ft”, \( WI \) is the well index in “RB/psi”, \( S_k \) is the skin factor in the cell penetrated by well at \( z = k \), \( k_x, k_y \) and \( k_z \) are the x, y and z permeabilities in “md”, \( s \) is the time variable \( (s = t_1, t_2, t_3, ..., t_n) \) in “day”, \( t_n \) is the time at which the sensitivity coefficients are intended, and \( P_d \) is the cell pressure drop in “psi”, which corresponds to the pressure solution of the reservoir model producing with rate \( q \) (STBO/day). Moreover, \( \hat{P}_{nd} \) is the pressure response by a unit source corresponding to a well at location \( (x_i, y_j) \) having different sink terms at each perforation. The sink terms are defined as follows

\[
Sink(i, j, k) = \frac{(WI)_k \delta(z - z_k)}{\sum_{k=1}^{l_k} (WI)_k}
\]

(A.7)

in which, \( \delta(z - z_k) \) is the Dirac Delta function in “ft^{-1}”. It should be noted that the integral of the sink terms over the wellbore is equal to unity. The Eclipse 100 reservoir simulator was used to estimated \( \hat{P}_{nd} \) and \( P_d \) required for implementing this algorithm.

The discretized from of equation (A.3) can be expanded as follows
term 1 =
\[-\frac{C_i}{\mu} \Delta y_n \Delta x_n \sum_{p=1}^{n} \frac{t_p - t_{p-1}}{2} \left\{ \begin{array}{c}
\left[ \frac{\partial P_d}{\partial x} \left(x_{i-1/2}, y_m, z_n, t_{p-1} \right) \right] \\
+ \left[ \frac{\partial P_d}{\partial x} \left(x_{i+1/2}, y_m, z_n, t_{p} \right) \right] \\
\frac{\partial \hat{P}_{nd}}{\partial (t_{n-s})} \left(x_{i-1/2}, y_m, z_n, t_{p-1} \right) \\
\frac{\partial \hat{P}_{nd}}{\partial (t_{n-s})} \left(x_{i+1/2}, y_m, z_n, t_{p} \right) \end{array} \right\} \]

(A.8)

This equation requires to calculate some derivatives as \( \frac{\partial P_d}{\partial x} \), \( \frac{\partial P_d}{\partial x} \), or \( \frac{\partial \hat{P}_{nd}}{\partial (t_{n-s})} \). Since the x derivative is discontinuous at cell interface, therefore we have to introduce the one-sided derivatives. Thus, for \( \frac{\partial P_d}{\partial x} \), for example, we have

\[ \frac{\partial P_d}{\partial x} \left(x_{i+1/2} \right) = \frac{2k_i}{k_i \Delta x_i + k_{i+1} \Delta x_i} \left[ P_{d,j+1} - P_{d,j} \right] \]  

(A.9)

\[ \frac{\partial P_d}{\partial x} \left(x_{i-1/2} \right) = \frac{2k_{i-1}}{k_{i-1} \Delta x_i + k_i \Delta x_{i-1}} \left[ P_{d,j} - P_{d,j-1} \right] \]  

(A.10)

The other derivatives will take the same form as equations (A.9) and (A.10). Still, there are some other terms in equation (A.8) that are required to be calculated. This includes \( \hat{P}_{nd} \left(x_i, y_m, z_n, t - s \right) \) for instance, in which s is representing a simulation time. However, we do not compute pressures at all times \( t_{n-s} \). Therefore, we should look for some interpolation methods. The method presented by (He, 1997) was a liner interpolation procedure. Therefore, a particular type of spline interpolation (i.e. Akima’s spline (Akima, 1970)) was also implemented here to estimate the required pressure values at \( t_{n-s} \) times (Figure A.1). In contrast to general cubic splines, Akima’s spline does not show the oscillation in the neighbourhood of an outlier (Anatolyevich, 2009). Using the spline interpolation we can simply make differentiation at any specified point within the
interval. This is implemented using a C# numerical library provided by Visual Numerics (2007).

