Appendix 4.3. Calibration

Initial values from the SAM, SNA and Pension Fund balance

\[ \text{IO}_{T0} \quad \text{Initial intermediate demand} \]

\[ \text{CZ}_{T0} \quad \text{Initial household demand} \]

\[ \text{CG}_{T0} \quad \text{Initial government demand} \]

\[ \text{INV}_{T0} \quad \text{Initial investment demand} \]

\[ \text{INVG}_{T0} \quad \text{Initial level of government investment} \]

\[ \text{SF}_{T0} \quad \text{Initial foreign savings} \]

\[ \text{KY}_{T0} \quad \text{Initial total capital income} \]

\[ \text{L}_{T0} \quad \text{Initial total labour income} \]

\[ \text{TRF}_{T0} \quad \text{Initial total government transfers to household} \]

\[ \text{TRP}_{T0} \quad \text{Initial indirect tax revenues} \]

\[ \text{TRY}_{T0} \quad \text{Initial income tax revenues} \]

\[ \text{E}_{T0} \quad \text{Initial level of export} \]

\[ \text{M}_{T0} \quad \text{Initial level of import} \]

\[ \text{TRGROW}_{T0} \quad \text{Initial foreign transfers to government} \]

\[ \text{TRHROW}_{T0} \quad \text{Initial foreign transfers to households} \]

\[ \text{PPC}_{T0} \quad \text{Initial workers pension contributions} \]

\[ \text{GPCZ}_{T0} \quad \text{Initial government pension contributions} \]
Parameters

Exogenous parameters

rss  Return to capital on the steady state  0.05
ro   Time preference               0.05
adjsh Share of adjustment costs out of investment 0.01

Demographic parameters

g  Steady state population growth rate
lg_T Labour force growth at time T
pg_T Pension age population growth at time T
popg_T Total population growth at time T
popgcum_T Cumulative growth of total population between T_0 and T
lgcum_T Cumulative growth of labour force between T_0 and T
pgcum_T Cumulative growth of pension age population between T_0 and T
tpop_T Total population size at time T
lpop_T Labour force size at time T
ppop_T Pension age population size at time T

Calibrated parameters

ty   Income tax rate
io   Leontief technical coefficients
αK   Capital value share in production function
\( \alpha_L \) Labour value share in the production function

\( a_F \) Efficiency parameter in the production function

\( \delta \) Depreciation rate

\( \phi \) Adjustment cost parameter

INVGshare Share of government investment in GDP

GPCshare Share of government pension contributions

**Calibration**

*Calculating initial level of output and capital and labour shares (assuming constant returns to scale)*

\[
XD_{t_0} = \frac{P_{t_0} * IO_{t_0} + PK_{t_0} * KY_{t_0} + PL_{t_0} * L_{t_0} + TRP_{t_0}}{PD_{t_0}}
\]

\[
\alpha_L = \frac{PL_{t_0} * L_{t_0}}{PK_{t_0} * KY_{t_0} + PL_{t_0} * L_{t_0}}
\]

\( \alpha_K = 1 - \alpha_L \)

*Cumulative demographic growth rates and size of population groups*

\[
l_{gcum_{T}} = \prod_{t_{0}}^{T} l_{T}
\]

\[
p_{gcum_{T}} = \prod_{t_{0}}^{T} p_{T}
\]

\[
.popgcum_{T} = \prod_{t_{0}}^{T} pop_{T}
\]

\[
l_{pop_{T}} = l_{pop_{T}} * l_{gcum_{T}}
\]
\[ p_{\text{pop}, t} = p_{\text{pop}, t_0} \times \text{pgcum}, t \]

\[ t_{\text{pop}, t} = t_{\text{pop}, t_0} \times \text{popgcum}, t \]

Labour supply equals to labour demand

\[ L_{S, t_0} = L_{T, t_0} \]

Adjusting the data for loss in output caused by the adjustment costs by subtracting the adjustment costs from private consumption

\[ C_{T_0} = C_{T_0} - \text{adjsh} \times \text{INV}_{T_0} \]

