Appendix E

Modelling Relationships between Geomorphic Parameters of Fluvial Meandering Channels

E.1 Introduction

The objective of this Appendix is to find relationships between different geomorphic parameters of fluvial meandering channels and to predict channel dimensions using the machine learning techniques presented in Chapter 4. These relationships can be used as prior information when modelling fluvial reservoirs (Chapters 6 and 7).

Rivers are natural systems that play very important role in human life and activity. Rivers provide vital resources: food, water; renewable energy and within their sediments it is possible to find economical mineral deposits. Therefore, the study of river geometry has been an important topic of research for many scientists.

The characterization of fluvial channel geometry has a significant impact on the interpretation of the sandstone bodies of fluvial origin, preserved in the
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geological record (Leeder, 1973; Miall, 1992; 1996; Bridge, 2003), since it leads to direct implication on the estimation of hydrocarbon reserves.

Most of the work published on characterization and prediction of river geometry has been based on physical or process based models and statistical models, showing physical relations and deterministic empirical equations among the geomorphic parameters of fluvial channels. Rhoads (1992) states that mathematical or process based models are too rigorous to be used in a general approach and that statistical models could be misled by sampling or measurement errors.

The next section of this Appendix, describes the relationships among channel geomorphic variables proposed by previous authors, using empirical, deterministic equations, under bivariate and multivariate perspectives. These previous published relationships are compared to multidimensional maps built using the Machine Learning Techniques described in Chapter 4 (Multilayer Perceptron and Support Vector Regression (SVR)). In section E.3 there is an example on the application of Support Vector Machine (SVM) to the classification problem of channels based on geometry. Section E.4 explains the application of SVR for describing the geometry of meandering channel deposits preserved in the geological record.

E.2 Meandering Fluvial Channel Geometry

Fluvial channel morphology is a product of complex interaction of erosion and deposition, which depends on the grain size of the sediments transported by the current, the discharge of water that runs through the channel, the slope and the composition of the materials that surround the channels. Fluvial channels have been classified according to their morphology. One of the most known attempts to classify channels was proposed by Leopold and Wolman (1957) who classified channel patterns based on their planview, with three basic classes: meandering channels, braided channels and straight channels. More recently Rosgen (1994) developed a very detailed classification of natural rivers based
on their morphology including not only geometrical parameters but also hydrological features that affect the river morphology.

Describing the behaviour of meandering channels, Ferguson (1975) points out that a sideway twisting course is often taken by a laterally restricted flow of fluid of a sufficient size and speed when constrained by gravity against a deformable medium. The term meandering is used to name the morphology generated by these movements of the streams. Meandering channels can be described as streams with sinuous trace whose length is normally equal or greater than 1.5 the down-valley distance, i.e. sinuosity >1.5 (sinuosity is considered the ratio of channel centreline length to the meander-belt axial length).

Allen (1982) recognized three types of meandering channels: (1) channel lying in bed-rock or ice, associated with little or no alluvium; these are called incised or non-alluvial channels (2) channel is in contact everywhere with alluvium; these channels have been known as free-meanders (Carlston, 1965); (3) intermediate type named confined meandering channels where the stream is partly determined by older rocks and partly by itself. Confined meanders have their lateral growth restricted by relatively resistant terrace deposits or bed rock. The second type of meandering channels that have contact with alluvium everywhere or the so called alluvial meanders, are the ones considered in this chapter, because the reservoir used in this thesis (Chapters 6 and 7) were deposited in such environment.

Free-meandering rivers are developed in alluvial plains with low slope where the sediments are characterized by a high ratio of mud/sand. These rivers form sinuous channels (see Figure E.1). This type of channels are characterised by deposition of sand inside the meanders. This is due to the lateral migration of the channel, these deposits are known as point bars, which can be composed by sandy or in some cases gravely sediments (Allen, 1982).
Figure E.1 illustrates an idealized meandering channel, where we can observe the geomorphic parameters that have been used to describe this type of channels: Channel Width ($w$), Channel Depth ($D$), Meander Length ($L$), Meander Belt Width ($MBW$), Curvature Radius ($R_c$) and Sinuosity ($P$). The channel centreline has a series of inflection which may be joined by a smooth curve, the meander-belt axis. Meander belt lies between tangents to the outsides of the curves or meanders of the active stream (see Figure E.1). The meander-belt margins are the tangent lines themselves, the meander belt width ($MBW$) being the distance between opposite margins. Leopold and Wolman (1960) and Carlston (1965) used meander amplitude ($A$), which is considered as the distance between opposite points of the channel centreline. A meander comprises two consecutive meander loops; the channel segment lying between every second inflection point comprises one meander. Each loop embraces a body of sediment called a point bar (Carlston, 1965). The meander length ($L$) is the distance measured along the meander belt axis between one inflexion point and the next but one downstream (Carlston, 1965; Zeller, 1967). Meander-loop radius ($R_c$) partially describes a meander loop shape; it is measured at the
channel centreline. Other useful parameter used to describe meander geometry is the sinuosity \((P)\) which is considered the ratio of channel length (centreline) to the meander-belt axial length.