Term 2: The term 2 of equation (A.1) involves the sensitivity of well indices \((WI)_k\) to reservoir parameters and skin factor. The well index of the penetrated cell by the vertical well, only depends on the \(k_x, k_y\) and skin factor of the cell. Therefore,

\[
\frac{\partial (WI)_k}{\partial \varphi_{r,m,n}} = 0 \quad (A.11)
\]

for all cells, and

\[
\frac{\partial (WI)_k}{\partial S_r} = 0 \quad (A.12)
\]

if \((r \neq k)\). However, for the cells penetrated by the well, where \(r = k\)

\[
\frac{\partial (WI)_k}{\partial S_k} = \frac{-(WI)_k}{\ln \left( \frac{r_{ik}}{r_w} \right) + s_k} \quad (A.13)
\]

and also,
\[
\frac{\partial (W_I)_k}{\partial k_{x,l,m,n}} = \frac{\partial (W_I)_k}{\partial k_{y,l,m,n}} = \frac{\partial (W_I)_k}{\partial k_{z,l,m,n}} = 0
\]  \hspace{1cm} (A.14)

for all gridblocks unless \((x_i, y_j, z_k)\), which are the penetrated cells by the well. However for the cell penetrated by the well,

\[
\frac{\partial (W_I)_k}{\partial k_{x,l,m,n}} = \frac{C_i \Delta z_k}{2 \mu \left( \ln \left( \frac{r_{ok}}{r_w} \right) + s_k \right)} \\
\times \left[ \frac{k_{x,i,j,k}}{k_{x,i,j,k}} \ln \left( \frac{r_{ok}}{r_w} \right) + s_k \right] \times \left( \frac{\Delta v_j^2 \sqrt{k_{x,i,j,k}k_{y,i,j,k}}}{k_{y,i,j,k} \Delta v_i^2 + k_{x,i,j,k} \Delta v_j^2} - \frac{k_{y,i,j,k}}{k_{y,i,j,k} + \sqrt{k_{y,i,j,k}}} \right)
\]  \hspace{1cm} (A.15)

and similarly,

\[
\frac{\partial (W_I)_k}{\partial k_{y,l,m,n}} = \frac{C_i \Delta z_k}{2 \mu \left( \ln \left( \frac{r_{ok}}{r_w} \right) + s_k \right)} \\
\times \left[ \frac{k_{x,i,j,k}}{k_{x,i,j,k}} \ln \left( \frac{r_{ok}}{r_w} \right) + s_k \right] \times \left( \frac{\Delta v_j^2 \sqrt{k_{x,i,j,k}k_{y,i,j,k}}}{k_{y,i,j,k} \Delta v_i^2 + k_{x,i,j,k} \Delta v_j^2} - \frac{k_{y,i,j,k}}{k_{y,i,j,k} + \sqrt{k_{y,i,j,k}}} \right)
\]  \hspace{1cm} (A.16)

The sensitivity coefficients for other reservoir parameters (e.g. \(k_y\), \(k_z\) and porosity) can be obtained by the same procedure as followed for \(k_x\) (see He, 1997).
Appendix B

Petro-Elastic modelling

Implementing a simple modelling approach by assuming isotropic and elastic rock materials leads to a favourable situation that the density and only two elastic moduli (i.e. shear and (adiabatic) bulk moduli) are sufficient to describe the elastic behaviour of the rock under sound wave propagation. The bulk and the shear moduli demonstrate the resistance of a medium to a change in confining stress or shear stress respectively. The elastic properties of a rock determine its P-wave velocity and S-wave velocity, both of which can be measured in the field. The P-wave velocity is determined by all three elastic properties while the S-wave velocity is determined only from the density and the shear modulus.