Calculating initial Pension Fund and related parameters

\[ \text{PF}_{T_0} = \text{PPC}_{T_0} + \text{GPC}_{T_0} \]

\[ \text{GPCshare} = \frac{\text{GPC}_{T_0}}{\text{PF}_{T_0}} \]

\[ \text{pc}_{T_0} = \frac{\text{PPC}_{T_0}}{L_{T_0} \times \text{PL}_{T_0}} \]

\[ \text{repr}_{T_0} = \frac{(\text{PF}_{T_0} / p_{\text{pop}, t_0})}{(L_{T_0} - \text{PPC}_{T_0}) / \text{lpop}_{T_0}} \]

Calculating savings and income

\[ \text{CE}_{T_0} = C_{T_0} \times \text{PD}_{T_0} \]

\[ \text{SH}_{T_0} = \text{INV}_{T_0} \times \text{PK}_{T_0} - \text{INVG}_{T_0} - \text{SF}_{T_0} \]
\[ S_{t_0} = SH_{t_0} + INV_{t_0} + SF_{t_0} \]

\[ Y_{t_0} = CE_{t_0} + SH_{t_0} + TRY_{t_0} + PPC_{t_0} \]

\[ NETLY_{TFIRST} = L_{TFIRST} \cdot PL_{TFIRST} - PF_{TFIRST} \]

*Calculating tax revenue and tax rates*

\[ TAXR_{t_0} = TRP_{t_0} + TRY_{t_0} \]

\[ tp = \frac{TRP_{t_0}}{XD_{t_0} \cdot PD_{t_0} - IO_{t_0}} \]

\[ ty = \frac{TRY_{t_0}}{Y_{t_0}} \]

*Calibrating technical coefficients*

\[ io = \frac{IO_{TFIRST}}{XD_{TFIRST} \cdot PD_{TFIRST}} \]

*Calibrating \( \lambda \) based on the assumption that the initial point is on the steady state path*

\[ \lambda_{TFIRST} = PK_{TFIRST} + 2 \cdot adjsh \cdot PK_{TFIRST} \]

*Calibrating initial capital stock based on the first order condition with respect to \( K \)*

\[ K_{t_0} = \frac{(LAMBDA_{t_0} \cdot adjsh \cdot PK_{t_0}) \cdot INV_{t_0} - \alpha K \cdot (XD_{t_0} \cdot PD_{t_0} - io \cdot XD_{t_0} \cdot PD_{t_0}) \cdot (1 - tp)}{(g \cdot rss) \cdot LAMBDA_{t_0}} \]

*Calibrating the adjustment parameter and the depreciation rate*
\[ \varphi = \text{adjsh} \times \frac{K_{\tau_0} \times PK_{\tau_0}}{INV_{\tau_0} \times PD_{\tau_0}} \]

\[ \delta = \frac{INV_{\tau_0}}{K_{\tau_0}} - g \]

**Calibrating the shift parameter in the output production function**

\[ aF = \frac{XD_{\tau_0}}{K_{\tau_0} \times L_{\tau_0} \times \text{adj}} \]

**Calculating dividends as a residual after value added tax, adjustment cost and labour income**

\[ \text{DIV}_{\tau_0} = (1 - tp) \times (PD_{\tau_0} \times XD_{\tau_0} - PD_{\tau_0} \times IO_{\tau_0}) - \varphi \times PD_{\tau_0} \times \frac{INV_{\tau_0}^2}{K_{\tau_0}} - PL_{\tau_0} \times L_{\tau_0} \]

**Calculating government revenues and spending**

\[ \text{GREV}_{\tau_0} = \text{TAXR}_{\tau_0} + \text{TRGROW}_{\tau_0} \]

\[ \text{TRF}_{\tau_0} = \text{TRF}_{\tau_0} - \text{PF}_{\tau_0} \]

\[ \text{GSPEND}_{\tau_0} = \text{CG}_{\tau_0} + \text{TRF}_{\tau_0} + \text{GPC}_{\tau_0} + \text{INVG}_{\tau_0} \]

**Calculating initial level of GDP**

\[ \text{GDP}_{\tau_0} = XD_{\tau_0} \times PD_{\tau_0} - \text{io} \times XD_{\tau_0} \times PD_{\tau_0} \]