**E.2.1 Models to describe meandering channel geometry**

**Bivariate analysis of meandering channels**

Many authors have found that the geometry of rivers depends fairly conservatively on discharge \((Q)\), thus most of the bivariate analyses of channel geometry have been based on finding relationships between channel geometrical parameters and discharge. Carlston (1965) described discharge as the volume of water and sediments that pass through a channel section in a period of time; in our case we are going to measure discharge in cubic meters per second \((\text{m}^3/\text{sec})\).

As highlighted by Allen (1982) and Rhoads (1992) among others the dependence of width \(w\), depth \(D\), and flow velocity \(U\) upon discharge \(Q\) is known for many rivers in terms of conventional power functions (linear in logarithms):

\[
w = aQ^m \quad (eq.5.1) \\
D = bQ^n \quad (eq.5.2) \\
U = cQ^p \quad (eq.5.3)
\]

By continuity, \(Q = wDU\), whence the intercept coefficients \((a,b,c)\) and the exponents \((m,n,p)\) conform to (Leopold and Maddock, 1953):

\[abc = 1 \text{ and } m + n + p = 1.\]

Some of the authors that have previously worked on finding relationships among the geomorphic parameters of river channels have determined that apart from discharge \((Q)\), river channel patterns and dimensions are related to other variables such as, sediment load, slope \((S)\) and drainage basin area \((Dury, 1967; Leopold and Wolman, 1957; Schumm, 1963; Langbein and Leopold,
1966; Schumm, 1967). Table E.1 shows some of the empirical equations proposed to describe the relationships among geomorphic variables.

Figure E.2 shows two plots made by Carlston (1965) and Chitale (1970) finding relationships among the geomorphic parameters of meandering rivers, in Figure 5.2 clouds of data points are described with linear equations.

Figure E.2. Bivariate plots of geomorphic variables: (a) Mean Annual Discharge $Q_{ma}$ vs Meander Length $L$; (b) Mean Annual Discharge $Q_{ma}$ vs Channel width $w$, these two plots use data from Chitale (1970) and Carlston (1965) with their associated power equations; (c) represents Meander Belt Width $MBW$ vs Meander Length $L$ with 838 data points, highlighted in red we can appreciate a region of possible outputs instead of a single value given by an empirical equation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>$eq$</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L=65.2Q^{0.5}$</td>
<td>E.4</td>
<td>Leopold and Wolman (1957)</td>
</tr>
<tr>
<td>$L=7.32w^{1.1}$</td>
<td>E.5</td>
<td>Leopold and Wolman (1957)</td>
</tr>
</tbody>
</table>
Table E.1: Bivariate empirical equations, predicting the geometry of fluvial channels. $L$: meander length; $Q$: annual mean discharge; $w$: bankfull width; $A$: meander amplitude; $R_c$: radius of curvature; $MBW$: meander belt width; $P$: sinuosity; $LR/LV$: tortuosity; $D$: bankfull depth; $D_{50}$: mean grain size of the bed load; $M$: content of mud and clay on the surrounding area of a channel and $S^*$: slope in feet per 10,000 ft length of channel.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L=10.9w^{1.1}$</td>
<td>E.6 Leopold and Wolman (1960)</td>
</tr>
<tr>
<td>$A=2.7w^{1.1}$</td>
<td>E.7 Leopold and Wolman (1960)</td>
</tr>
<tr>
<td>$L=4.7R_c^{0.98}$</td>
<td>E.8 Leopold and Wolman (1960)</td>
</tr>
<tr>
<td>$L=Q^{0.46}$</td>
<td>E.9 Carlston (1965)</td>
</tr>
<tr>
<td>$MBW=65.8Q^{0.47}$</td>
<td>E.10 Carlston (1965)</td>
</tr>
<tr>
<td>$w=7Q^{0.46}$</td>
<td>E.11 Carlston (1965)</td>
</tr>
<tr>
<td>$P=3.5(w/D)^{-0.22}$</td>
<td>E.12 Schumm (1963)</td>
</tr>
<tr>
<td>$P=0.94M^{0.25}$</td>
<td>E.13 Schumm (1963)</td>
</tr>
<tr>
<td>$LR/LV=1.429(D_{50}/D)^{-0.077}$</td>
<td>E.14 Chitale (1970)</td>
</tr>
<tr>
<td>$S^*=0.052(w/D)^{-0.085}$</td>
<td>E.15 Chitale (1970)</td>
</tr>
<tr>
<td>$MBW/w=48.299(w/D)^{-0.471}$</td>
<td>E.16 Chitale (1970)</td>
</tr>
<tr>
<td>$S^*=0.453(D_{50}/D)^{-0.110}$</td>
<td>E.17 Chitale (1970)</td>
</tr>
<tr>
<td>$w=6.8D^{1.34}$</td>
<td>E.18 Leeder (1973)</td>
</tr>
<tr>
<td>$L=1.63MBW$ for $MBW$ between 3.7 and 13700 m</td>
<td>E.19 Williams (1986)</td>
</tr>
<tr>
<td>$A=0.054L^{1.53}$ for $L$ between 10 and 23200 m</td>
<td>E.20 Williams (1986)</td>
</tr>
<tr>
<td>$W=0.27MBW^{0.52}$ for $MBW$ between 3 and 13700 m</td>
<td>E.21 Williams (1986)</td>
</tr>
</tbody>
</table>