B.1 Gassman’s substitution theory

The elastic moduli of the saturated rocks are comprised of fluid, mineral, and dry rock elastic moduli which generally depend on volume fraction, density and geometric details of the comprising phases (Oldenziel, 2003). The Gassman’s substitution method (Gassmann, 1951) is the widely used equation for petro-elastic modelling of the saturated rocks that predicts how the rock moduli change with a change of pore fluids (Avseth et al., 2005). The Gassmann’s equation implements a spherical model to describe the organisation of the lithological components in a rock sample (Veeken, 2007). The Gassmann’s equation can be written as follows
\[ K_{\text{sat}} = K_{\text{dry}} \left( \frac{\sigma_{\text{eff}}}{1 - \frac{\mu_{\text{dry}}}{\mu_{\text{sat}}}} \left( \frac{1 - \frac{\mu_{\text{dry}}}{\mu_{\text{sat}}}}{\sigma_{\text{eff}} K_{m}} \right)^2 \right) \]  

\[ \mu_{\text{sat}} = \mu_{\text{dry}} \]  

in which:

- \( K_{\text{sat}} \) = the saturated bulk modulus
- \( K_{m} \) = the bulk modulus of mineral matrix
- \( K_{f} \) = the bulk modulus of pore fluid
- \( K_{\text{dry}} \) = the bulk modulus of the dry frame (porous rock frame)
- \( \varphi \) = porosity
- \( P_{\text{eff}} \) = average fluid pressure
- \( T \) = reservoir temperature
- \( \sigma_{\text{eff}} \) = effective stress
- \( C \) = a vector containing API gravity, salinity and solution gas-oil ratio
- \( \mu_{\text{dry}} \) = dry rock shear modulus
- \( \mu_{\text{sat}} \) = saturated rock shear modulus

The Gassmann’s equation is not a generalised equation and is valid only under the certain assumptions as follows:

- The reservoir rock should be macroscopically isotropic and homogeneous (The Gassmann’s equation should be modified for any mixed minerals; this can be achieved by application of the effective moduli concepts. Brown and Korringa (1975) and Brown and Gurevich (2004) also discussed the a set of modifications for anisotropic conditions).
- Hooke’s law holds (i.e. linear proportionality of the stress and strain).
- The rock’s pores are interconnected and there are no isolated pore structures and micro-fractures in the rocks.
- Fluid pressure is uniform in the pore space
- The rock and fluid system is undrained.
• The filling fluid has low viscosity (This violates the Gassmann’s equation applicability in heavy oil reservoirs).

• There is no coupling between grains and the fluids (i.e. in the low frequencies of wave propagation the induced pore pressure is equilibrated throughout the pore space during the seismic time period).

• There is no physical and chemical interaction between the rock and the fluids.

The Gassmann's relations are usually appropriate for 3-D surface seismic frequencies (< 100 HZ) which fulfills the low frequency condition (Avseth et al., 2005; Mavko et al., 2009). However, this might not be the general case for the high frequency log data ($10^4$ HZ). This is due to the dispersive nature of the wave in the high frequencies (i.e. the velocity increase with frequency) which leads to the pore pressure not to remain totally relaxed (Batzle et al., 2001).

B.2 Pore fluid elastic properties

The equations to calculate the seismic properties of reservoir fluid were presented by Batzle and Wang (1992) that relates those properties to the pressure, the temperature and the reservoir properties of the fluid. The corresponding set of equations are briefly summarized in Eclipse reservoir simulation mammal as well (Schlumberger, 2009a).