As previously shown, many authors have analysed the relationships among many channel geomorphic variables in order to predict and interpret the behaviour of a fluvial channel due to changes in hydrological variables, but all of these empirical equations are fitted in order to describe that clouds of points, all of these equations present ranges of errors (Leopold and Wolman, 1957, 1960; Carlston, 1965; Chitale, 1970; Williams, 1986) because the equations do not consider the whole scatter “cloud” of points. All these data points should be considered in the analysis of parameters relationships because all of them came from measurements of actual rivers.
Multivariate analysis of meandering channels

In the previous discussion about bivariate analysis of channel geomorphic variables it is fairly clear that most of these variables are related and some works like Chitale (1970) and Schumm (1963) show the use of variables like width-depth ratio in order to find relationships among three or four variables. In Table E.2 we can see some of the equations proposed to describe multivariate relationships of geomorphic variables.

Williams (1986) introduced some equations that correlate width, depth and sinuosity (equations 19 and 20, Table e.2). Introducing more variables implies that the correlation coefficient obtained from these equations is lower than the correlation coefficient of bivariate equations (Rhoads, 1992). *i.e.* curse of dimensionality (Section 4.4.2).

<table>
<thead>
<tr>
<th>Equation</th>
<th>eq</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w = 9D^{1.23}P^{2.3} )</td>
<td>E.22</td>
<td>Williams (1986)</td>
</tr>
<tr>
<td>( D = 0.09w^{0.55}P^{-1.46} )</td>
<td>E.23</td>
<td>Williams (1986)</td>
</tr>
<tr>
<td>( \ln(w_D) = \alpha_1 + \gamma_{11} \ln(D_D) + \gamma_{13} \ln(S) + \beta_{11} \ln(Q_{v2D}) + \beta_{12} \ln(Q_v) + \beta_{13} S_b + \epsilon_1 )</td>
<td>E.24</td>
<td>Rhoads (1992)</td>
</tr>
<tr>
<td>( \ln(D_D) = \alpha_{21} + \gamma_{21} \ln(w_D) + \gamma_{23} \ln(S) + \beta_{21} \ln(Q_{v2D}) + \beta_{22} \ln(Q_v) + \beta_{24} S_c + \epsilon_2 )</td>
<td>E.25</td>
<td>Rhoads (1992)</td>
</tr>
<tr>
<td>( \ln(S) = \alpha_3 + \gamma_{31} \ln(w_D) + \gamma_{32} \ln(D_D) + \beta_{31} \ln(Q_{v2D}) + \beta_{32} \ln(Q_v) + \beta_{33} S_v + \epsilon_3 )</td>
<td>E.26</td>
<td>Rhoads (1992)</td>
</tr>
</tbody>
</table>

Table E.2: Multivariate equations, proposed by William (1986) and Rhoads (1992) to predict the geometry of fluvial channels. Symbols: \( w \) is bankfull width; \( D \) is bankfull depth; \( P \) is sinuosity; \( D_D \) is dimensionless depth; \( W_D \) is dimensionless width, \( S \) is slope; \( Q_v \) is the ratio of the 5 year flood to the mean annual discharge; \( Q_{vD} \) is the dimensionless mean annual discharge and \( S_v \) is the valley slope.

In an attempt to characterize the changes in geometry of fluvial meandering channels Howard and Hemberger (1991) generated a suite of 40 geomorphic variables measured in 57 sections of freely meandering channels from 33 rivers. They examined these variables by two multivariate statistical techniques, factor analysis and discriminant analysis to determine the principal modes by
which meandering channels differ from another, and how meander geometry varies with time.

Some of the equations proposed by Rhoads (1992) can be seen in Table E.2 (equations 21, 22 and 23), one of the problems of using these equations is that it is necessary to create dimensionless variables (e.g. dimensionless width $W_d = w/D_{50}$). Rhoads (1992) highlights that a potential problem using dimensionless variables is the occurrence of spurious correlation.

Channel geomorphic parameters using MLT

The use of artificial neural networks in hydrology is gaining popularity, Govindaraju (2000a, 2000b) has expressed that the application of these machine learning technique has given a new insight to old problems in hydrology, we can find in these two Govindaraju papers application of artificial neural networks on ground water management, water quality and precipitation predictions made through artificial neural networks.

Bhattacharya and Solomatine (2000) found that using artificial neural networks in the prediction of river discharge based on river stages, reduces the difference between the predicted and the actual discharge by 10% compared to the conventional methods of predicting river discharge. Finding relationships of channel geomorphic parameters is a multivariate and non-linear problem. The advantages of using machine learning techniques as data driven modelling tools, are shown in this Appendix.

As mentioned before, previous work related to the prediction of channel geometry or finding relationships among geomorphic parameters of fluvial channels, base their results on empirical and deterministic equations that can give one single output. We can use a compilation of these equations like Table E.1 or the table presented by Bridge (2003) to establish ranges of possible combination of parameters, as was proposed by Arnold (2008), but as it is observed in Figure E.2 (c) we are still missing realistic information acquired from nature.
Data Description

In this thesis, 838 data points of channel geometry and hydrological conditions were collected from published work by other authors. Most of the data used here came from the database compiled by Crane (1983), adding some other datapoints (Leeder, 1973, Williams, 1986, Chitale, 1970 and van den Berg, 1995). The geomorphic variables used in this chapter are presented in Table E.3. A table with the entire channel data points used in this thesis was presented in Appendix A.

The logarithm of these parameters values were used to train our machine learning algorithms, since it is easy to observe how the logarithm of these parameters are directly related to each other, as observed in Figure E.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Mean Discharge</td>
<td>$Q_{ma}$</td>
<td>(m$^3$/sec)</td>
</tr>
<tr>
<td>Meander Belt Width</td>
<td>$MBW$</td>
<td>(m)</td>
</tr>
<tr>
<td>Bankfull Channel Width</td>
<td>$W$</td>
<td>(m)</td>
</tr>
<tr>
<td>Bankfull Channel Depth</td>
<td>$D$</td>
<td>(m)</td>
</tr>
<tr>
<td>Steam slope</td>
<td>$S$</td>
<td></td>
</tr>
<tr>
<td>Meander Amplitude</td>
<td>$A$</td>
<td>(m)</td>
</tr>
<tr>
<td>Meander Length</td>
<td>$L$</td>
<td>(m)</td>
</tr>
<tr>
<td>Sinuosity</td>
<td>$P$</td>
<td></td>
</tr>
</tbody>
</table>

Table E.3 Geomorphic variables used to model their interrelationships.

Two popular algorithms in machine learning were used and compared the results obtained with both algorithms, we start using artificial neural network, specifically Multilayer Perceptron (MLP), and Support Vector regression (SVR).

E.2.2 Validity Domain

As it is not realistic to make prediction outside than the space surrounding data points, it is necessary to obtain a region or validity domain, where the relations found can be considered as realistic or valid.
In this thesis the validity domain is delineated by tuning the width of the kernel function of the General Regression Neural Network, which is part of the nonparametric kernel regression models (Fan and Gijbels 1996; Kanevski, 1999). The sum of the kernel widths (Section 4.6.2) selected for each data point, after tuning, is plotted together in a grid and the limit of the validity domain can be selected, based on the closeness to the datapoints. The sum of the kernel widths in a plot of Discharge $Q$ vs channel width $w$ is shown in Figure E.3.

Figure E.3: Estimation of Validity Domain (Uncertainty) using the Kernel width Sum from General Regression Neural Networks, the cut-off value of kernel sum to select our validity domain was 2.00 (limit between blue and dark blue colours).