B.2.1 Elastic properties of oil

The bulk modulus of oil is dependent upon API gravity (e.g. the oil API may be about 5 for very heavy oils and about 100 for light condensates) and the gas-oil ratio. This is given in KPa and is represented by

$$K_o = \rho_o \times v_o^2$$  \hspace{1cm} (B.3)

where, $\rho_o$ is oil density in “g/cc” and $v_o$ is the velocity of sound in oil. Wang et al. (1990) and Wang (1988), based on several laboratory measurements, showed that the sonic velocity in the dead oil reservoir fluid (m/s) is given by be following correlation

$$v_o = 2096 \times \sqrt{\frac{\rho_{o,R}}{2.6 - \rho_{o,R}}} - (3.7 \times T) + (4.64 \times P) + 0.0115 \times \left\{ 4.12 \frac{1.08}{\sqrt{\rho_{o,R}}} - 1 \right\} \times T \times P$$ \hspace{1cm} (B.4)
where, $T$ is the temperature in \(^{\circ}\text{C}\), $P$ is the pressure in \(\text{MPa}\), $\rho_{o,R}$ is the density of oil in \(\text{g/cm}^3\) at reference conditions (15.6\(^{\circ}\text{C}\) and atmospheric pressure).

The oil density at reservoir condition, $\rho_o$, can be calculated by applying the pressure and temperature correction to the oil density at the tank conditions, $\rho_{o,R}$. The pressure correction is

$$\rho_{o,p} = \rho_{o,R} + \left(0.00277 P - 1.71 \times 10^{-7} P^3\right)\left(\rho_{o,R} - 1.15\right)^2 + 3.49 \times 10^{-4} P$$ \hspace{1cm} (B.5)

and then after correcting for temperature we have

$$\rho_o = \rho_{o,p} / \left(0.972 + 3.81 \times 10^{-4} (T + 17.78)^{1.175}\right)$$ \hspace{1cm} (B.6)

For an oil with constant composition, the pressure and temperature effect are largely independent. The pressure dependence is fairly small but the temperature change is rather large. For the live-oil cases the dissolved gas can reduce the fluid velocities. This could change the reservoir reflection coefficients by more than a factor of two (Batzle and Wang, 1992). For the live-oil fluid the solution gas represents the gas components of the fluid. The only modification to the live oil velocity correlation is to replace $\rho_{o,R}$ by the pseudo-density, $\rho^*$, defined as follows

$$\rho^* = \frac{\rho_{o,R}}{B_o} \left(1 + 0.001 \times R_S\right)^{-1}$$ \hspace{1cm} (B.7)

in which, $B_o$ is the oil formation volume factor (Lit/Lit) and $R_S$ (Lit/Lit) is the maximum solution gas at bubble point pressure. $R_S$ is a function of pressure, temperature and the fluid composition and is defined as follows (Standing, 1962)

$$R_S = 0.02123 G \left(P \times \exp\left[0.02878 \text{API} - 0.00377 T\right]\right)^{1.205}$$ \hspace{1cm} (B.8)

In the reservoir simulation application these steps may be reversed since the fluid pressure and the “reservoir density” are obtained as solution vectors provided by the
simulator at various time steps. Therefore, by applying the temperature correction to the reservoir oil density one can obtain

\[
\rho_{o,p} = \rho_o \times \left(0.972 + 3.81 \times 10^{-4} (T + 17.78)^{1.175}\right)
\]  

and then using the pressure correction, the reference density of the live-oil, \(\rho_{o,R}\), can be eventually estimated as a solution to the following quadratic equation

\[
\rho_{o,p} = \rho_{o,R} + \left(0.00277P - 1.71 \times 10^{-7} P^3\right)\left(\rho_{o,R} - 1.15\right)^2 + 3.49 \times 10^{-4} P
\]  

However, this equation has two solutions, one of which will usually be a negative value and is discarded. Figure B.1 shows the calculated bulk modulus of sound in different oil system as a function of pressure, temperature and composition. The figure (Figure B.1) shows that the bulk modulus of the oil reduces by increasing of the reservoir temperature, reducing of reservoir pressure or the specific gravity. The bulk modulus of more complex fluid systems (e.g. gas-condensate) is discussed in details in Chapter 7. In such systems, the composition of the fluid changes as the pressure reduces below the dew point pressure (retrograde condensation) and the oil phase condens es out of the gas phase.