Kanevski (1999) used this technique to define a validity domain for soil contamination maps. Actually, this function represents density of sampling points in space. Outside the interpolation region this function is negligible. From the results obtained (Figure 5.3) the kernel sum value of 2.0 was used as the limit of the validity domain.
E.2.3 Multilayer Perceptron (MLP) Model

Channel data from Carlston (1965) and Chitale (1970) were combined to work with MLP. These sets of data were described by different empirical equations, as shown in Table E.1 and Figure E.2. The variables considered were: channel width \((w)\), mean annual discharge \((Q_{ma})\) and meander length \((L)\) in order to find relationships in three-dimensional space.

**Tuning MLP**

The multilayer perceptron with lowest training and testing errors found, uses 2 hidden layers with a combination of 5 neurons in each hidden layer, 2 input variables (mean annual discharge and meander belt width) and one output (meander length). The network was tuned by analysing the training and testing errors (Section 4.5) when changing the combination of neurons and hidden layers (Kanevski et al., 2009). Increasing the number of neurons and the number of hidden layers makes the network more capable of finding relations among complex data, but making very complex combination of neurons will increase training and testing errors. The optimal network will have the lowest training and testing errors, Figure E.4 shows the error analysis that allows us to select the optimal network for this case.

**Results**

Figure E.5 is a representation of the map obtained when plotting the three variables mean annual discharge \((Q_{ma})\), channel width \((w)\) and meander length \((L)\), we can see how these three parameters are directly related in a three-dimensional space.
Figure E.4: Error (eq. 4.2) analysis for tuning MLP (Inp=number of inputs; HL\textsubscript{1} and HL\textsubscript{2} number of neurons in the first and second hidden layers; and Out= number of outputs) the optimal network is composed by 2 inputs 2 hidden layers of 5 neurons each and 1 output (meander length). The lowest training error highlighted in orange do not correspond with the lowest testing error.

The equations, comparing channel width (w) vs Discharge (Q\textsubscript{ma}), obtained by Chitale (1970) and Carlston (1965) were plotted in the same map. These equations are not powerful enough to describe the relationships and their uncertainties among these three parameters, when compared with the map obtained using MLP. It is important to highlight that although the optimal structure of neurons was chosen and the limit of the extrapolation was set using a technique that considers data distribution, like general regression neural networks, some unrealistic artefacts can be created by MLP (red ellipse in Figure E.5). This map is generated within the validity domain obtained in section E.2.2, which represents the uncertainty in the value of channel width (w) or output, given an input (Discharge, Q\textsubscript{ma}).
Figure E.5: 3-D map relating Mean annual discharge ($Q_{ma}$), width ($w$) and meander length ($L$), using multilayer perceptron, the regression proposed by Chitale (1970) and Carlston (1965) to characterize their data are plotted here. The red ellipse highlights an unrealistic artefact produced by MLP. The linear regression from Chitale (1970) in black and Carlston (1965) in red do not consider the meander Length ($L$), they only consider Discharge ($Q_{ma}$) and channel width ($w$). Using this two regression lines do not allow space for uncertainty and do not consider possible combination of $Q_{ma}$ and $w$ observed in nature.

E.2.4 Support Vector Regression (SVR) Model

Figure E.6 is the surface generated after tuning the SVR to obtain the optimal combination of hyper-parameters, which is explained later in this section. Figure E.6 illustrates how the three variables (channel width, annual mean discharge and meander length) are directly related. The complexity of the map structure, tells that there is a non-linear relationship between these parameters. The data used for SVR was the same used for the MLP model, within the same validity domain.

Tuning SVR

Kanevski et al. (2009) highlight that choosing the hyper-parameters is the most important step in SVR. The SVR hyper-parameters to tune will be: the type of
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kernel, kernel bandwidth $\sigma$, the trade-off parameter $C$ and $\varepsilon$ (equations 4.41 and 4.42 respectively).

Figure E.6: 3-D map relating Mean annual discharge ($Q_{ma}$), width ($w$) and meander length ($L$), using support vector regression. Optimal model after tuning the hyperparameters $\sigma=0.8$; $\varepsilon=0.05$; $C=15$ (using Gaussian Kernel).

The general approaches to select the hyper-parameters are: trial-error, cross-validation of the hyper-parameters and the training and testing error cubes (eq. 4.2) (Kanevski et al., 2009) and Bayesian inference (Sollich, 2002; Demyanov et al., 2008). In this thesis the hyper-parameters were tuned by cross-validation (Kanevski et al., 2009) after analysing the cubes generated using the training and testing errors vs the tuning parameters ($\sigma$, $\varepsilon$ and $C$). A grid search on the hyper-parameters between values: $\sigma=[0.1 - 5]$, $\varepsilon=[0.01 - 1]$ and $C=[1 – 50]$, was used to generate training and testing error cubes. The combination of hyper-parameters with the lowest training and testing errors was selected.

Comparing MLP vs SVR Results
Comparing the surfaces obtained using the MLP (Figure E.5) and the one obtained using the SVR (Figure E.6) we can observe the same trend (meander length $L$ increasing with channel width $w$) in both surfaces but there are no unrealistic artefacts in the models generated with SVR, unlike the results obtained using MLP. The results observed using SVR are not strongly biased by internal trends (extrapolation of results following the trend of close data values, giving low weight to more distanced data points) e.g. red ellipse in Figure E.5, which generates this kind of artefacts.

**Higher dimensional space**

The 838 data points were used to build a more complicated combination of the related variables like the mean annual discharge ($Q_{ma}$), channel width ($w$), meander length ($L$) and channel depth ($D$), and build a four dimensional space where we can observe the relationships between these variables. These relationships can be appreciated in Figure E.7 (a, b, c, d), where four 3-dimensional surfaces illustrate the relationships among these four variables.

Not all the data points (out of the 838) have all the measurements available. In this section semi-supervised SVR (Chapter 4) with unlabelled data was used, in order generate these four 3-dimensional surfaces.

When comparing Figure E.6 and Figure E.7 we can observe the same general trend where all the variables are directly related, but the structure of the surfaces is more complex, when the number of data points increases. This shows a more realistic representation of the relationships between the geomorphic parameters. Red ellipses were drawn to highlight areas, where the third dimension is not following the general trend, showing that under certain conditions the relations among the geomorphic parameters could change e.g. in Figure E.7 (b) the general trend shows that increasing channel width ($w$) and channel depth ($D$) results in increasing meander length ($L$), but if we look at the upper right corner we can see that increasing $w$ and $D$ results in a reduction of $L$. These features can be observed in all these four maps and were picked easily by using SVR.
Figure E.7 (a) Map of the relationships among mean annual discharge ($Q_{ma}$), Channel depth ($D$) and meander length ($L$). The regression 2D line $Q_{ma}$ vs $D$ obtained from van den Berg (1995) data is plotted here.

Figure E.7 (b) Map of the relationships among mean annual discharge ($Q_{ma}$), Channel width ($w$) and channel depth ($D$). The regression 2D lines $Q_{ma}$ vs $w$, proposed by Chitale (1970) and Carlston (1965) are plotted here.
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Figure E.7 (c) Map of the relationships among mean annual discharge ($Q_{ma}$), channel width ($w$) and meander length ($L$). The regression 2D regression lines $Q_{ma}$ vs $w$, proposed by Chitale (1970) and Carlston (1965) are plotted here.

Figure E.7 (d) Map of the relationships among channel width ($w$) channel depth ($D$) and meander length ($L$), notice that the data from the bottom left corner came from flume experiments. The 2D regression lines $w$ vs $D$, obtained from Williams (1986) and Leeder (1973) data are plotted here.

Figure E.7: SVR Four 3-D surfaces relating mean annual discharge ($Q_{ma}$), channel width ($w$), channel depth ($D$), and meander length ($L$).
E.3 Fluvial Channel Classification

Rosgen (1994) states that the idea behind establishing a categorization of river systems through channel geometry is to:

- Predict river behaviour
- Develop hydraulic and sediment relations for a given morphological channel type.
- Provide mechanism to extrapolate fluvial channel information among similar fluvial systems
- Generate a consistent and reproducible frame of reference of communication for those working with river systems.

In this Section a comparison between a classification system for river channels based on a parameter developed by van den Berg (1995) and the use of SVM as an alternative method to discriminate between braided, meandering and straight channels, is presented.