![Figure B.1: The bulk modulus of oil as a function of temperature, pressure, and composition (Batzle and Wang, 1992)](image-url)
**B.2.2 Elastic properties of water**

For brines, the salinity has a great impact on the bulk modulus. The bulk modulus of water is given (in MPa) by

\[
K_w = \rho_w \times v_w^2
\]  
\hspace{1cm} \text{(B.11)}

where, \(v_w\) is the velocity of sound waves in water (in m/s) and is calculated as follows

\[
v_w = v_{pw} + C_s S + \left(780 - 10P + 0.16P^2\right)S^{1.5} - 1820 \times S^2
\]  
\hspace{1cm} \text{(B.12)}

\[
C_s = 1170 - 9.6T + 0.055T^2 - 8.5 \times 10^{-5}T^3 + 2.6P - 0.0029TP - 0.0476P^2
\]  
\hspace{1cm} \text{(B.13)}

in which, \(S\) is the salinity of the water (in ppm/10\(^6\), that is, as a fraction of 1), \(P\) is the pressure in MPa., \(T\) is temperature in °C and \(v_{pw}\) is the velocity of sound in pure water (in m/s) given by

\[
v_{pw} = \sum_{i=0}^{4} \left(\sum_{j=0}^{3} w_{ij} T^i P^j\right)
\]  
\hspace{1cm} \text{(B.14)}

The included constant \(w_{ij}\) coefficients are listed in Table B.1. It should be noted that the bulk modulus of brine decreases linearly with the increase of gas in solution (Mavko et al., 2009).

**Table B.1: The bulk modulus of oil as a function of temperature, pressure, and composition (Batzle and Wang, 1992)**

| \(w_{00}\) | 1402.85 |
| \(w_{02}\) | 3.437 \times 10^{-3} |
| \(w_{10}\) | 4.871 |
| \(w_{12}\) | 1.739 \times 10^{-4} |
| \(w_{20}\) | -0.04783 |
| \(w_{22}\) | -2.135 \times 10^{-8} |
| \(w_{30}\) | 1.487 \times 10^{-4} |
| \(w_{32}\) | -1.455 \times 10^{-8} |
| \(w_{40}\) | -2.197 \times 10^{-11} |
| \(w_{42}\) | 5.230 \times 10^{-11} |
| \(w_{01}\) | 1.524 |
| \(w_{03}\) | -1.197 \times 10^{-5} |
| \(w_{11}\) | -0.0111 |
| \(w_{13}\) | -1.628 \times 10^{-9} |
| \(w_{21}\) | 2.747 \times 10^{-4} |
| \(w_{23}\) | 1.237 \times 10^{-8} |
| \(w_{31}\) | -6.503 \times 10^{-7} |
| \(w_{33}\) | 1.327 \times 10^{-10} |
| \(w_{41}\) | 7.987 \times 10^{-10} |
| \(w_{34}\) | -4.614 \times 10^{-13} |
B.2.3 Elastic properties of gas

The bulk modulus of gas is dependent on the gas composition (which is usually represented by gas specific gravity, $\gamma_G$) and can be formulated using the inverse of the coefficient of the adiabatic compressibility. This is given (in MPa) by

$$K_g = \frac{P\chi}{1 - \left(\frac{P}{Z} f\right)}$$ \hspace{1cm} (B.15)

where,

$$\chi = 0.85 + \frac{5.6}{P_r + 2} + \frac{27.1}{(P_r + 3.5)^2} - 8.7e^{-0.65(P_r + 1)}$$ \hspace{1cm} (B.16)

$$P_r = \frac{P}{4.892 - (0.4048\gamma_G)}$$ \hspace{1cm} (B.17)

$$T_r = \frac{T_a}{94.72 + (170.75\gamma_G)}$$ \hspace{1cm} (B.18)