E.3.1 Fluvial Channel Classification Systems

Classification of channels has been developed for many authors since 1899 when Davis, first divided the streams into young, mature and old stage, based on the relative stage of adjustment. According to channel morphology; one of the most known attempts to classify channels was proposed by Leopold and Wolman (1957). They classified channel patterns based on their planview with three basic classes: meandering channels, braided channels and straight channels. Many other authors have developed their own classifications including features like erosion, deposition rates (Schumm, 1963). Vegetation, sinuosity, braiding patterns, levee formations and meander scrolls (Culbertson et al., 1967). Smith and Smith (1980) treat anastomosed channels as a different type of channel systems separating them from the braided channels realm. More recently Rosgen (1994) developed a very detailed classification of natural rivers based on their morphology including not only geometrical parameters but also hydrological features that affect the river morphology.
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In this section the classification system proposed by van den Berg (1995) is used to illustrate the application of Support Vector Machines in classifying channels and identifying stability fields of channel geometry.

Van den Berg (1995) based his work on 228 data points obtained from several rivers around the world. He proposed a method that enables to predict the equilibrium condition for the occurrence of braided and high sinuosity rivers in unconfined alluvial plains. Van den Berg (1995) identified an independent discriminant function that separates the occurrence of braided and meandering rivers. This function was generated from two channel parameters: mean grain size of the bed material and a potential *specific stream power* parameter. The specific stream power is estimated as:

\[ W = \tau_0 u = \rho g Q_{bf} S_c / w \]  
(eq. E.27)

Where \( \tau_0 \) is the bed shear stress, \( u \) is the depth averaged flow velocity, \( \rho \) is the density of the water, \( g \) is the acceleration due to gravity, \( Q_{bf} \) is the bankful discharge, \( S_c \) is the channel slope and \( w \) is the channel width.

Figure E.8 shows a plot of the Mean Annual Discharge (\( Q \)) and the Mean grain size (\( D_{50} \)) in millimetres of the river bed material. This Figure shows certain separation of stability fields between Braided Meandering (Sinuosity > 1.5) and Straight Channels (Sinuosity< 1.5)

Figure E.9 is a plot of the Mean grain size vs. the *specific stream power* (\( W_v \)) measured in W/m\(^2\) obtained by van den Berg (1995). Figure E.9 presents as well the main results of the van den Berg (1995) work, which is the discriminant equation that separates the fields of meandering and braided channels.
Appendix E

Figure E.8: Plot of Mean Grain Size ($D_{50}$) in millimetres vs Mean Annual Discharge ($Q$) in m$^3$/s. This plot suggests a separation of stability fields between braided, meandering and straight channels. From van den Berg (1995).

$$Wv = 2.1 Sv \sqrt{Q_{bf}} \quad (\text{kW/m}^2)$$

(eq. E.28)

Figure E.9: Plot of Mean Grain Size ($D_{50}$) in millimetres vs the Specific Stream Power ($Wv$) in W/m$^2$. And the discriminant equation (eq E.28). This plot present a separation of stability fields between braided and meandering channels (using Schumm, 1985 classification). From van den Berg (1995).
It is important to highlight that eq. E.28 is only for sand-bed rivers. In van den Berg (1995) paper there is another discriminant equation for gravel-bed rivers.

### E.3.2 Support Vector Machine (Classification) Model

Figure E.10 shows a map of stability fields of channel geometry separating braided, meandering and straight channels with fuzzy limits. This plot was built using SVM classification (Chapter 4) with three classes and the van den Berg (1995) data presented Figure E.8. This is a 3-D classification problem where the variables are Discharge, mean grain size and the type of channels.

![Figure E.10: SVM classification map showing the stability fields for braided, meandering and straight channels, based on the data used by van den Berg (1995) and showed in Figure E.9.](image-url)
Parameters of SVM were tuned using the methodology presented in Chapter 4 for SVM classification ($\sigma=20$ and $C=100$). Just with the plot in Figure E.10 it is possible to recognize the stability fields for each group of channels as well as the transition zones between these types of channels.

Figure E.11 is a SVM classification map that shows the stability field for braided, meandering and straight channels, using the specific stream power parameter proposed by van den Berg (1995) and the Mean grain size of the river bed ($D_{50}$).

![SVM classification map](image)

Figure E.11: SVM classification map showing the stability fields for braided, meandering and straight channels, based on the data used by van den Berg (1995) and showed in Figure E.9. Dashed line is the representation of equation E.28 as shown in Figure E.9.

Figure E.11 is comparable with Figure E.9 which is the graph presented by van den Berg (1995) to discriminate between stability fields of braided and meandering channels. In Figure E.11 it is possible to observe the difference between braided and meandering, but also it is possible to detect a stability field for straight channels, which was not observed in the work presented by van den
Berg (1995). Implying that SVM is able to detect stability fields not observed by the methodology presented by van den Berg (1995).

E.4 Meandering Belts Preserved in the Geological Record

As mentioned in most of the chapters of this thesis, one of them most important tasks in modelling reservoirs developed in fluvial settings is determining the geometry of the sandbodies. In this section, an example on how to use geological prior information related to the geometry of fluvial deposits in reservoir modelling is presented.

E.4.1 Support Vector Regression (SVR) Model

Now that information about meandering channel geometry has been collected from different sources (Appendix A) and that SVR have been used to estimate the geometry of channel (Section E.2). It is possible to use all these data to generate SVR maps to identify realistic combination of geomorphic parameters (channel thickness and width and meander wavelength and amplitude). The realistic combination of parameters can be used to generate single meandering channel belts within geological models.

Four 3-dimensional maps were generated using SVR, in order to find a relationship among the four geomorphic parameters mentioned before, using the Appendix A data. Figure E.12 illustrates the 3D representation of these maps.

Figure E.12 A is an example where all the variables in general are directly related, with some trends, like that after reaching a maximum of the combination of three variables two of the variables could increase and the third one reduces. For example Figure E.12 (A) where channel thickness, channel width and meander amplitude continuously increase until meander amplitude reaches around 4000 m, after this point channel thickness and width continue to increase but meander amplitude begins to reduce.
Figure E.12: 3-D map showing the relationship between the geomorphic variables that controls the geometry of meandering channels. (A) Thickness-Width-Amplitude; (B) Thickness-Width-Wavelength; (C) Thickness-Wavelength-Amplitude; (D) Width-Wavelength-Amplitude. Note that the colour scale is related to the third variable in each plot.

**Geomodelling Application**

These 3-D surfaces that have been built using SVR can be used in the geomodelling process of meandering channels in order to generate meandering channels whose combination of parameters have been observed in nature, assuring realistic facies geometry.

Rojas *et al.* (2011) used these 3-D surfaces to generate geomodels of sinuous channels. They compared the production response and oil in place (STOIIP) estimation of models generated with unrealistic and realistic combination of channel parameters and the truth case. It was demonstrated that in some cases
an unrealistic combination of parameters could lead to a better history match of the production data, which could be considered as a problem in the future development of a reservoir, since the geometry of the reservoir is not realistic. This was mainly due to the small number of models evaluated (30 models) and to the connectivity of the geobodies presented by the unrealistic models (B) was better than the connectivity presented by the realistic models.

Figure E.13 shows some of the results obtained from Rojas et al. (2011) where it is possible to observe the different models and their production response as well as the STOIIP estimated for each model.

E.5 Summary

In this Appendix it is shown how to model multidimensional relationships between geomorphic variables for fluvial channels, using machine learning techniques. Our results are compared to the results obtained by bivariate or multivariate statistical models. Our results predict fluvial channel geometries that are within the field of possible realistic forms. The MLT models can be used to predict possible changes in the geometry of the fluvial channels if for any reason any of the geomorphic variables changes.

The models generated here using SVR can play an important role in the risk analysis of fluvial flooding or in the construction of channelized streams. These models can be used to detect when a channelized or natural stream could change its geometry due to changes in the hydrological parameters, e.g. flooding due to increase in flow discharge. These MLT models can be used in the prediction of sand deposits geometry preserved in the geological record, which has a great impact on volume estimation of mineral resources.

It is suggested using an SVR approach rather than MLP since SVR models determine more efficiently the trends observed in data. Some unrealistic artefacts were generated in MLP models; these could mislead an interpretation or prediction related to changes in channel geometry.
An SVR model for prediction and characterization of channel geometry was built based on a large number of datapoints (838). This model was constrained by 5 geomorphic variables. Obviously, it is possible to increase the number of variables like percentage of vegetation, grain size bedload, percentage of mud around the channels or any other variable that could control the geometry of fluvial channels.
Just like parametric regression models, machine learning models are not supported by a strong physical and theoretical interpretation of a specific problem (e.g. physical relationship between channel sinuosity and mean annual discharge). The results obtained are driven by the data. However, the results can be reviewed by the user who can interpret them based on understanding of the problem physics and make sure that the obtained results are within a realistic field.

Modelling geological prior information using intelligent techniques in a four-dimensional space of the observed characteristics of geological bodies improves realism in reservoir modelling.

Channel geometry has a great impact on the volume estimation and the reservoir production (Rojas et al., 2011), which are key factors in the reservoir management process. Simulation of realistic channel geometry will improve the reliability of the reservoir model predictions and will reduce the uncertainty in reservoir evaluation. However, it is necessary to consider other factors like number of models and facies connectivity (Chapters 5 and 6).