$$f = c d m + a$$ \hspace{1cm} (B.19)

$$z = (a P_r) + b + (c d)$$ \hspace{1cm} (B.20)

$$a = 0.03 + 0.00527(3.5 - T_r)^3$$ \hspace{1cm} (B.21)

$$b = 0.642T_r^2 - 0.007T_r^4 - 0.52$$ \hspace{1cm} (B.22)

$$c = 0.109(3.85 - T_r)^2$$ \hspace{1cm} (B.23)

$$d = e^{\left[0.45b + 0.56 - \frac{1}{T_r} + \frac{P_r^{1/2}}{T_r}\right]}$$ \hspace{1cm} (B.24)
Appendix B

\[ m = -1.2 \left( 0.45 + 8(0.56 - \frac{1}{T_r})^2 \right) \frac{P_r^{0.2}}{T_r} \]  \hspace{1cm} (B.25)

\[ \gamma_G = \frac{\rho_{r,\text{surface}}}{\rho_{r,sc}} \]  \hspace{1cm} (B.26)

in which, \( T_a \) is the absolute temperature in Kelvin, \( T_r \) and \( P_r \) are the dimensionless reduced temperature and pressure, \( z \) is the gas compressibility factor, \( \gamma_G \) is the gas specific gravity measured at standard condition and \( \chi \) is the ratio of the heat capacity at constant pressure to the heat capacity at constant volume. The approximation used in here is dependent upon the correlation used for determination of the gas compressibility factor and is valid as long as \( P_r \) and \( T_r \) are not both within 0.1 of unity (Batzle and Wang, 1992; Avseth et al., 2005; Mavko et al., 2009).

Figure B.2 shows the calculated bulk modulus of sound in different hydrocarbon gas system as a function of pressure, temperature and composition. The figure (Figure B.2) shows that the bulk modulus of the gas reduces by increasing of the reservoir temperature, by reducing the reservoir pressure or by reducing the specific gravity.

\[ \text{Figure B.2: The bulk modulus of gas as a function of temperature, pressure, and composition (Batzle and Wang, 1992).} \]

**B.3 Effective moduli of multi-mineral and/or multi-fluid media**

In the presence of multi-mineral rocks or multi-fluid saturations the effective elastic moduli of the media is implemented to fulfil the Gassmann’s equation requirements. The effective (or composite) medium theory relates the properties of the composite
medium to the properties, proportion, and arrangement (including shape) of the constituents (Berryman and Milton, 1991). However, in absence of the knowledge of structural or geometrical configuration of the constituent particles, the accurate effective modulus cannot be predicted and the general practice is to only define the two upper and lower bounds for the effective moduli. The Voigt “upper bound” (Voigt, 1928) and the Reuss “lower bound” (Reuss, 1929) averaging of the elastic moduli are shown in Figure B.3.

![Figure B.3: The schematic representation of the upper and lower bounds on the effective elastic bulk modulus (Mavko et al., 2009).](image)

The Voigt upper bound of the effective elastic modulus, $M_v$ of N phases is defined as follows

$$M_v = \sum_{i=1}^{N} f_i \times M_i$$  \hspace{1cm} (B.27)

in which, $f_i$ and $M_i$ are the volume fraction and the elastic modulus of the i-th medium respectively. This average sometimes is called isostain average (Oldenziel, 2003; Avseth et al., 2005; Mavko et al., 2009) and assumes that all constituents have the same strain. The Reuss lower bound of the effective elastic modulus, $M_R$, is

$$\frac{1}{M_R} = \sum_{i=1}^{N} \frac{f_i}{M_i}$$  \hspace{1cm} (B.28)
The Reuss average is called isostress average (Oldenziel, 2003; Avseth et al., 2005; Mavko et al., 2009) and assumes that all constituents have the same stress. If the constituents of the medium are all fluids, the Voigt and Reuss averages are still valid. However, volume faction is replaced by the fluid saturation of each phase. Sometimes the Voigt-Reuss-Hill average (Hill, 1963) is used as an estimation of the effective modulus of the media. This is simply the arithmetic average of the Voigt upper bound and the Reuss lower bound.

**B.4 Dry frame moduli**

The dry frame bulk and shear moduli are calculated by the equations set out by MacBeth (2004), for the clean sandstone, and MacBeth and Ribeiro (2007), for the shaly sandstones. The clay content usually softens the dry rock frame and results in reducing of the elastic moduli and the sound velocities (Tosaya and Nur, 1982; Klimentos, 1991; MacBeth and Ribeiro, 2007). The proposed equations are three-parameter formula, which describe the sensitivity of the elastic moduli for a (shaly-) sandstone rock frame under isotropic stress.

\[
K_{\text{dry}}(\sigma_{\text{eff}}) = \frac{k_{\infty} - \sigma_{\text{eff}}}{1 + E_{k} \times e^{\frac{\sigma_{\text{eff}}}{P_{k}}}} \tag{B.29}
\]

\[
\mu_{\text{dry}}(\sigma_{\text{eff}}) = \frac{\mu_{\infty} - \sigma_{\text{eff}}}{1 + E_{\mu} \times e^{\frac{\sigma_{\text{eff}}}{P_{\mu}}}} \tag{B.30}
\]

\[
S_{k} = \frac{E_{k}}{1 + E_{k}} \tag{B.31}
\]

\[
S_{\mu} = \frac{E_{\mu}}{1 + E_{\mu}} \tag{B.32}
\]

where, \(P_{k}\) and \(P_{\mu}\) are the characteristic pressure constants and determine the rollover point beyond which the rock frame attains its state of relative insensitivity. \(K_{\infty}\) and \(\mu_{\infty}\) are the background high pressure asymptotes (MacBeth, 2004). The \(S\)-factors \((S_{\mu} \text{ and } S_{k})\) are related to overall stiffness of pore soft structures and are used to determine the \(E_{k}\) and \(E_{\mu}\). Thus, the higher number of flat cracks will tend to increase these values.
MacBeth (2004) defined the typical values of these parameters for specific reservoir rocks based on the 179 laboratory measurements of core and outcrop samples with low to moderate porosity and a range of clay fractions and cementation.

Besides, the effective stress (or effective pressure), $\sigma_{\text{eff}}$ (or $P_{\text{eff}}$), used in the definition of the stress sensitivity coefficients is defined as

$$
\sigma_{\text{eff}} \text{ (or } P_{\text{eff}} \text{)} = \sigma_{ob} - \alpha P_{\text{pore}} 
$$

Where, $\alpha$ is the Biot coefficient (Biot and Willis, 1957), $\sigma_{\text{eff}}$ is the effective stress, $\sigma_{ob}$ is overburden stress and $P_{\text{pore}}$ is the pore pressure. The Biot coefficient describes the ration of pore volume change ($\Delta V_p$) to total bulk volume change ($\Delta V$) under dry or drained conditions (Mavko et al., 2009). This is closer to unity for the higher porosity facies. However, many other factors as degree of cementation and clay content may lower the value of the Biot coefficient.

### B.5 Effective density of saturated medium

The effective density is given by the effective-porosity weighted average of the fluid and the mineral density and is written as follows

$$
\rho = (1 - \varphi) \rho_m + \rho_f (1 - S_w) \varphi + \rho_s S_w \varphi 
$$

$$(B.34)$$

$$
\rho_m = \sum \rho_{m,i} \times f_i 
$$

$$(B.35)$$

in which, $\rho$ is the effective density of the saturated formation, $\rho_f$ is the fluid density, $\varphi$ is the effective porosity, $\rho_m$ is the effective density of a multi-mineral formation and the $f_i$ is the volume fraction of each mineral constituting the rock mineral.


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